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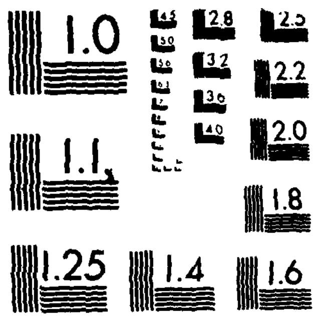

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NAVAL POSTGRADUATE SCHOOL
Monterey, California

AD-A141 940

THESIS

APPLICATION OF SENSITIVITY ANALYSIS
TO AERODYNAMIC PARAMETERS OF A
BANK-TO-TURN MISSILE

by

Tiago da Silva Ribeiro

December 1983

Thesis Advisor: Daniel J. Collins

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This thesis is an application of parameter sensitivity analysis to aerodynamic parameters of a Bank-to-Turn missile. In the deve'opment a brief review of trajectory sensitivity theory is presented. A linear analysis is performed using an Uncoupled Pitch Channel Autopilot and a Coupled Roll-Yaw Channel Autopilot of the missile taken as model. Finally, a nonlinear analysis is given for the system. Comparisons between the linear and nonlinear cases are outlined.

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Application of Sensitivity Analysis to Aerodynamic Parameters of a Bank-to-Turn Missile

by

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ABSTRACT

This thesis is an application of parameter sensitivity analysis to aerodynamic parameters of a Bank-to-Turn missile. In the development a brief review of trajectory sensitivity theory is presented. A linear analysis is performed using an Uncoupled Pitch Channel Autopilot and a Coupled Roll-Yaw Channel Autopilot of the missile taken as model. Finally, a nonlinear analysis is given for the system. Comparisons between the linear and nonlinear cases are outlined.

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TABLE OF SYMBOLS AND ABBREVIATIONS

BTT	Bank-to-Turn
CBTT	Coordinate Bank-to-Turn
C_ℓ	rolling moment coefficient
C_{ℓ_β}	slope of curve of rolling coefficient, C_ℓ vs β
$C_{\ell_{\delta R}}$	change in C_ℓ per degree roll control incidance, δ_R
$C_{\ell_{\delta Y}}$	change in C_ℓ per degree yaw control incidence, δ_Y
C_m	pitching moment coefficient
C_{m_α}	slope of curve of pitching moment coefficient C_m vs α
$C_{m_{\delta p}}$	change in C_m per degree pitch control incidance, δ_p
C_N	normal force coefficient
C_{N_α}	slope of curve normal force coefficient C_N vs α
$C_{N_{\delta p}}$	change in C_N per degree pitch control incident, δ_p
C_n	yawing moment coefficient
C_{n_β}	slope of curve of yawing moment coefficient, C_n vs β
$C_{n_{\delta Y}}$	change in C_n per degree yaw control incidence, δ_Y
$C_{n_{\delta R}}$	change in C_n per degree roll control incidence, δ_R
C_Y	side force coefficient
C_{Y_β}	slope of curve of side force coefficient C_Y vs β
$C_{Y_{\delta Y}}$	change in C_Y per degree yaw control incidence, δ_Y
$C_{Y_{\delta R}}$	change in C_Y per degree roll control incidence, δ_R
d	reference length for coefficients = 2ft
I_{YY}	moment of inertia about $\bar{Y}_B$ axis

I_{zz} moment of inertia about $\bar{z}_B$ axis

I_{xx} moment of inertia about $\bar{x}_B$ axis

KYP CBTT autopilot coordinator branch gain

p roll rate about $\bar{x}_B$

$\dot{p}$ roll acceleration about $\bar{x}_B$

P_e constant or equilibrium roll angular rate

$\bar{q}$ dynamic pressure

q pitch rate about y

$\dot{q}$ pitch angular acceleration about y

Q_e constant or equilibrium pitch angular rate

r yaw angular rate about $\bar{z}_B$

r_c yaw angular rate command (coordination command)

$\dot{r}$ yaw angular acceleration about $\bar{z}_B$

S reference area for coefficients $= \pi$ ft²

u velocity component in $\bar{x}_B$ direction

v velocity component in $\bar{y}_B$ direction, assumed to be constant

V constant missile flight path velocity

$\bar{V}$ missile velocity vector

w velocity component in z direction

$\bar{X}_B$ body-fixed roll axis, along axis of symmetry, positive forward

$\bar{Y}_B$ body-fixed pitch axis, positive starboard

$\bar{Y}_V$ vehicle axis in local horizontal direction, approximated as inertial axis

$\bar{Z}_B$ body-fixed yaw axis

$\bar{z}_v$ vehicle axis in downward direction along local gravity vector, approximated as inertial axis

η_Z achieved normal acceleration in $\bar{z}_B$ direction

η_{Z_c} commanded normal acceleration in $\bar{z}_B$ direction

η_Y achieved normal acceleration in $\bar{y}_B$ direction

η_Z achieved normal acceleration in $\bar{z}_V$ direction

η_Y achieved normal acceleration in $\bar{y}_V$ direction

η_c normal acceleration command from guidance computer in $\bar{z}_V$ direction plus anti-gravity bias command

η_{Z_c} normal acceleration guidance command in $\bar{z}_V$ direction

η_{Y_c} normal acceleration guidance command in $\bar{y}_V$ direction

ϕ_c roll attitude command from guidance computer, zero degrees in -z direction and 90 degrees in $\bar{y}_V$ direction

ϕ roll attitude, zero degrees in $-\bar{z}_V$ direction and 90 degrees in $\bar{y}_V$ direction

ϕ_e roll attitude error, $\phi_c - \phi$

θ Elevation Euler Angler, second rotation, $\int(q\cos\phi - r\sin\phi)dt$

ψ Azimuth Euler Angle, first rotation about $\bar{y}_V$, $\int(q\sin\phi + r\cos\phi)dt$

δ_P pitch control incidence (positive tail incidence produces negative pitiching moment)

δ_{P_c} commanded pitch control incidence, δ_P

δ_Y yaw control incidence (positive tail incidence produces negative yawing moments)

δ_{Y_c} commanded yaw control incidence, δ_Y

δ_R roll control incidence (positive tail incidence produces

positive rolling moment)

δ_{R_c} commanded roll control incidence

α_e constant or equilibrium angle-of-attack

α angle-of-attack

$\dot{\alpha}$ angle-of-attack rate

$\bar{\alpha}$ modified form of estimated angle-of-attack for autopilot coordination command

β angle of sideslip

$\dot{\beta}$ sideslip angular rate

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I. PARAMETERS SENSITIVITY ANALYSIS OF A BANK-TO-TURN MISSILE

The determination of changes in system performance due to changes in parameters is of great importance in engineering analysis and design. Sensitivity questions arise when model uncertainty is present, or when a range of operating conditions is contemplated.

The questions of parameters sensitivity particularly arise in the fields of engineering where models are idealized, inexactly identified, or the systems themselves are subject to unpredictable changes with time due to enviromental, material property or operational influences so that there is always a discrepancy between the physical reality and the mathematical model.

Sensitivity analysis provides the engineer with methods for investigating or minimizing such parameter deviations.

In general, the diagram of a system can be represented by a single block as given on Fig. 1.1.

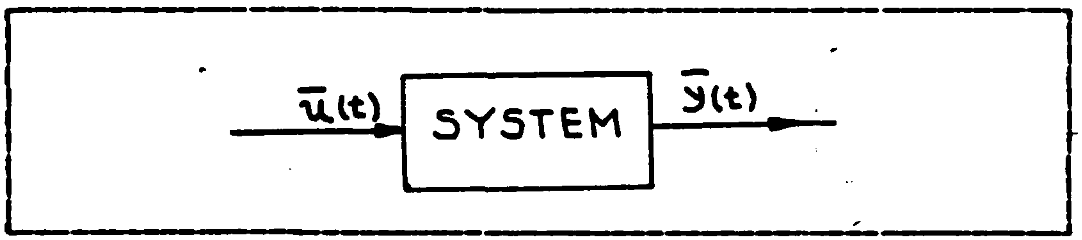

Figure 1.1 General Representation of a Dynamic System.

From a mathematical point of view, what we call a system is the explicity or implicity given relationship between the input signal u and output signal y. In general, u and y can be vectors. The character of this relationship is commonly called the structure of the system.

For example, the structure of the system may be characterized by the order of a differential or difference equation, linearity or nonlinearity, the order of the numerator and denominator of a rational transfer function and the quantitative properties of the system parameters.

Typical parameters are initial conditions, time-invariant or time-variant coefficients, natural frequencies and sampling periods.

The change of the state or the change of the output variable with time, can be caused by: (1) the influence of input signals, (2) the change of parameters. These quantities are shown in Fig. 1.2.

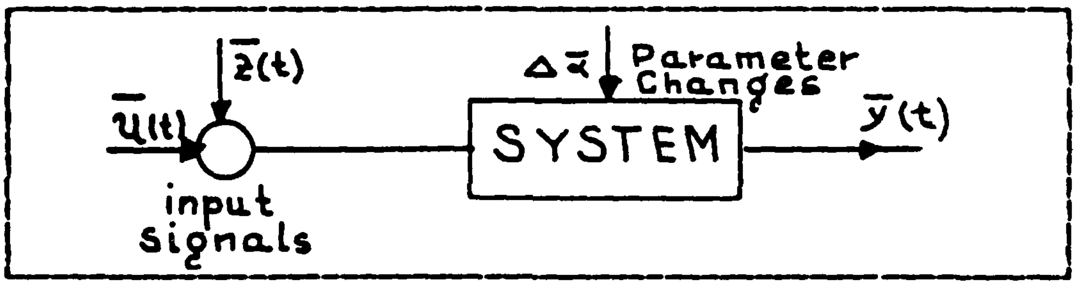

Figure 1.2 Quantities Affecting the Dynamics of a System.

This work addresses the application of the sensitivity analysis to the variation of aerodynamic parameters of a Bank-to-Turn missile.

PARAMETER SENSITIVITY is the effect of parameter changes on the dynamics of a system, say, the time response, the state, or any other quantity characterizing the system dynamics.

In order to have a realistic model to work with a NASA Contractor Report [Ref. 2] was adopted as a reference for application of the sensitivity analysis on the variation of aerodynamic parameters of a Bank-to-Turn missile. Appendix A, B and C give the detailed aerodynamic assumption of the system.

One brief explanation of the sensitivity theory and analysis is given as a guideline to better understand the subsequent work.

An analysis of the sensitivity of the aerodynamic parameters applied to the case of the linear uncoupled pitch and coupled roll-yaw autopilot is performed.

The pitch and roll-yaw autopilots are coupled as given in Fig.C.1 in the appendix C. A nonlinear parameter sensitivity analysis is performed.

Conclusions about the influence of parameters of concern are given and a comparison between the linear and nonlinear case is done. Comments and recommendations for the future are outlined in order to delineate the continuity of the present work.

II. SENSITIVITY THEORY

A. INTRODUCTION

The basis of all sensitivity considerations in the case of time-invariant parameter variations is the so-called sensitivity functions. Dynamic systems can be characterized in several ways:in the time domain, in the frequency domain, or in terms of a perfomance index. There is evidently an adequate number of ways to define the sensitivity functions of a dynamic system. The definition that is actually used depends on the form of the mathematical model as well as on the purpose under consideration.

The sensitivity functions can be classified into the following three categories:

(1) Sensitivity functions in the time domain
(2) Sensitivity functions in the frequency domain, and
(3) Perfomance-index sensitivity

In this chapter we will outline just the sensitivity functions in the time-domain of the continuous systems. Aplications of this analysis follow throughout this work.

The reader can find detailed informations about the others two categories of sensitivity functions in the [Ref. 1].

B. TRAJECTORY SENSITIVITY FUNCTION OF CONTINUOUS SYSTEMS

A continuous, possibly nonlinear system of nth order can, in general, be described in the state space by a vector differential equation as seen in Eqn.2.1.

$$\dot{\bar{X}} = \bar{f}(\bar{X},t,\bar{u},\bar{\alpha}_o) \ , \ \bar{X}(t_o) = \bar{X}^o \qquad (2.1)$$

Here $\bar{X}$ is an n x 1 state vector, $\bar{f}$ an n x 1 vector function, $\bar{u}$ an input vector, $\bar{\alpha}_o$ a nominal r x 1 parameter vector, and $\bar{X}$ is the n x 1 initial condition vector or initial state. Eqn.2.1 is called the NOMINAL STATE EQUATION.

Assuming that the parameter vector deviates from the nominal value $\bar{\alpha}_o$ by $\Delta\bar{\alpha}_o$, we have the Eqn.2.2.

$$\dot{\bar{X}} = \bar{f}(\bar{X},t,\bar{u},\bar{\alpha}), \ \bar{X}(t_o) = \bar{X}^o \qquad (2.2)$$

with the initial conditions $\bar{X}^o$ unchanged. This equation is called the ACTUAL STATE EQUATION.

Now it is assumed that Eqn.2.2 has a unique solution, $\bar{X} = \bar{X}(t,\bar{\alpha})$ for all admissible initial conditions and parameter values.

$\bar{X}$ is of course a function of $\bar{u}$, $\bar{X}^o$ and t_o as well. However , this dependence is not needed for the following considerations and will, therefore, be dropped for ease of notation. Furthermore, the solution $\bar{X}$ is assumed to be a bounded continuous function in t and $\bar{\alpha}$.

If the parameter takes on its NOMINAL value $\bar{\alpha}_o$, the nominal solution $\bar{X}_o = \bar{X}(t,\bar{\alpha}_o)$ is obtained. If, on the other hand the ACTUAL solution is given by $\bar{X} \mathrel{\hat{=}} \bar{X}(t,\bar{\alpha})$, then the parameter-induced change of the state vector is given by

$$\Delta\bar{X}(t,\bar{\alpha}) \mathrel{\hat{=}} \bar{X}(t,\bar{\alpha}) - \bar{X}(t,\bar{\alpha}_o) \qquad (2.3)$$

A first-order approximation of $\Delta \bar{X}$ can be obtained by using Taylor expansion in the form of Eqn.2.4.

$$\Delta \bar{X}(t,\bar{\alpha}) = \sum_{j=1}^{r} \frac{\partial \bar{X}}{\partial \alpha_j} \Big|_{\bar{\alpha}_0} \Delta \alpha_j \qquad (2.4)$$

This equation can be viewed as a definition of the parameter-induced trajectory deviation.

The subscript $\bar{\alpha}_0$ shall indicate that the partial derivative expressed by ∂ is taken at nominal parameter values.

Let the state $\bar{X}$ of a continuous system be a continuous function of a time-invariant parameter vector $\bar{\alpha}$ = $\{\alpha_1, \alpha_2, \ldots, \alpha_r\}^T$. Then the partial derivative can be defined as

$$\bar{\lambda}_j(t,\bar{\alpha}_0) \triangleq \frac{\partial(x,\bar{\alpha})}{\partial \alpha_j} \Big|_{\bar{\alpha}_0} \qquad (2.5)$$

$i=1,\ 2,\ \ldots,\ n$

$j=1,\ 2,\ \ldots,\ r$

Eqn.2.5 is called the trajectory sensitivity vector with respect to the jth parameter.

Note that the trajectory sensitivity vector is of the same dimension as the state vector, namely, n. Its components are the TRAJECTORY SENSITIVITY FUNCTIONS as

$$\bar{\lambda}_{ij}(t,\bar{\alpha}_0) \triangleq \frac{\partial x_i(t,\bar{\alpha})}{\partial \alpha_j} \Big|_{\bar{\alpha}_0} \qquad (2.6)$$

Eqn.2.6 is the partial derivative of the ith state variable in relation to the jth parameter as

$$\bar{\lambda}_j = \{\lambda_{1j}, \lambda_{2j}, \ldots, \lambda_{nj}\}^T = \left\{\frac{\partial x_1}{\partial \alpha_j}, \ldots, \frac{\partial x_n}{\partial \alpha_j}\right\}^T_{\bar{\alpha}_0} \qquad (2.7)$$

Hence, all n x r trajectory sensitivity functions form the trajectory sensitivity matrix as given in Eqn.2.8 or 2.9.

$$\bar{\lambda} = \{\bar{\lambda}_1, \ldots, \bar{\lambda}_r\} \triangleq \frac{\partial \bar{x}}{\partial \bar{\alpha}} \Big|_{\bar{\alpha}_0} \qquad (2.8)$$

$$\bar{\lambda} = \begin{bmatrix} \frac{\partial x_1}{\partial \alpha_1} & \cdots & \frac{\partial x_1}{\partial \alpha_r} \\ \vdots & & \vdots \\ \frac{\partial x_n}{\partial \alpha_1} & \cdots & \frac{\partial x_n}{\partial \alpha_r} \end{bmatrix} \qquad (2.9)$$

The columns of $\bar{\lambda}$ are the trajectory sensitivity vectors $\bar{\lambda}_j$. Here $\bar{\lambda}$ is the Jacobian matrix of the state vector with respect to the parameter vector $\bar{\alpha}$, taken at the nominal parameter values.

With these definitions the parameter-induced *change* of the trajectory can be taken as

$$\Delta \bar{x}(t, \bar{\alpha}) = \bar{\lambda}(t, \bar{\alpha}_0)\, \Delta\bar{\alpha} = \sum_{j=1}^{r} \bar{\lambda}_j \, \Delta \alpha_j \qquad (2.10)$$

Where $\bar{\alpha} = \bar{\alpha}_0 + \Delta\bar{\alpha}$, which is the ACTUAL parameter vector of the system.

C. TRAJECTORY SENSITIVITY EQUATIONS OF CONTINUOUS SYSTEMS

Lets consider the general continuous system described as previously by the state equation (Eqn.2.1).

$$\dot{\bar{X}} = \bar{f}(\bar{X},t,\bar{u},\bar{\alpha}),\ \bar{X}(t_o) = \bar{X}^0 \tag{2.11}$$

Where $\bar{X}$ denotes the n-dimensional state vector, $\bar{\alpha}$ the r-dimensional parameter vector, $\bar{f}$ an n-dimensional vector function, and $\bar{u}$ the input vector independent of $\bar{\alpha}$. It is assumed that the continuity conditions are fulfilled and that $\bar{\alpha}$ is time-invariant.

Considering α-parameters and taking the partial derivative of $\bar{X}$ (Eqn.2.11) with respect to α_j, one obtains, by the application of the chain rule

$$\frac{\partial \dot{\bar{x}}}{\partial \alpha_j} = \frac{\partial \bar{f}}{\partial \bar{x}} \frac{\partial \bar{x}}{\partial \alpha_j} + \frac{\partial \bar{f}}{\partial \alpha_j},\ \frac{\partial \bar{x}^0}{\partial \alpha_j} = 0 \tag{2.12}$$

The derivative of the initial conditions vector $\bar{X}^0$ with respect to α_j is zero, since $\bar{X}^0$ does not depend on $\bar{\alpha}$.

If $\bar{\alpha}$ is r-dimensional, there are r equations of the form of Eqn.2.12. If we now interchange the sequence of taking the derivative with respect to time t and α_j, and then let $\bar{\alpha}$ approach to $\bar{\alpha}_0$, one obtains the following equation

$$\dot{\bar{\lambda}}_j = \frac{\partial \bar{f}}{\partial \bar{x}}\Big|_{\bar{\alpha}_0} \bar{\lambda}_j + \frac{\partial \bar{f}}{\partial \alpha_j}\Big|_{\bar{\alpha}_0},\ \bar{\lambda}(0) = 0 \tag{2.13}$$

Here $\bar{\lambda}_j = \frac{\partial \bar{x}}{\partial \alpha_j}\Big|_{\bar{\alpha}_0}$ is the trajectory sensitivity vector with respect to the jth parameter.

The Eqn.2.13 is called the sensitivity equation in the state space or the TRAJECTORY SENSITIVITY EQUATION.

The above shows that for α -parameters all initial conditions of the trajectory sensitivity equations are equal to zero.

Now consider the output vector equation, as given by

$$\bar{y} = \bar{g}(\bar{x}, t, \bar{u}, \bar{\alpha}) \tag{2.14}$$

In a procedure similar to that above one obtains the algebraic sensitivity equation as seen in Eqn.2.15.

$$\bar{\sigma}_j = \frac{\partial \bar{x}}{\partial \alpha_j}\Big|_{\bar{\alpha}_0} = \frac{\partial \bar{g}}{\partial \bar{x}}\Big|_{\bar{\alpha}_0} \bar{\lambda}_j + \frac{\partial \bar{g}}{\partial \alpha_j}\Big|_{\bar{\alpha}_0} \tag{2.15}$$

Which relates the output sensitivity vector $\bar{\sigma}_j$ = $\frac{\partial \bar{y}}{\partial \alpha_j}\Big|_{\bar{\alpha}_0}$ to the trajectory sensitivity vector $\bar{\lambda}_j$. This equation is called the VECTOR OUTPUT SENSITIVITY EQUATION.

Using the trajectory sensitivity matrix $\bar{\lambda}$ and the output sensitivity matrix $\bar{\sigma}$, the above result can be rewritten in the following general form

$$\dot{\bar{\lambda}} = \frac{\partial \bar{f}}{\partial \bar{x}}\Big|_{\bar{\alpha}_0} \bar{\lambda} + \frac{\partial \bar{f}}{\partial \bar{\alpha}}\Big|_{\bar{\alpha}_0}, \bar{\lambda}^0 = 0 \tag{2.16}$$

$$\dot{\bar{\sigma}} = \frac{\partial \bar{g}}{\partial \bar{x}}\Big|_{\bar{\alpha}_0} \bar{\lambda} + \frac{\partial \bar{g}}{\partial \bar{\alpha}}\Big|_{\bar{\alpha}_0} \tag{2.17}$$

These equations are called the STATE SENSITIVITY EQUATIONS of the system. It is seen that these equations are linear whether the original system is linear or nonlinear.

If, in particular, the original system is linear, the state equations, take the form

$$\dot{\bar{X}} = \bar{A}\,\bar{X} + \bar{B}\,\bar{u} \quad , \bar{X}(t_o) = \bar{X}^o \qquad (2.18)$$

$$\dot{\bar{Y}} = \bar{C}\,\bar{X} + \bar{D}\,\bar{u} \qquad (2.19)$$

Where, in general,

$\bar{A} = \bar{A}(\alpha)$, $\bar{B} = \bar{B}(\alpha)$, $\bar{C} = \bar{C}(\alpha)$, $\bar{D} = \bar{D}(\alpha)$,
$\bar{X} = \bar{X}(t,\alpha)$, and $\bar{Y} = \bar{Y}(t,\alpha)$.

Note, however, that $\bar{u}$ is not a function of α if $\bar{u}$ is defined as an external input of the system.

Now taking the partial derivatives with respect to α_j , reversing the order of differentiations with respect to time and α_j , and letting $\bar{\alpha}$ approach to $\bar{\alpha}_o$, the TRAJECTORY SENSITIVITY EQUATIONS are obtained

$$\dot{\bar{\lambda}}_j = \bar{A}_o \lambda_j + \frac{\partial \bar{A}}{\partial \alpha_j}\Big|_{\bar{\alpha}_o} \bar{X}^o + \frac{\partial \bar{B}}{\partial \alpha_j}\Big|_{\bar{\alpha}_o} \bar{u}(t) \quad , \bar{\lambda}(t_o) = 0 \qquad (2.20)$$

Where $\bar{A}_o = \bar{A}(\alpha_o)$, $\bar{X}_o = \bar{X}(t, \alpha_o)$, and $j = 1, 2, \ldots, r$. The initial condition vector $\bar{\lambda}_j(t_o)$, is again zero since $\bar{X}(t_o)$ does not depend on $\bar{\alpha}$.

By similar procedure the VECTOR SENSITIVITY EQUATION becomes

$$\bar{\sigma}_j = \bar{C}_o \bar{\lambda}_j + \frac{\partial \bar{C}}{\partial \alpha_j}\Big|_{\bar{\alpha}_o} \bar{x}_o + \frac{\partial \bar{D}}{\partial \alpha_j}\Big|_{\bar{\alpha}_o} \bar{u}(t) \tag{2.21}$$

Where $\bar{C}_o = \bar{C}(\alpha_o)$ and $j=1,2,\ldots, r$.

In the case of linear systems, the vector sensitivity equations have the same A matrix as the nominal state equations and hence the same characteristic matrix sI - A. They differ from the nominal original state equation only in the driving function and the initial conditions. The latter are all zero. The driving functions can be obtained by solving the nominal state equations.

Here, in the linear case, the sensitivity equations can be joined to the system equations , forming the so-called COMBINED SYSTEM as given in Eqns.2.22 and 2.23.

$$\begin{bmatrix} \dot{\bar{x}}_o \\ \dot{\bar{\lambda}} \end{bmatrix} = \begin{bmatrix} A_o & 0 \\ \frac{\partial \bar{A}}{\partial \alpha_j}\Big|_{\bar{\alpha}_o} & \bar{A}_o \end{bmatrix} \begin{bmatrix} \bar{x}_o \\ \bar{\lambda}_j \end{bmatrix} + \begin{bmatrix} \bar{B}_o \\ \frac{\partial \bar{B}}{\partial \alpha_j}\Big|_{\bar{\alpha}_o} \end{bmatrix} \bar{u}(t) \tag{2.22}$$

$$\begin{bmatrix} \dot{\bar{x}}_o \\ \dot{\bar{\sigma}}_j \end{bmatrix} = \begin{bmatrix} \bar{C}_o & 0 \\ \frac{\partial \bar{C}}{\partial \alpha_j}\Big|_{\bar{\alpha}_o} & \bar{C}_o \end{bmatrix} \begin{bmatrix} \bar{x}_o \\ \bar{\lambda}_j \end{bmatrix} + \begin{bmatrix} \bar{D}_o \\ \frac{\partial \bar{D}}{\partial \alpha_j}\Big|_{\bar{\alpha}_o} \end{bmatrix} \bar{u}(t) \tag{2.23}$$

The simultaneous solution of these differential equations using the same standard procedure for each of the

above matrix equations yields the output vector, the state vector, and the corresponding output and trajectory sensitivity vectors σ_j, and λ_j. If there are r parameter variations, r systems of equations of the above type have to be solved. Since the homogeneous part of the original differential equation is identical with the homogeneous part of the sensitivity equations with respect to all parameters.

A graphical interpretation of Eqns.2.22 and 2.23 is given in Fig.2.1.

Since the driving function of the sensitivity model contains the nominal state, the measuring circuit for the trajectory sensitivity functions consists of a connection of the nominal original system and the sensitivity model as illustrated by Fig.2.1.

If the actual input $\bar{u}$ is applied to this structure the trajectory and output sensitivity vectors $\bar{\lambda}_j$ and $\bar{\sigma}_j$ can be measured at the points 1 and 2 respectively in the Fig.2.1.

In order to measure all r sensitivity vectors simultaneously, r sensitivity models are required.

D. STRUCTURAL METHOD

The basis for the measurement of the trajectory sensitivity functions is the STRUCTURAL INTERPRETATION of the trajectory sensitivity equation.

The physical system represented by the trajectory sensitivity equations is called the trajectory sensitivity model or the sensitivity model in the state space.

Regardless of the nature of the original system the sensitivity model in the state space is always linear. The graphical illustration of sensitivity equation can be given by Fig.2.2. The system described by the sensitivity equation is refered to as the sensitivity model of the original system. For each output, a system has as many sensitivity

models as parameters of interest. Both the nominal original system and the corresponding sensitivity models form the COMBINED SYSTEM. The sensitivity model is always linear. If $Y(t,\alpha)$ is the output of a system, the corresponding sensitivity functions $\overline{\sigma}_j(t,\overline{\alpha}_0) = \frac{\partial Y}{\partial \alpha_j}\Big|_{\alpha_0}$ are the outputs of the corresponding sensitivity models. Consequently, in order to measure the output sensitivity functions simultaneously,the nominal original system and the sensitivity models have to be measured at the outputs of the sensitivity models.

The double frame used for the original system in Fig.2.2 is to indicate that, in general, the system may be nonlinear, whereas the sensitivity models are always linear. In the nonlinear systems the Eqn.2.12 applies and different programs have to be set up for the original and the sensitivity equations.

For some applications, the sensitivity functions with respect to all parameters are required simultaneously. If there are r parameters of interest, r sensitivity models would be needed. This implies a rather extensive computer time.

A method of determining all output sensitivity functions of a system simultaneously by a single sensitivity model is available , which is also called the method of SENSITIVITY POINTS. This method has application just for linear systems. Detailed explanation about the application of the sensitivity points theory can be found in the [Ref. 1].

B. OUTPUT SENSITIVITY FUNCTION OF CONTINUOUS SYSTEMS

Consider the input-output behavior of a continuous, possibly nonlinear, single-variable system described by an ordinary differential equation of the type

$$f\{ y^{(n)}, y^{(n-1)}, \ldots, y, t, \alpha_0 \} = 0 \tag{2.24}$$

With the initial condition $y(t_0) = y_i{}^0$, $i=0, 1, \ldots n-1$. y denotes the output signal, t the time, and α a single time-invariant or slowly varying -parameter that has nominal value α_0.

In general, f is a function of the input u as well. However, if u is an external input which does not depend on α_0, the dependence of f on u is not relevant in further considerations.

Let us suppose that the above NOMINAL differential equation has the unique solution given below

$$Y_0 = Y(t, \alpha_0) \tag{2.25}$$

Which one shall call the NOMINAL SOLUTION.

Let us now assume that the parameter changes from α_0 to $\alpha = \alpha_0 + \Delta\alpha$, where $\Delta\alpha$ is time-invariant or slowly varying with time. α is called the actual parameter value. The corresponding ACTUAL DIFFERENTIAL EQUATION can, then, be written as

$$f = \{ y^{(n)}, y^{(n-1)}, \ldots, y, t, \alpha \} = 0 \tag{2.26}$$

Note that by this change of α_0 into α the initial conditions remain unchanged, namely $y(t_0) = y_i{}^0$.

The corresponding solution is given as

$$y = y(t, \alpha) \tag{2.27}$$

Which we shall call the ACTUAL (or *PERTURBED*) *SOLUTION*.

It is assumed that $y(t, \alpha)$ is of the same type as $y(t, \alpha_o)$, and $y(t, \alpha)$ deviates infinitesimally from $y(t, \alpha_o)$ if α deviates infinitesimally from α_o. The conditions for fulfilling this requirement are given in the mathematical literature. For our purpose, it is sufficient to know that y is continuous in α if f is continuous in y which is true for all continuous systems and $\alpha_o \neq 0$.

With the above assumptions, the actual solution $y(t, \alpha_o + \Delta\alpha)$ can be expanded into a Taylor series around α_o, yielding the Eqn. 2.28.

$$y(t,\alpha) = y(t,\alpha_o) + \frac{\partial y}{\partial \alpha}\Big|_{\alpha_o} \Delta\alpha + \frac{1}{2}\frac{\partial^2 y}{\partial \alpha^2}\Big|_{\alpha_o} \Delta\alpha^2 + \ldots \tag{2.28}$$

If $\Delta\alpha << \alpha_o$, the Taylor series can be truncated at the linear term.

This gives the Eqn.2.29.

$$y(t,\alpha) = y(t,\alpha_o) + \frac{\partial y}{\partial \alpha}\Big|_{\alpha_o} \Delta\alpha \tag{2.29}$$

For finite values of $\Delta\alpha$, this expression can be considered a first-order approximation of $y(t, \alpha)$.

The actual output can be written as

$$y(t,\alpha) \triangleq y(t,\alpha_0) + \sigma(t,\alpha_0)\,\Delta\alpha \tag{2.30}$$

Where $y(t,\alpha_0)$ is the nominal output and $\sigma(t,\alpha_0)$ is the parameter-induced output error.

With $\sigma(t,\alpha_0)$ defined as $\left.\frac{\partial y(t,\alpha_0)}{\partial \alpha}\right|_{\alpha_0}$. The parameter induced output error is in these terms defined as

$$y(t,\alpha) \triangleq \sigma(t,\alpha_0)\,\Delta\alpha \tag{2.31}$$

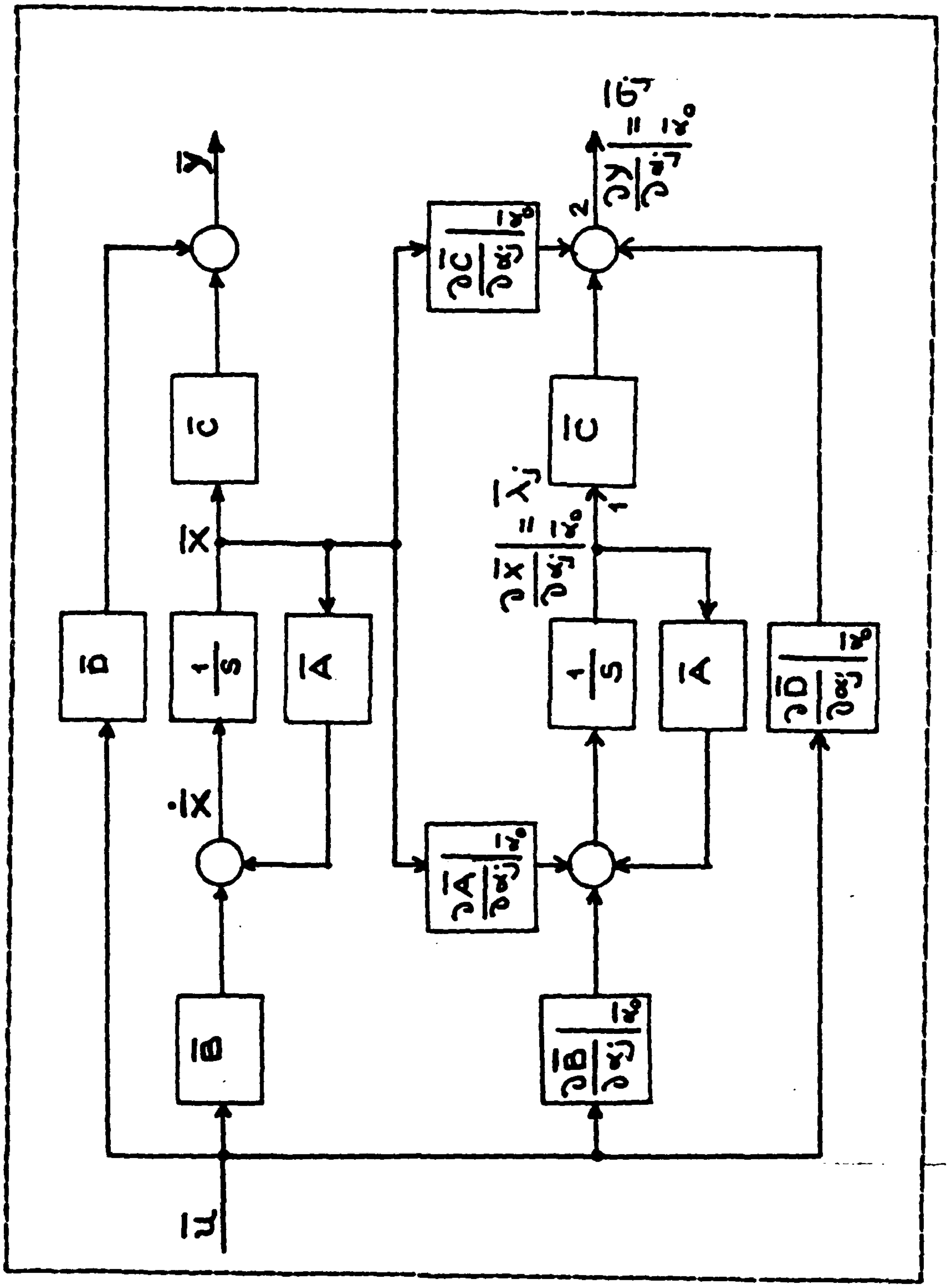

Figure 2.1 Graphical Interpretation of Eqns.2.22 and 2.23.

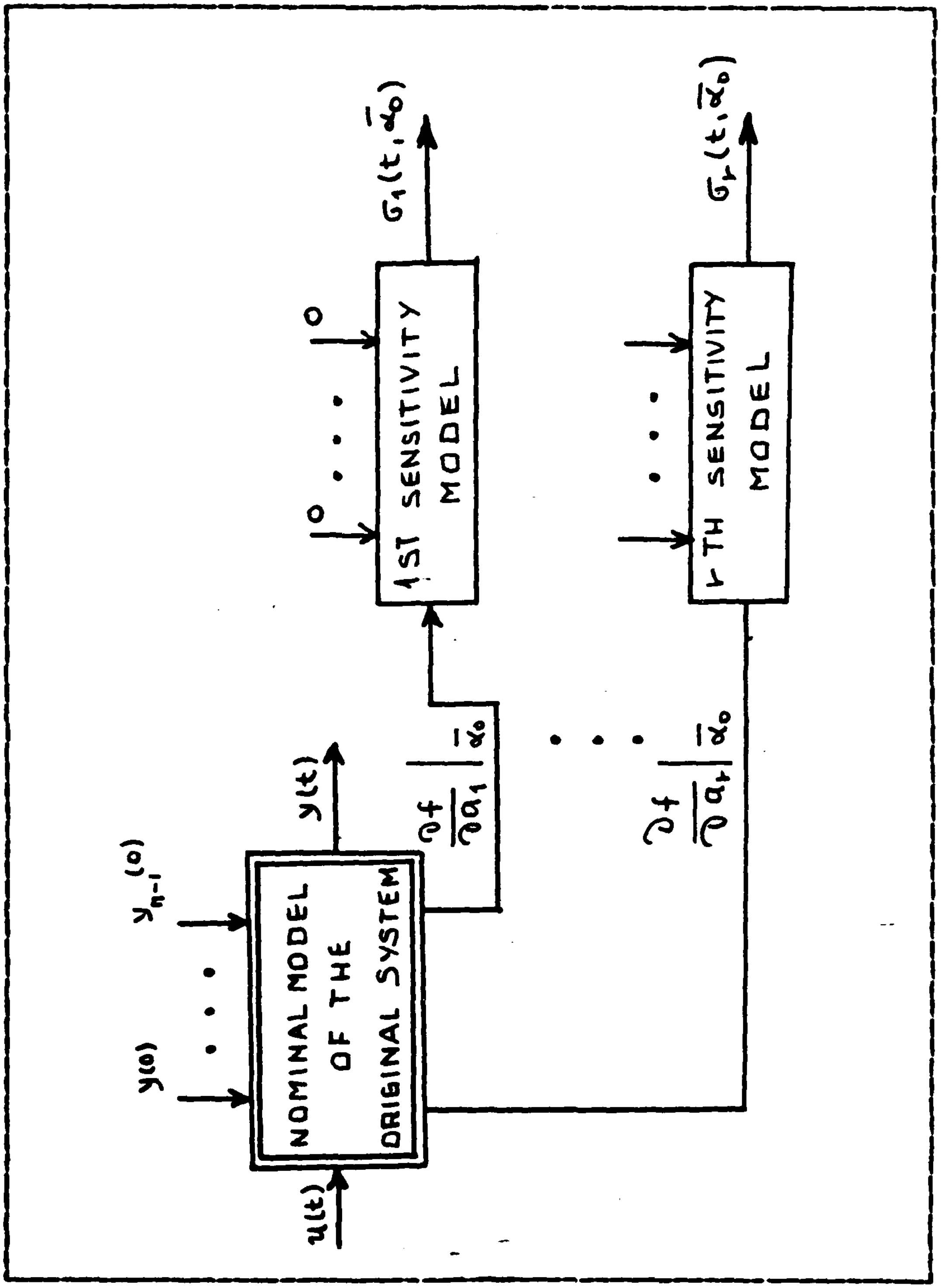

Figure 2.2 Strutural Diagram of the Combined System.

III. APPLICATION OF SENSITIVITY ANALYSIS TO LINEAR SYSTEMS

A. INTRODUCTION

The sensitivity theory described previously will be applied to the case of an uncoupled pitch autopilot and to a roll-yaw coupled autopilot presented in the appendix C. For each case, the linear equations of the nominal system are presented in state variable form and the correspondent trajectory sensitivity equations are derived. A sensitivity analysis of the systems is performed.

B. UNCOUPLED PITCH AUTOPILOT ANALYSIS

1. Linear Equations of the Nominal System

From the block diagram of the Fig.B.1 in the appendix B one can obtain the following nominal state equations of the uncoupled pitch autopilot.

$$\dot{X}1 = C2\ A3\ X2 + C2\ A4\ X3 \qquad (3.1)$$

$$\dot{X}2 = X1 - K\ C1\ A1\ X2 - K\ C1\ A2\ X3 \qquad (3.2)$$

$$\dot{X}3 = -\ C3\ X3 + C3\ Conv\ X6 \qquad (3.3)$$

$$\dot{X}4 = -\ C1\ C4\ A1\ X2 - C1\ C4\ A2\ X3 - C4\ X4 \qquad (3.4)$$

$$\dot{X}5 = C7\ X4 - C5\ X5 - C6\ NZC \qquad (3.5)$$

$$\dot{X}6 = C9/Conv + C2\ C8/conv\ A3\ X2 \qquad (3.6)$$

$$+ C2\ C8/Conv\ A4\ X3 - C7\ C8\ X4 + (C5\ C8 - C9)\ X5$$
$$+ C6\ C8\ NZC$$

For the purpose of using the linear method these state equations must be presented in matrix form as given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & c_2 A_3 & c_2 A_4 & 0 & 0 & 0 \\ 1 & -K_1 C_1 A_1 & -K C_1 A_2 & 0 & 0 & 0 \\ 0 & 0 & -c_3 & 0 & 0 & c_3 \mathrm{conv} \\ 0 & -c_1 c_4 A_1 & -c_1 c_4 A_2 & -c_4 & 0 & 0 \\ 0 & 0 & 0 & c_7 & -c_5 & 0 \\ \frac{c_9}{\mathrm{conv}} & \frac{c_2 c_8 A_3}{\mathrm{conv}} & \frac{c_2 c_8 A_4}{\mathrm{conv}} & -c_7 c_8 & c_5 c_8 - c_9 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -c_6 \\ c_6 c_8 \end{bmatrix} N_{zc} \qquad (3.7)$$

The correspondence of the state vectors is:

$X1 = q$, $X2 = \alpha$, $X3 = \delta_p$, $X4 = X$, $X5 = Y$, $X6 = \delta_{p_c}$

Definition of the constants C1 through C9 are given in appendix B. The parameters of interest for this system are given by A1, A2, A3, and A4 which are :

$A1 = C_{N_\alpha}$, $A2 = C_{N\delta_p}$, $A3 = C_{m_\alpha}$, $A4 = C_{m\delta_p}$.

This nomenclature is used here to easily apply the sensitivity theory and to avoid the inconvenience of carrying symbols and constants.

2. Sensitivity Equations

To apply the procedure developed in chapter 2, consider the trajectory sensitivity equation (Eqn.2.13).

$$\dot{\bar{\lambda}}_j = \bar{A}_o \bar{\lambda}_j + \frac{\partial \bar{A}}{\partial \alpha_j}\bigg|_{\bar{\alpha}_o} \bar{x}_o + \frac{\partial \bar{B}}{\partial \alpha_j}\bigg|_{\bar{\alpha}_o} \bar{u}(t), \quad \bar{\lambda}_j(t_o) = 0 \qquad (3.8)$$

From Eqn.3.7 one can see that

$$\bar{A}_o = \begin{bmatrix} 0 & c_2 A_3 & c_2 A_4 & 0 & 0 & 0 \\ 1 & -k_1 c_1 A_1 & -k c_1 A_2 & 0 & 0 & 0 \\ 0 & 0 & -c_3 & 0 & 0 & c_3 conv \\ 0 & -c_1 c_4 A_1 & -c_1 c_4 A_2 & -c_4 & 0 & 0 \\ 0 & 0 & 0 & c_7 & -c_5 & 0 \\ \frac{c_9}{conv} & \frac{c_2 c_8 A_3}{conv} & \frac{c_2 c_8 A_4}{conv} & -c_7 c_8 & c_5 c_8 - c_9 & 0 \end{bmatrix} \qquad (3.9)$$

$$\bar{B} = [\,0 \quad 0 \quad 0 \quad 0 \quad -c_6 \quad c_6 c_8\,]^T \qquad (3.10)$$

The partial derivatives $\frac{\partial A}{\partial \alpha_j}\Big|_{\alpha_o}$ and $\frac{\partial B}{\partial \alpha_j}\Big|_{\alpha_o}$ are evaluated considering the parameters of interest A1, A2, A3 and A4, that are respectively the aerodynamic coefficients present in the pitch channel.

Applying the partial derivatives with respect to the parameters we have the following matrices

$$\frac{\partial A}{\partial A_1} = \begin{bmatrix} 0 & 0 & C_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{C_2 C_8}{ConV} & 0 & 0 & 0 \end{bmatrix} \qquad (3.11)$$

$$\frac{\partial A}{\partial A_2} = \begin{bmatrix} 0 & C_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{C_2 C_8}{ConV} & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (3.12)$$

$$\frac{\partial A}{\partial A_3} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -KC_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -C_1 C_9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (3.13)$$

$$\frac{\partial A}{\partial A_4} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -KC_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -C_1 C_9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (3.14)$$

and

$$\frac{\partial \bar{B}}{\partial \alpha_j} = 0$$

because $\bar{B}$ is independent of the parameters of interest.

In terms of components one can see

$$\lambda_{ij} = \frac{\partial x_i}{\partial \alpha_j} \Big|_{\bar{\alpha}_0} \tag{3.15}$$

$i=1,2,\ldots,6$ $j=1,2,3$ and 4

Here, four models are necessary to study the effect of parameter variations. These models are shown in Fig.3.1.

For instance, when one apply the theory for one parameter of interest, say A1, the sensitivity equations can be obtained from

$$\begin{bmatrix} \dot{\lambda}_{11} \\ \dot{\lambda}_{12} \\ \dot{\lambda}_{13} \\ \dot{\lambda}_{14} \\ \dot{\lambda}_{15} \\ \dot{\lambda}_{16} \end{bmatrix} = \begin{bmatrix} A_0 \end{bmatrix} \begin{bmatrix} \lambda_{11} \cdots \lambda_{14} \\ \vdots \quad\quad \vdots \\ \lambda_{61} \cdots \lambda_{64} \end{bmatrix} + \begin{bmatrix} \frac{\partial A}{\partial A_1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \tag{3.16}$$

Similar procedure can be done for the other parameters A2, A3, A4.

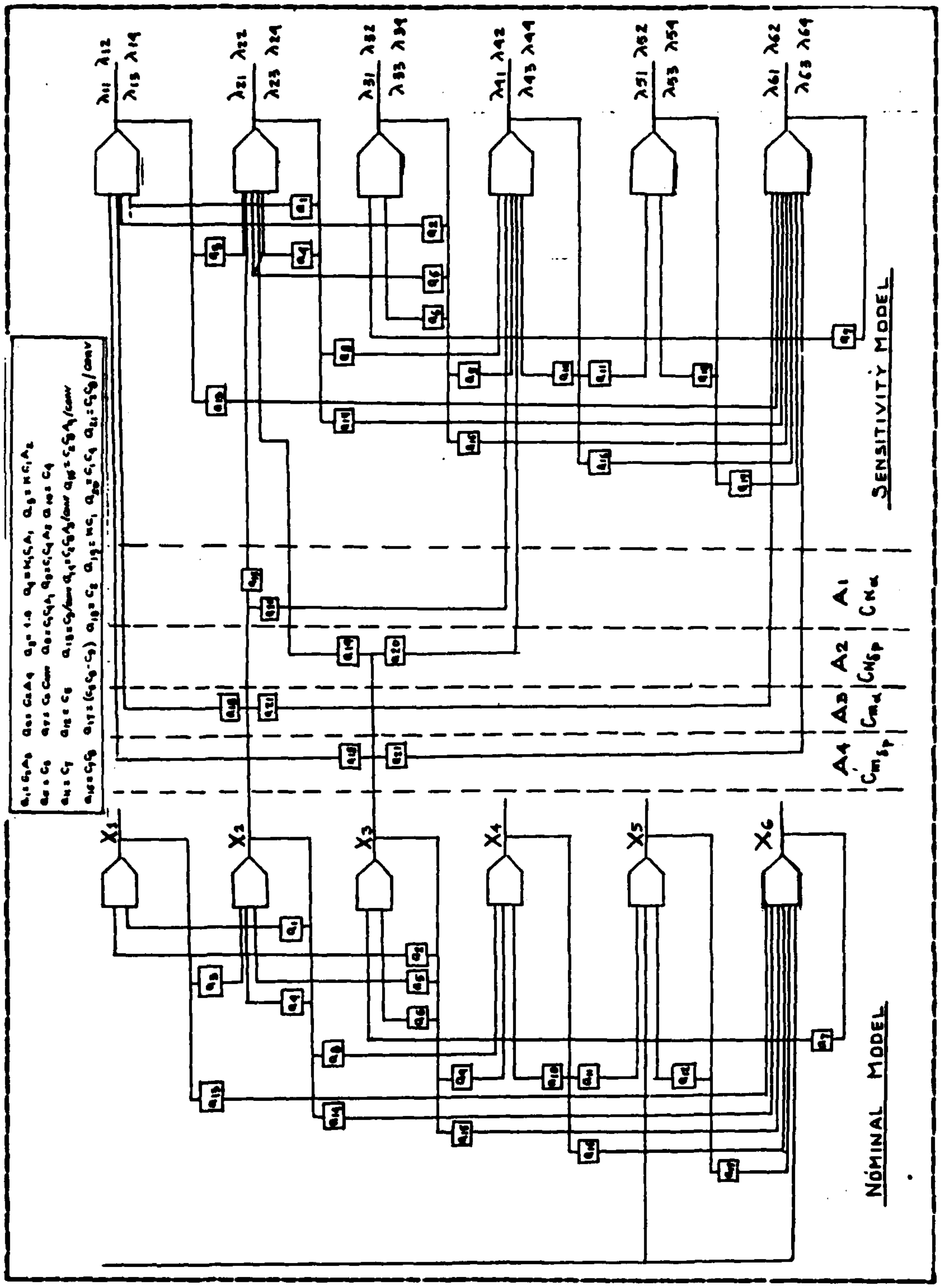

Figure 3.1 Nominal and Sensitivity Models.

C. SENSITIVITY ANALYSIS

A computer program using [Ref. 4] was written in order to simulate simultaneously the nominal and the sensitivity models as shown in the appendix D. A step was applied as input of the nominal system.

The number of equations solved are 6 for the nominal model and 24 for the sensitivity model. Each parameter was varied simultaneously 10% from all the nominal values.

D. ANALYSIS OF -PARAMETER VARIATIONS

The results are plotted in Fig.3.2 to 3.6 and Table I Each state variable output is plotted separately with the correspondent four output sensitivity functions. By means of these plots,the following observations can be made:

The plot of $\lambda_{1j} = \frac{\partial X_1}{\partial A_j}\Big|_{A_{j0}}$, j=1,2,3, and 4 in fig.3.2 indicates that a parameter change ΔA_{j0}(j=1,2,3, and 4) primarily affect the rise time and overshoot of X1(t) since $\lambda 11$, $\lambda 12$, $\lambda 13$, $\lambda 14$, are largest at the time where these effects in X1 occur.

The plot of $\lambda_{2j} = \frac{\partial X_2}{\partial A_j}\Big|_{A_{j0}}$, j=1,2,3, and 4 in Fig.3.3 indicates that X2(t) is little affected due to ΔA_{10} and ΔA_{20} and strongly affected in the rise time due to parameter variations ΔA_{30} and ΔA_{40}. The overshoot and steady state are little affected due to parameter variations ΔA_j (j=1,2,3, and 4).

The plot of $\lambda_{3j} = \frac{\partial X_3}{\partial A_j}\Big|_{A_{j0}}$, j=1,2,3, and 4 in Fig.3.4 indicates that X3(t) is little affected in the rise time due to parameter variations A (j=1,2,3,and 4). The overshoot and steady state are little affected due to parameter variations ΔA_{10} and ΔA_{20} and strongly affected due to ΔA_{30} and ΔA_{30}.

The plot of $\lambda_{4j} = \frac{\partial X4}{\partial A_j}|_{A_{j0}}$, j=1,2,3, and 4 in Fig.3.5 indicates that X4(t) is little affected in the rise time and overshoot due to parameter variations ΔA_{10} and ΔA_{20} and strongly affected due to ΔA_{30} and ΔA_{40}.

The plot of $\lambda_{5j} = \frac{\partial X5}{\partial A_j}|_{A_{j0}}$, j=1,2,3, and 4 in Fig.3.6 indicates that X5(t) is little affected in the rise time due to parameter variations ΔA_{j0} (j=1,2,3, and 4). The overshoot is little affected due to ΔA_{10} and ΔA_{20} and strongly affected due to ΔA_{30} and ΔA_{40}.

The plot of $\lambda_{6j} = \frac{\partial X6}{\partial A_j}|_{A_{j0}}$, j=1,2,3, and 4 in Fig.3.7 indicates that X6(t) is little affected in the rise time due to parameter variations ΔA_{j0} (j=1,2,3, and 4). The overshoot and steady state are little affected due to ΔA_{10} and ΔA_{20} and strongly affected due to ΔA_{30} and ΔA_{40}.

Table I shows the above analysis in a condensed way to give a general picture of all states and sensitivity functions with the correspondent effect as a function of time.

E. PARAMETER-INDUCED OUTPUT ANALYSIS

If $\Delta\alpha \ll \alpha_0$,the actual output can be written as (Eqn.2.29).

$$y(t,\alpha) \triangleq y(t,\alpha_0) + \sigma(t,\alpha_0)\,\Delta\alpha \qquad (3.17)$$

The computer program as shown in the Appendix D gives the actual output when 10%, 30% and 40% of variation from the nominal value of each parameter is assumed. Figs.3.8 through 3.13 show the actual output when each parameter is varied 10% from the nominal value.

Fig.3.8 indicates that the overshoot and rise time of the actual and nominal output of X1 are strongly affected which is in agreement with the previous analysis.

Fig.3.9 indicates that the steady state and overshoot of the actual and nominal output of X2 are primarily affected which is in agreement with the previous analysis.

Fig.3.10 indicates that the steady state and overshoot of the actual and nominal output of X3 are primarily affected which is in agreement with the previous analysis.

Fig.3.11 indicates that the overshoot of the actual and nominal output of X4 is primarily affected which is in agreement with the previous analysis.

Fig.3.12 indicates that overshoot of the actual and nominal output of X5 are primarily affected which is in agreement with the previous analysis.

Fig.3.13 indicates that the steady state and overshoot of the actual and nominal output of X6 are primarily affected ,having little effect in the rise time which is in agreement with the previous analysis.

Figs.3.14 through 3.19 and Figs.3.20 through 3.25 show respectively the actual output for 30% and 40% of the nominal value.

From the plots one can see that for small parameter variations the parameter-induced output error is negligible and when the variation becomes large as 30% or 40% one note that the error becomes pronounced and that modeling is starting to break down. This agrees with the assumption made in the derivation of the Eqn.2.28.

F. COUPLED ROLL-YAW AUTOPILOT ANALYSIS

1. Linear Equations of the Nominal System

From the block diagram of the Fig.B.1 in the appendix B one can get the following state equations of the coupled roll-yaw autopilot.

$$\dot{X}1 = B\ Conv\ (A4\ X3 + A3\ X8 + A5\ X11) \tag{3.18}$$

$$\dot{X}2 = C\ Conv\ (A7\ X3 + A8\ X8 + A6\ X11) \tag{3.19}$$

$$\dot{X}3 = -\ X1 - (ALPHAB/Conv)\ X2 \tag{3.20}$$

$$+\ KB\ A\ A2\ X3 + KB\ A\ A1\ X11$$

$$\dot{X}4 = -\ 8\ X4 - 17.6\ X12 + 17.6\ PHC \tag{3.21}$$

$$\dot{X}5 = -\ (0.755/Conv)\ X2 - C\ D\ A7\ X3 \tag{3.22}$$

$$+\ (0.755 - 8\ D)\ X4 - 5\ X5 - C\ D\ A8\ X8 - C\ D\ A6\ X11$$
$$-\ 17.6\ X12 + 17.6\ PHC$$

$$\dot{X}6 = C\ E\ A7\ X3 - 6\ X6 + E\ C\ A8\ X8 + C\ E\ A6\ X11 \tag{3.23}$$

$$\dot{X}7 = -\ F\ KC\ (0.755/Conv)\ X2 \tag{3.24}$$

$$-(D + E)\ F\ KC\ C\ A7\ X3 + F\ KC\ (0.755 - 8\ D)\ X4$$
$$+\ KC(15 - 5\ F)\ X5 + KC(6F - 15)\ X6 - 15\ X7$$
$$-\ (D + E)\ F\ KC\ C\ A8\ X8 - (D + E)\ F\ KC\ C\ A6\ X11$$
$$-\ F\ KC\ D\ 17.6\ X12 + F\ KC\ D\ 17.6\ PHC$$

$$\dot{X}8 = 188.4\ Conv\ X7 - 188.4\ X8 \tag{3.25}$$

$$\dot{X}9 = (K1\ A\ A2/t1)\ X1 - X9\ /\ T1 + (K1\ A\ A1/T1)\ X11 \tag{3.26}$$

$$\dot{X}10 =- (K2/Conv)\ X1 - (K2\ H\ ALPHAB/Conv\ X2 \tag{3.27}$$

$(k2/10)(K1\ A\ A2/T1) + B\ A4 - H\ ALPHAB\ C\ A7\ X3$

$+ (K2/10)(B\ A3 - H\ ALPHAB\ C\ A8\ X8 + K2(1$

$- 1/(10\ T1))\ X9 + (K2/10)((K1\ A\ A1)/T1 + B\ A5$

$- H\ ALPHAB\ C\ A6)\ X11$

$$\dot{X}11 = 188.4\ Conv\ X10 - 188.4\ X11 \tag{3.28}$$

$$\dot{X}12 = X2\ /\ Conv \tag{3.29}$$

Eqn.3.30 gives the matrix representation of the state variables above mentioned.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \\ \dot{x}_{10} \\ \dot{x}_{11} \\ \dot{x}_{12} \end{bmatrix} = \begin{bmatrix} 0 & \overset{1}{0} & (c_1A_1) & 0 & 0 & 0 & 0 & (c_1A_3) & 0 & 0 & (c_1A_5) & 0 \\ 0 & 0 & (c_2A_7) & 0 & 0 & 0 & 0 & (c_2A_8) & 0 & 0 & (c_2A_6) & 0 \\ -1 & c_3 & (c_4A_2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (c_4A_1) & 0 \\ 0 & 0 & 0 & -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -17.6 \\ 0 & -c_5 & (-c_7A_7) & c_8 & -5 & 0 & 0 & (-c_7A_8) & 0 & 0 & (-c_7A_6) & -c_8 \\ 0 & 0 & (c_9A_7) & 0 & 0 & -6 & 0 & (c_9A_8) & 0 & 0 & (c_9A_6) & 0 \\ 0 & c_{10} & (-c_{11}A_7) & c_{12} & c_{13} & c_{14} & -15 & (c_{11}A_8) & 0 & 0 & (c_{11}A_6) & c_{12} \\ 0 & 0 & 0 & 0 & 0 & 0 & c_{13} & (-188.4) & 0 & 0 & 0 & 0 \\ 0 & 0 & (c_{14}A_2) & 0 & 0 & 0 & 0 & 0 & -\frac{1}{T1} & 0 & (c_{14}A_1) & 0 \\ c_{15} & c_{16} & \begin{pmatrix} c_{17}A_2 \\ +c_{18}A_4 \\ -c_{19}A_7 \end{pmatrix} & 0 & 0 & 0 & 0 & \begin{pmatrix} c_{20}A_3 \\ -c_{21}A_8 \end{pmatrix} & c_{22} & 0 & \begin{pmatrix} c_{23}A_1 \\ +c_{24}A_5 \\ -c_{25}A_6 \end{pmatrix} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_{13} & -188.4 & 0 \\ 0 & \frac{1}{conv} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 17.6 \\ c_8 \\ 0 \\ c_{12} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{Pwc} \tag{3.30}$$

The correspondence of the states is:

$X1 = r$, $X2 = p$, $X3 = \beta$,

$X4 = X$, $X5 = Y1$, $X6 = X1$,

$X7 = \delta_{R_c}$, $X8 = \delta_R$, $X9 = Y$,

$X10 = \delta_{Y_c}$, $X11 = \delta_Y$, $X12 = \phi$

The definition of the constants C1 through C25 are given in appendix B. The parameters of interest for this system are given by A1 through A8 which are :

$A1 = C_{Y_{\delta Y}}$, $A2 = C_{Y_\beta}$, $A3 = C_{n_{\delta R}}$, $A4 = C_{n_\beta}$, $A5 = C_{n_{\delta Y}}$,
$A6 = C_{\ell_{\delta Y}}$, $A7 = C_{\ell_\beta}$, $A8 = C_{\ell_{\delta R}}$.

2. <u>Sensitivity Equations</u>

As showed in the previous analysis here one can apply the same procedure using the TRAJECTORY SENSITIVITY EQUATION as given in Equ.2.13.

$$\dot{\bar{\lambda}}_j = \bar{A}_o \bar{\lambda}_j + \frac{\partial \bar{A}}{\partial \alpha_j}\Big|_{\bar{\alpha}_o} \bar{X}_o + \frac{\partial \bar{B}}{\partial \alpha_j}\Big|_{\bar{\alpha}_o} \bar{u}(t) \quad , \bar{\lambda}_j(t_o) = 0 \qquad (3.31)$$

For the purpose of this procedure again one can see in Eqn.3.30 the correspondent matrices A_o and B_o.

The partial derivatives $\frac{\partial \bar{A}}{\partial A_j}\Big|_{\bar{\alpha}_o}$ and the $\frac{\partial \bar{B}}{\partial A_j}\Big|_{\bar{\alpha}_o}$ are evaluated considering the parameter A of interest. In this case they are A1,A2,...,A8, that are respectively the aerodynamic coefficients present in the coupled roll-yaw autopilot.

Once again, applying the partial derivatives with respect to the parameters one can obtain eight matrices respectively as found ,similarly, in the previous case of the uncoupled pitch autopilot.

Here, one can see that $\frac{\partial \bar{B}}{\partial \alpha_j} = 0$ because $\bar{B}$ is independent of the parameters of interest.

In terms of components one obtains

$$\lambda_{ij} = \frac{\partial x_i}{\partial \alpha_j} \Big|_{\overline{\alpha}_o} \tag{3.32}$$

$i=1,2,\ldots\ldots 12$

$j=1,2,\ldots\ldots 8$

Here,one can see that eight models are necessary to study the effect of parameter variations. These models are shown in Fig.3.27. For one parameter of interest, say A1, one have the following sensitivity equations :

$$\begin{bmatrix} \dot{\lambda}_{11} \\ \dot{\lambda}_{21} \\ \dot{\lambda}_{31} \\ \dot{\lambda}_{41} \\ \dot{\lambda}_{51} \\ \dot{\lambda}_{61} \\ \dot{\lambda}_{71} \\ \dot{\lambda}_{81} \\ \dot{\lambda}_{91} \\ \dot{\lambda}_{101} \\ \dot{\lambda}_{111} \\ \dot{\lambda}_{121} \end{bmatrix} = \begin{bmatrix} \\ \overline{A}_o \\ \\ \end{bmatrix} \begin{bmatrix} \lambda_{11} & \cdots & \lambda_{18} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \lambda_{121} & \cdots & \lambda_{128} \end{bmatrix} + \begin{bmatrix} \\ \frac{\partial \overline{A}}{\partial A_1} \\ \\ \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \end{bmatrix} \tag{3.33}$$

Similar procedure can be made for the others parameters A2 through A8.

G. SENSITIVITY ANALYSIS

In order to simulate simultaneouly the nominal and the sensitivity models,a computer program was written as shown in the appendix E.

For analysis purpose,each parameter was varied simultaneously 10% from the nominal value.

1. **Analysis of α-Parameter Variations**

The plots of $\lambda_{1j} = \frac{\partial X_1}{\partial A_j}|_{A_{j0}}$, j=1,2,...,8 in Fig.3.27 and 3.28 indicate that a parameter change ΔA_{j0}, (j=1,2,...,8) primarily affect the overshoot of X1(t).

The plots of $\lambda_{2j} = \frac{\partial X_2}{\partial A_j}|_{A_{j0}}$, j=1,2,...,8 in Fig.3.29 and 3.30 indicate that a parameter change ΔA_{j0}, (j=1,2,...,8) primarily affect the overshoot of X2(t).

The plots of $\lambda_{3j} = \frac{\partial X_3}{\partial A_j}|_{A_{j0}}$, j=1,2,...,8 in Fig.3.31 and 3.32 indicate that a parameter change ΔA_{j0}, (j=1,2,...,8) primarily affect the overshoot of X3(t).

The plots of $\lambda_{4j} = \frac{\partial X_4}{\partial A_j}|_{A_{j0}}$, j=1,2,...,8 in Fig.3.33 and 3.34 indicate that a parameter change ΔA_{j0}, (j=1,2,...,8) primarily affect the overshoot of X4(t).

The plots of $\lambda_{5j} = \frac{\partial X_5}{\partial A_j}|_{A_{j0}}$, j=1,2,...,8 in Fig.3.35 and 3.36 indicate that a parameter change ΔA_{j0}, (j=1,2,...,8) primarily affect the overshoot of X5(t).

The plots of $\lambda_{6j} = \frac{\partial X_6}{\partial A_j}|_{A_{j0}}$, j=1,2,...,8 in Fig.3.37 and 3.38 indicate that a parameter change ΔA_{j0}, (j=1,2,...,8) primarily affect the overshoot of X6(t).

The plots of $\lambda_{7j} = \frac{\partial X_7}{\partial A_j}|_{A_{j0}}$, j=1,2,...,8 in Fig.3.39 and 3.40 indicate that a parameter change ΔA_{j0}, (j=1,2,...,8) primarily affect the overshoot of X7(t).

The plots of $\lambda_{8j} = \frac{\partial X_8}{\partial A_j}|_{A_{j0}}$, j=1,2,...,8 in Fig.3.41 and 3.42 indicate that a parameter change ΔA_{j0}, (j=1,2,...,8) primarily affect the overshoot of X8(t).

The plots of $\lambda_{9j} = \frac{\partial X_9}{\partial A_j}|_{A_{j0}}$, j=1,2,...,8 in Fig.3.43 and 3.44 indicate that a parameter change ΔA_{j0}, (j=1,2,...,8) primarily affect the overshoot and rise time of X9(t).

The plots of $\lambda_{10j} = \frac{\partial X_{10}}{\partial A_j}|_{A_{j0}}$, j=1,2,...,8 in Fig.3.45 and 3.46 indicate that a parameter change ΔA_{j0}, (j=1,2,...,8) primarily affect the rise time of X10(t).

The plots of $\lambda_{11j} = \frac{\partial X_{11}}{\partial A_j}|_{A_{jo}}$, j=1,2,...,8 in Fig.3.47 and 3.48 indicate that a parameter change ΔA_{jo}, (j=1,2,...,8) primarily affect the overshoot of X11(t).

The plots of $\lambda_{12j} = \frac{\partial X_{12}}{\partial A_j}|_{A_{jo}}$, j=1,2,...,8 in Fig.3.49 and 3.50 indicate that a parameter change ΔA_{jo}, (j=1,2,...,8) primarily affect the overshoot of X12(t).

Table II and III show the above analysis in a condensed way to give a general picture of all states and output sensitivity functions with the correspondent effect as function of time.

2. Parameter-Induced Output Analysis

As Shown previously, if $\Delta\alpha << \alpha_o$, the actual output can be written as

$$y(t,\alpha) \triangleq y(t,\alpha_o) + \sigma(t,\alpha_o)\,\Delta\alpha \qquad (3.34)$$

The computer program given in appendix E was written for simulating the system when 10%, 30%, and 40% of variation from the nominal value of each parameter is assumed. Fig.3.51 through 3.62 give the actual output when each parameter is varied 10% from the nominal value.

Fig.3.63 and 3.64 give the actual output for 30% of variation from the nominal value of each parameter assumed. Fig.3.65 and 3.66 give the actual output for 40% of variation from the nominal value of each parameter assumed. These plots show the output of the state variables X3 and X11 that present strong deviations just to give the behavior of the system when parameter variations are not small. One notes that modeling is starting to break down.

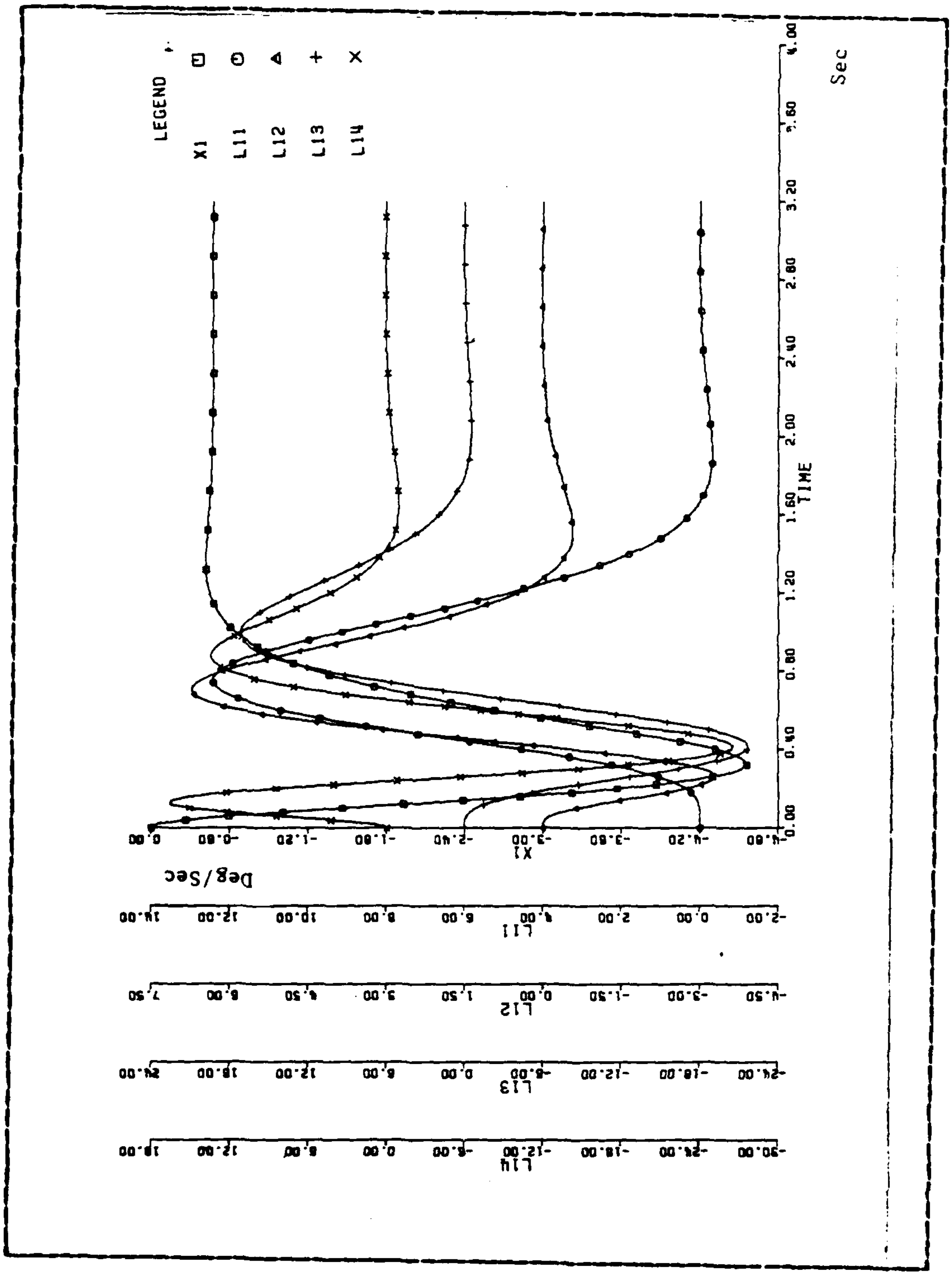

Figure 3.2 Sensitivity of X1 with Respect to A1,A2,A3,A4.

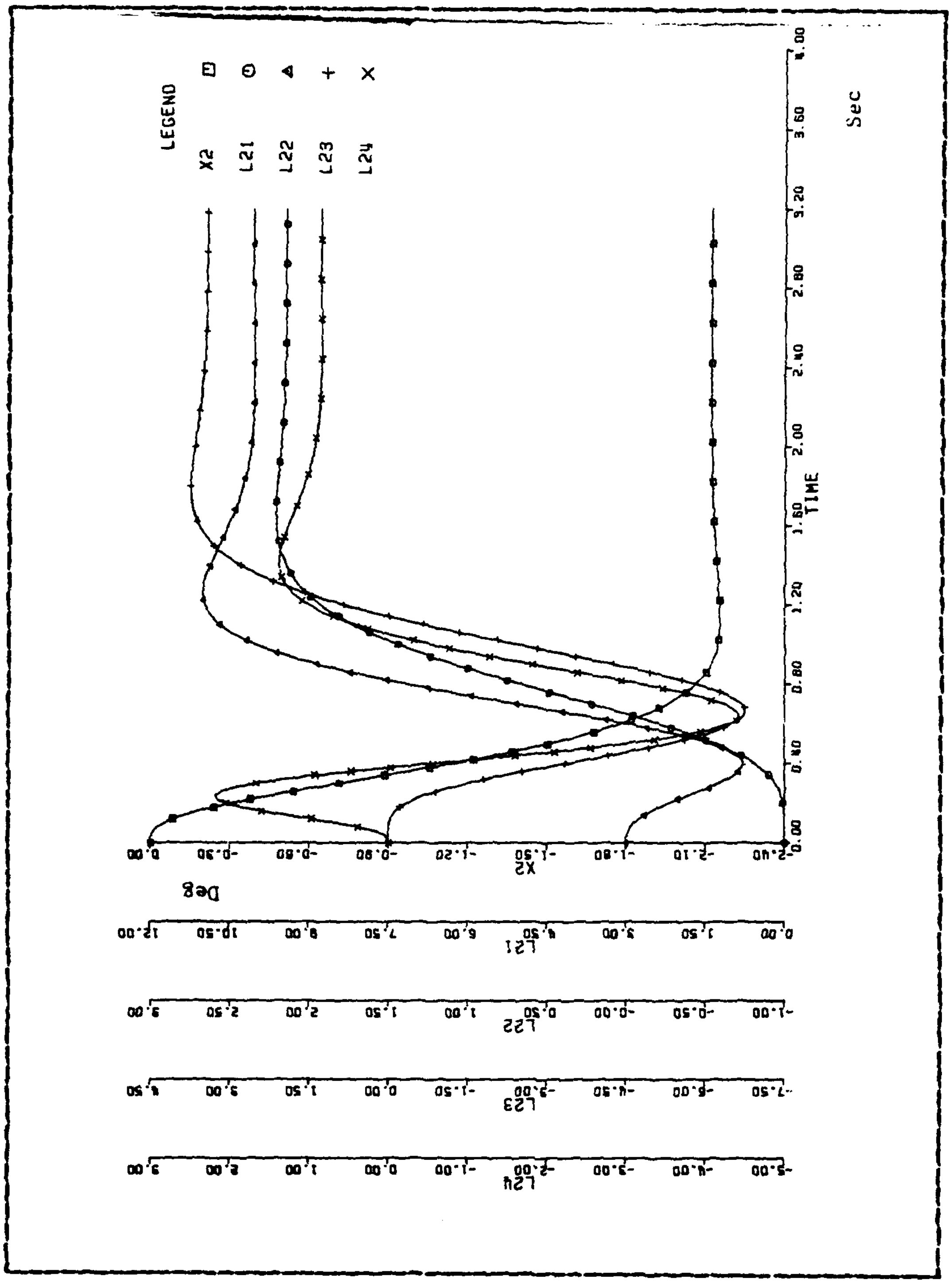

Figure 3.3 Sensitivity of X2 with Respect to A1,A2,A3,A4.

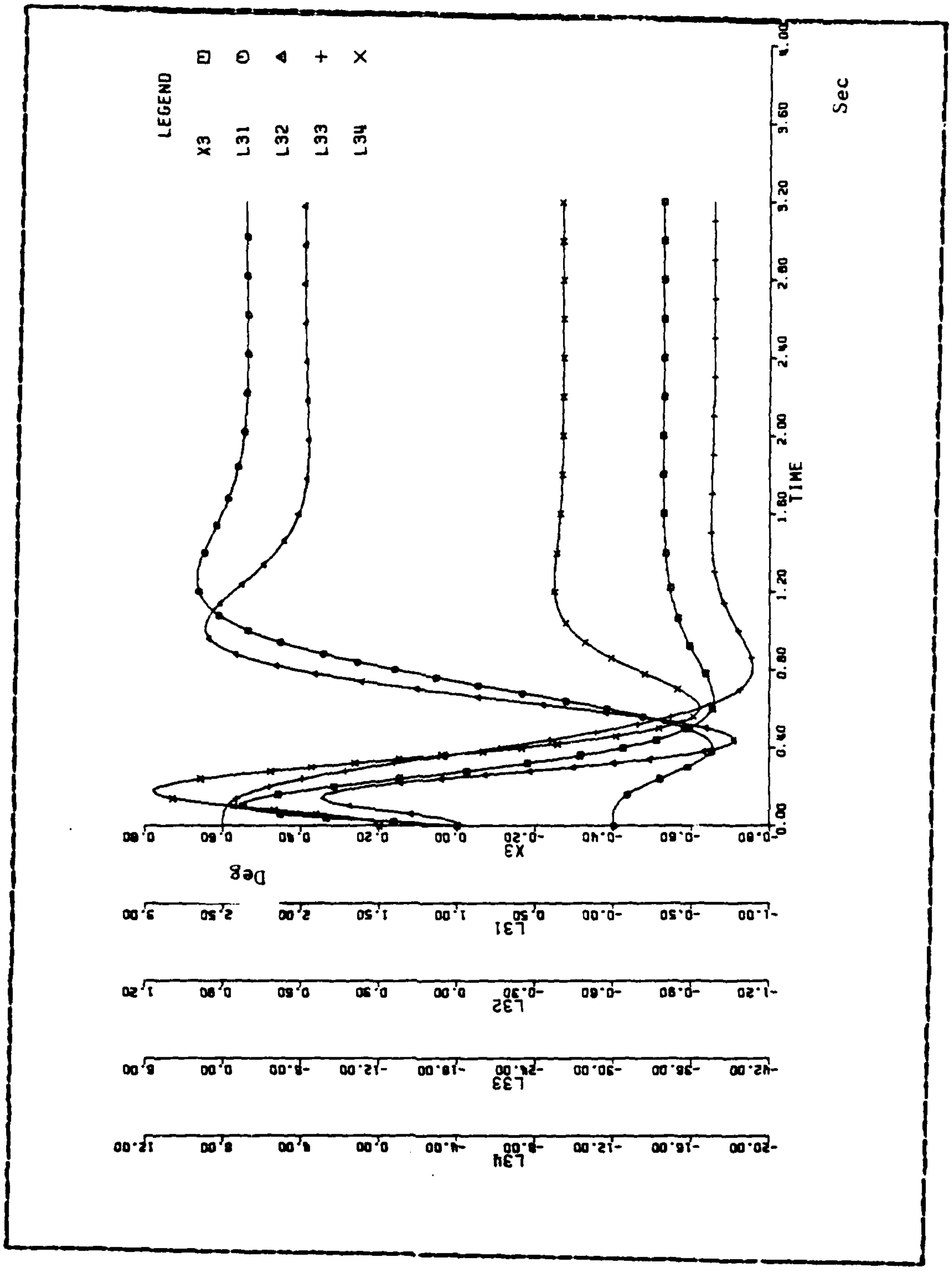

Figure 3.4 Sensitivity of X3 with Respect to A1,A2,A3,A4.

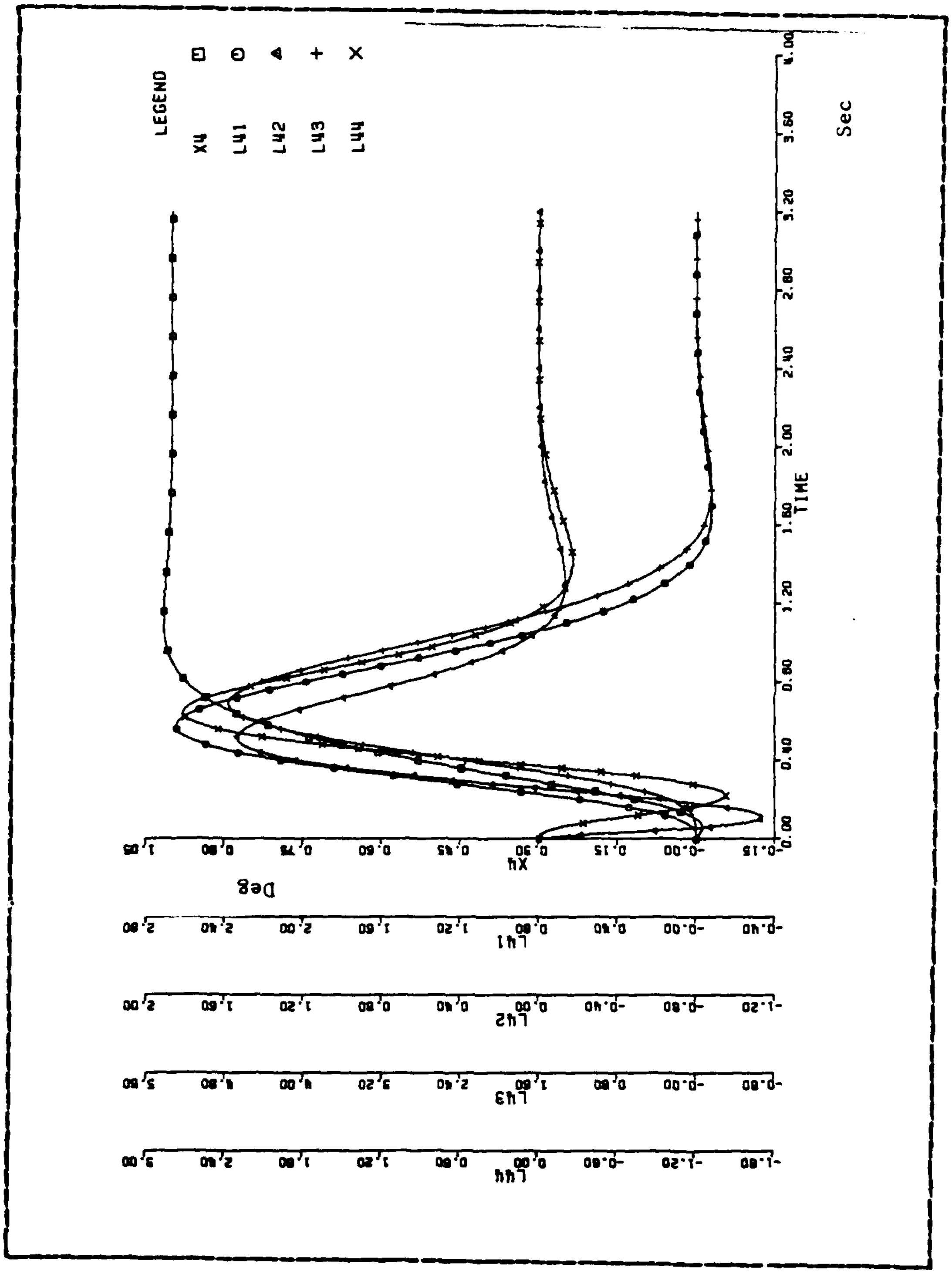

Figure 3.5 Sensitivity of X4 with Respect to A1,A2,A3,A4.

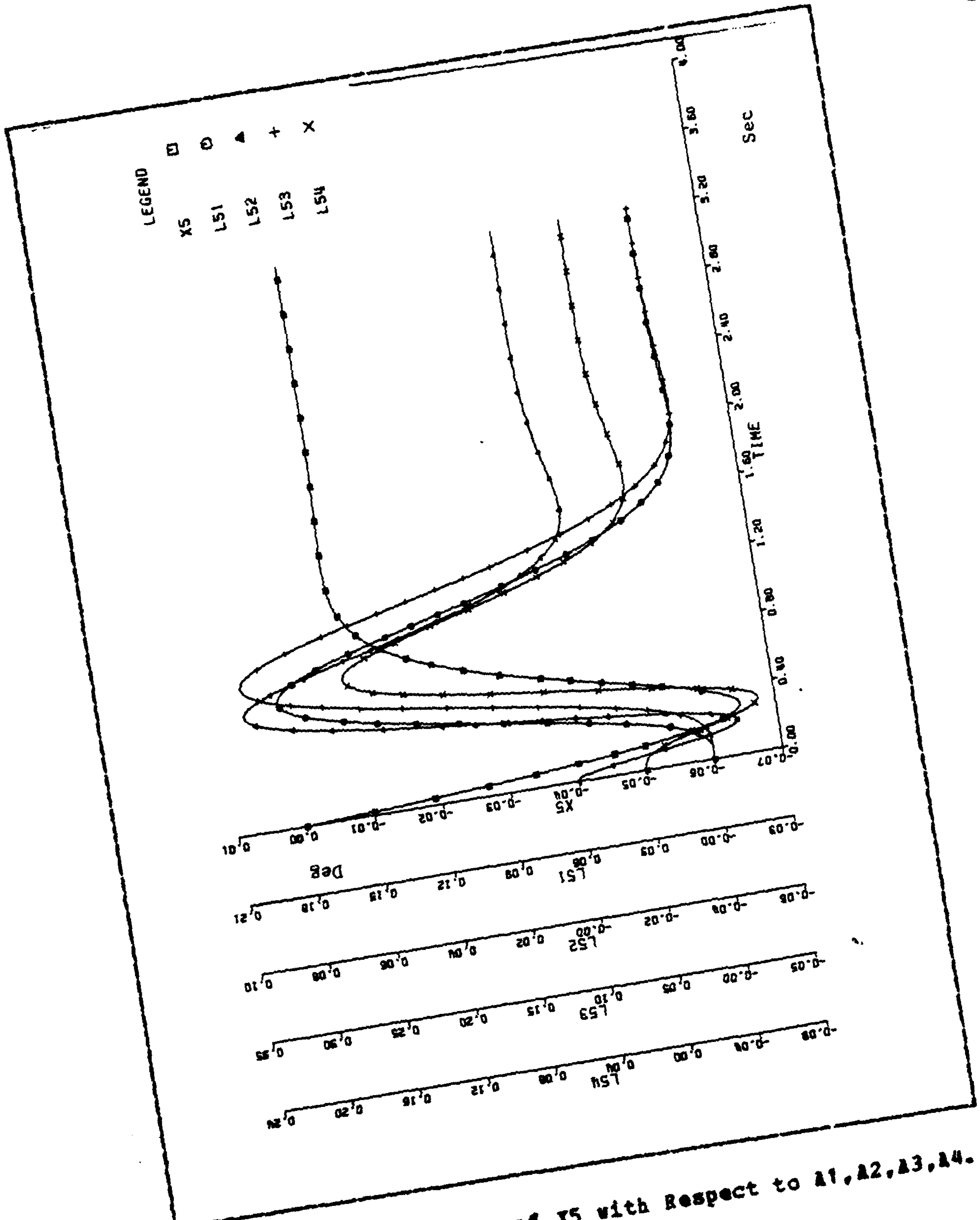

Figure 3.6 Sensitivity of X5 with Respect to A1,A2,A3,A4.

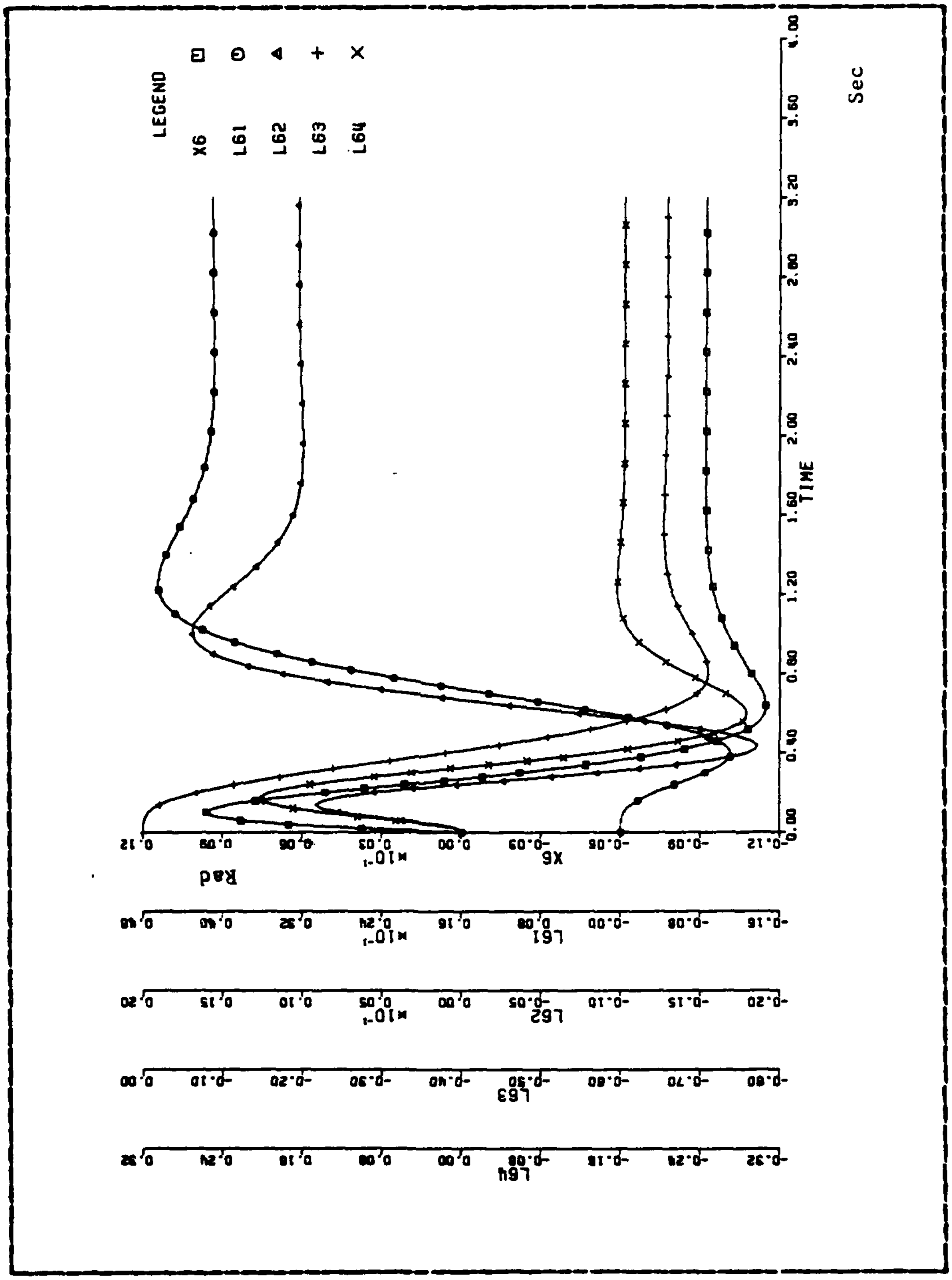

Figure 3.7 Sensitivity of X6 with Respect to λ1,λ2,λ3,λ4.

TABLE I

Influence of Parameters

X_1	λ_{11}	λ_{12}	λ_{13}	λ_{14}
RISE TIME	SE	SE	SE	SE
OVERSHOOT	SE	SE	SE	SE
STEADY STATE	NE	NE	NE	NE

X_2	λ_{21}	λ_{22}	λ_{23}	λ_{24}
RISE TIME	LE	LE	SE	SE
OVERSHOOT	LE	LE	LE	LE
STEADY STATE	LE	LE	LE	LE

X_3	λ_{31}	λ_{32}	λ_{33}	λ_{34}
RISE TIME	LE	LE	LE	LE
OVERSHOOT	LE	LE	SE	SE
STEADY STATE	LE	LE	SE	SE

X_4	λ_{41}	λ_{42}	λ_{43}	λ_{44}
RISE TIME	LE	LE	SE	SE
OVERSHOOT	LE	LE	SE	SE
STEADY STATE	NE	NE	NE	NE

X_5	λ_{51}	λ_{52}	λ_{53}	λ_{54}
RISE TIME	LE	LE	LE	LE
OVERSHOOT	LE	LE	SE	SE
STEADY STATE	NE	NE	NE	NE

X_6	λ_{61}	λ_{62}	λ_{63}	λ_{64}
RISE TIME	LE	LE	LE	LE
OVERSHOOT	LE	LE	SE	SE
STEADY STATE	LE	LE	SE	SE

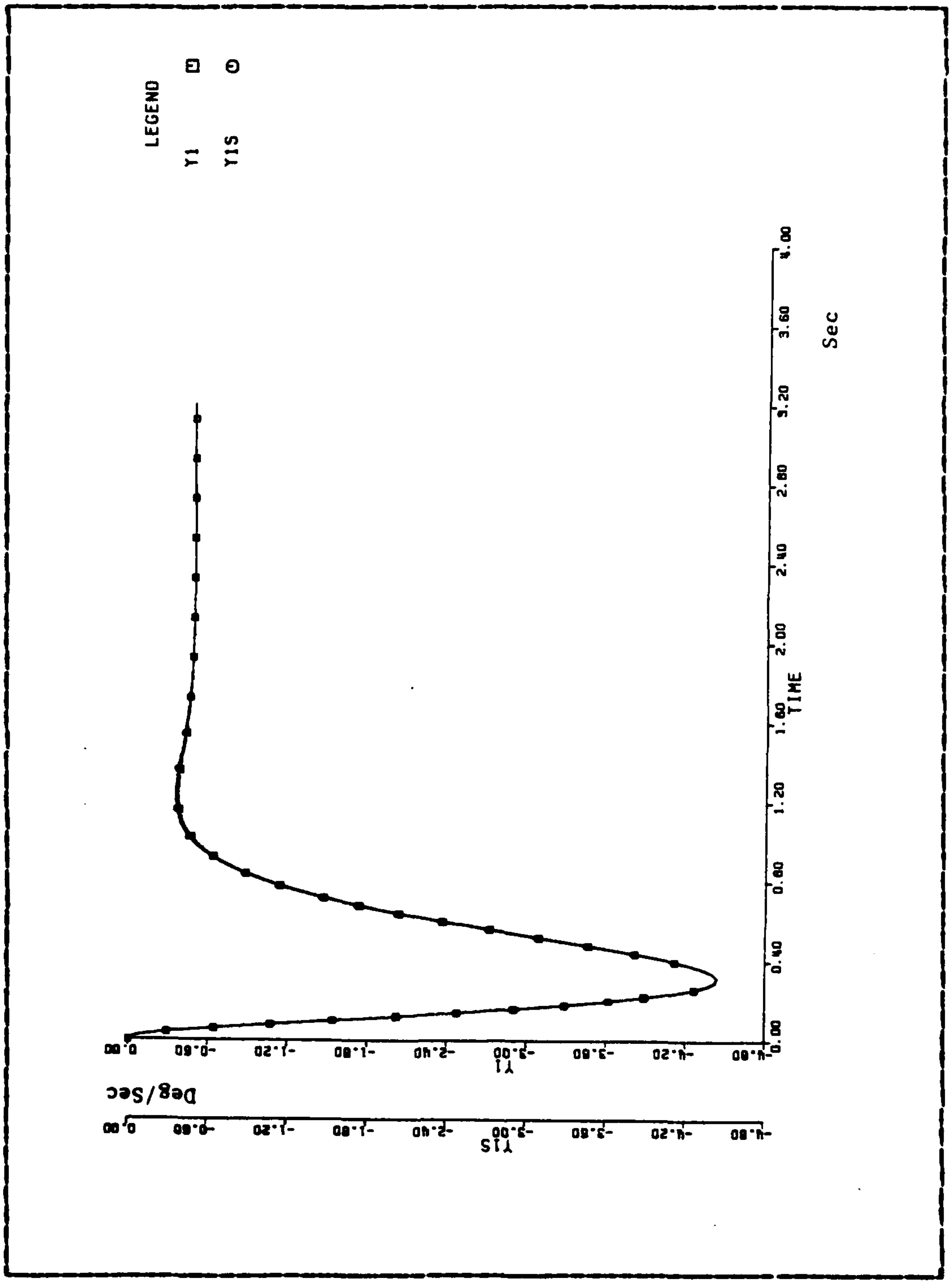

Figure 3.8 Actual and Nominal Output of X1 (10% variation).

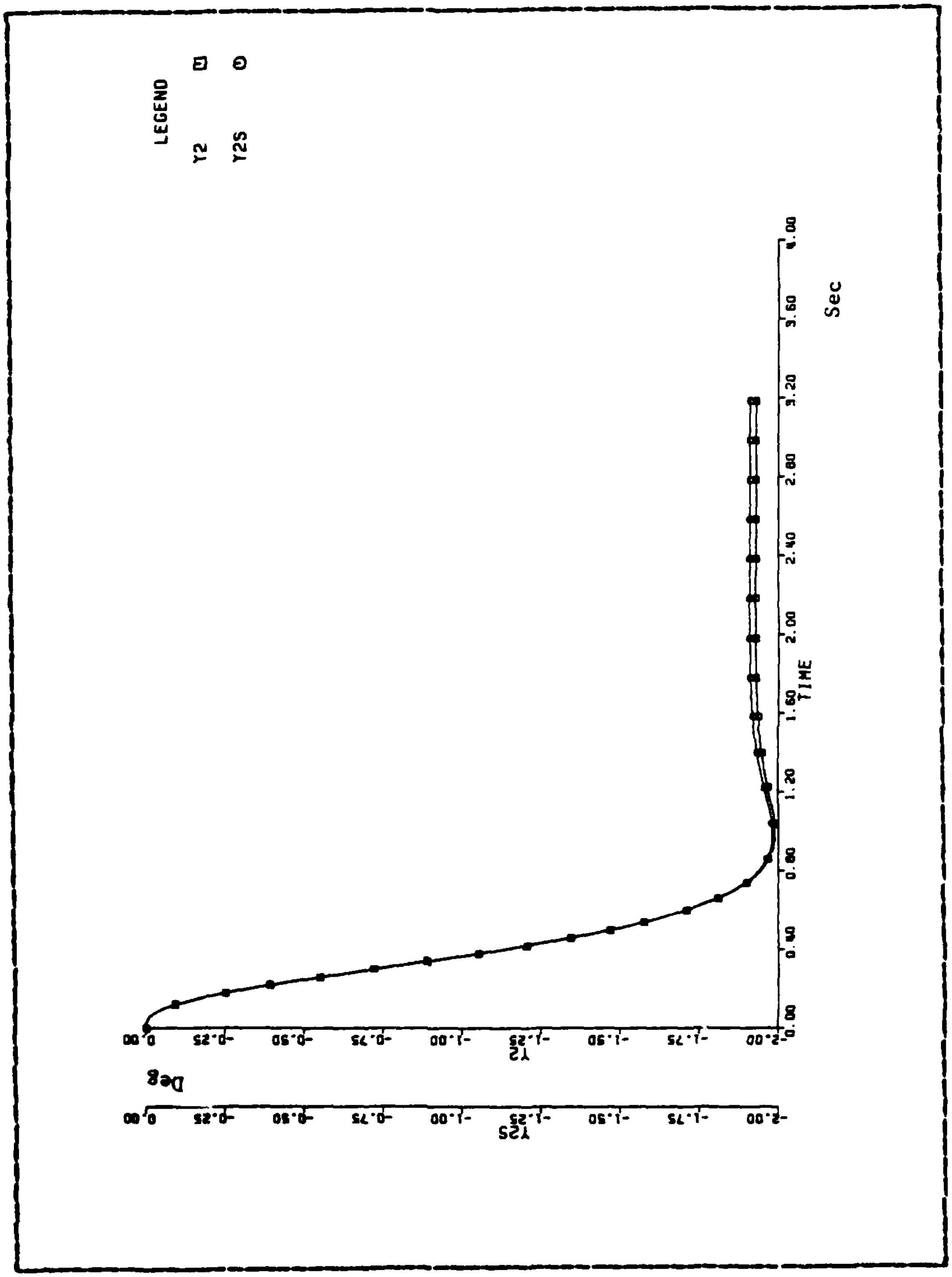

Figure 3.9 Actual and Nominal Output of X2 (10% variation).

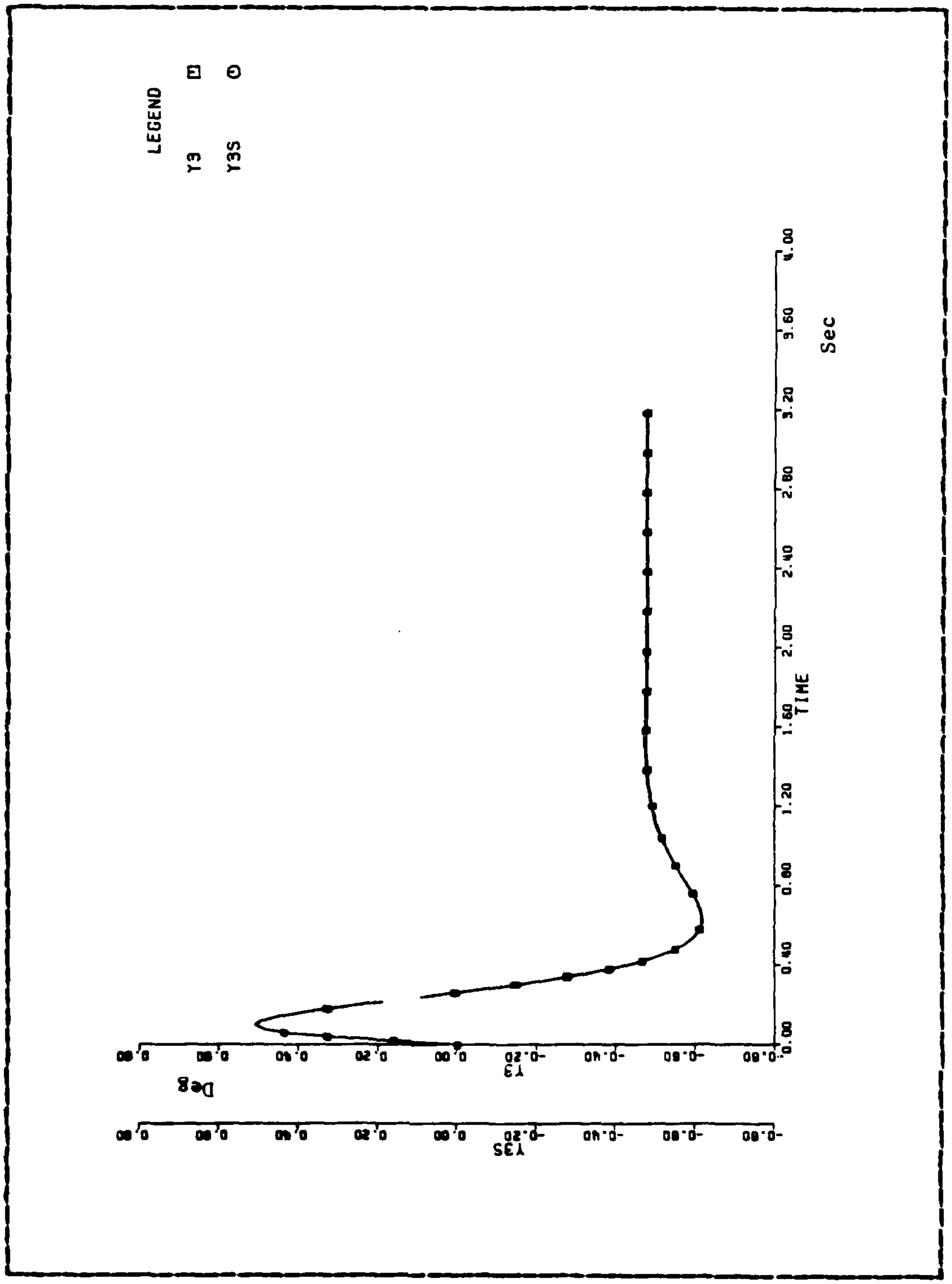

Figure 3.10 Actual and Nominal Output of X3 (10% variation).

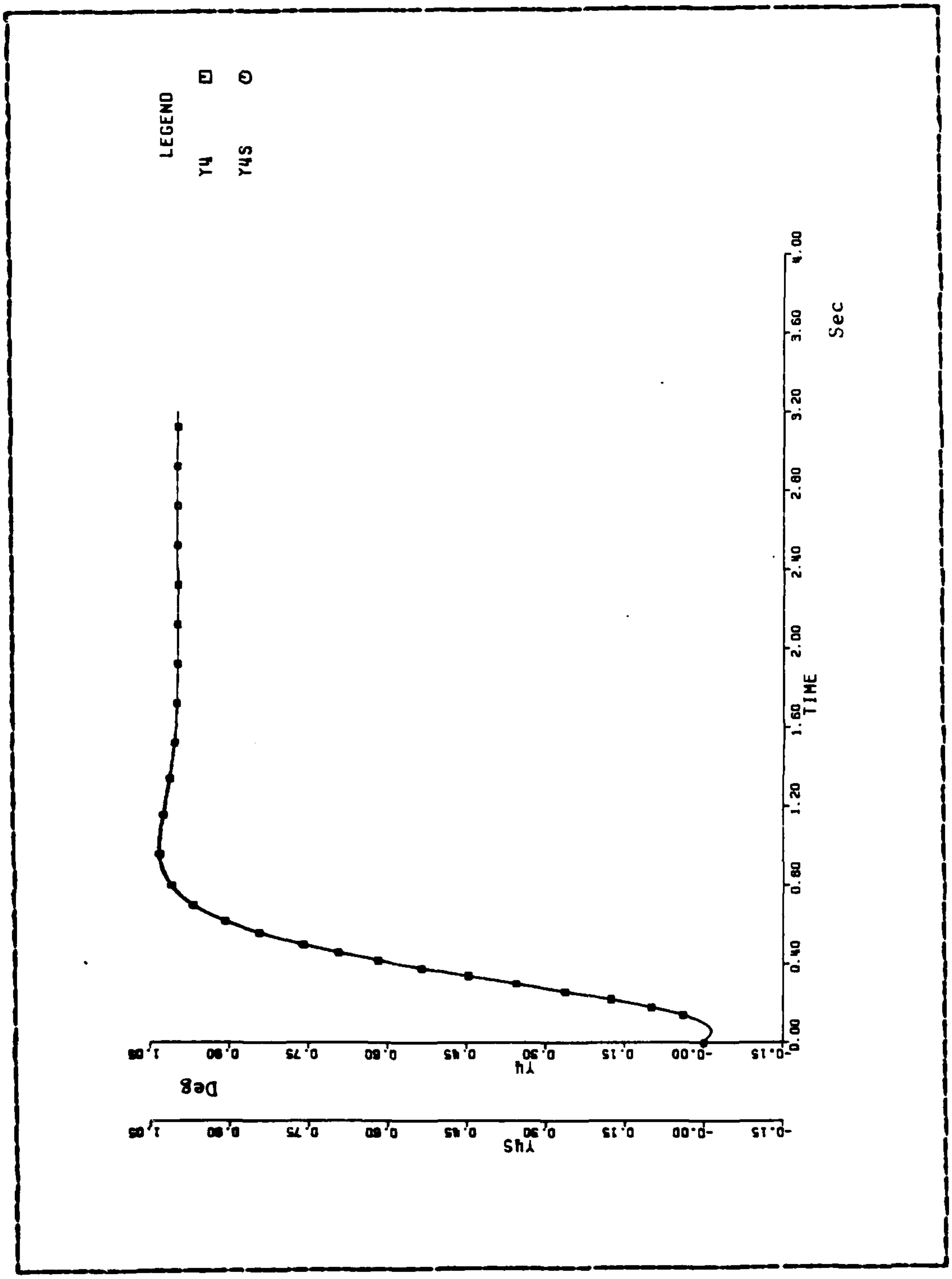

Figure 3.11 Actual and Nominal Output of X4 (10% variation).

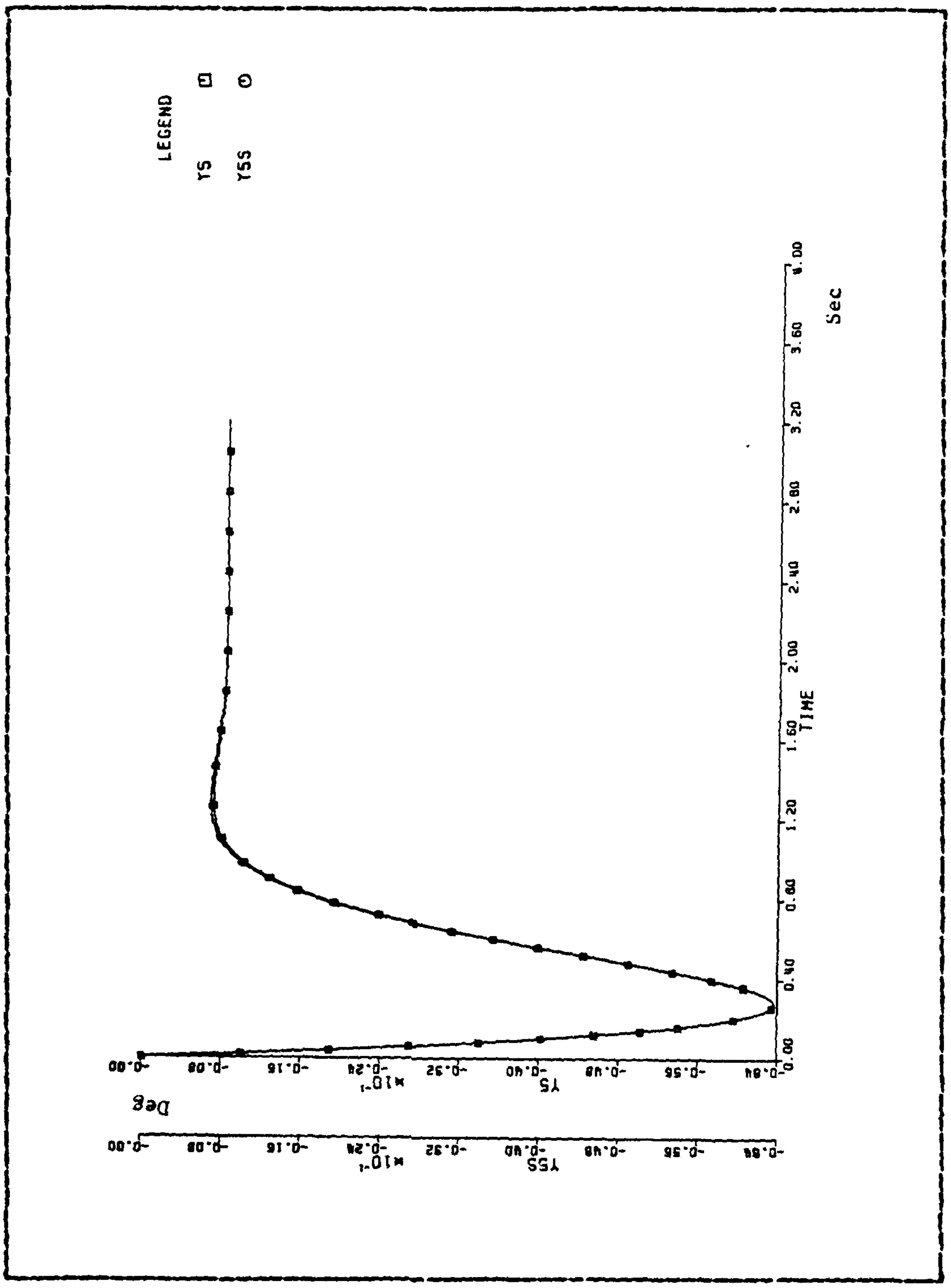

Figure 3.12 Actual and Nominal Output of X5 (10% variation).

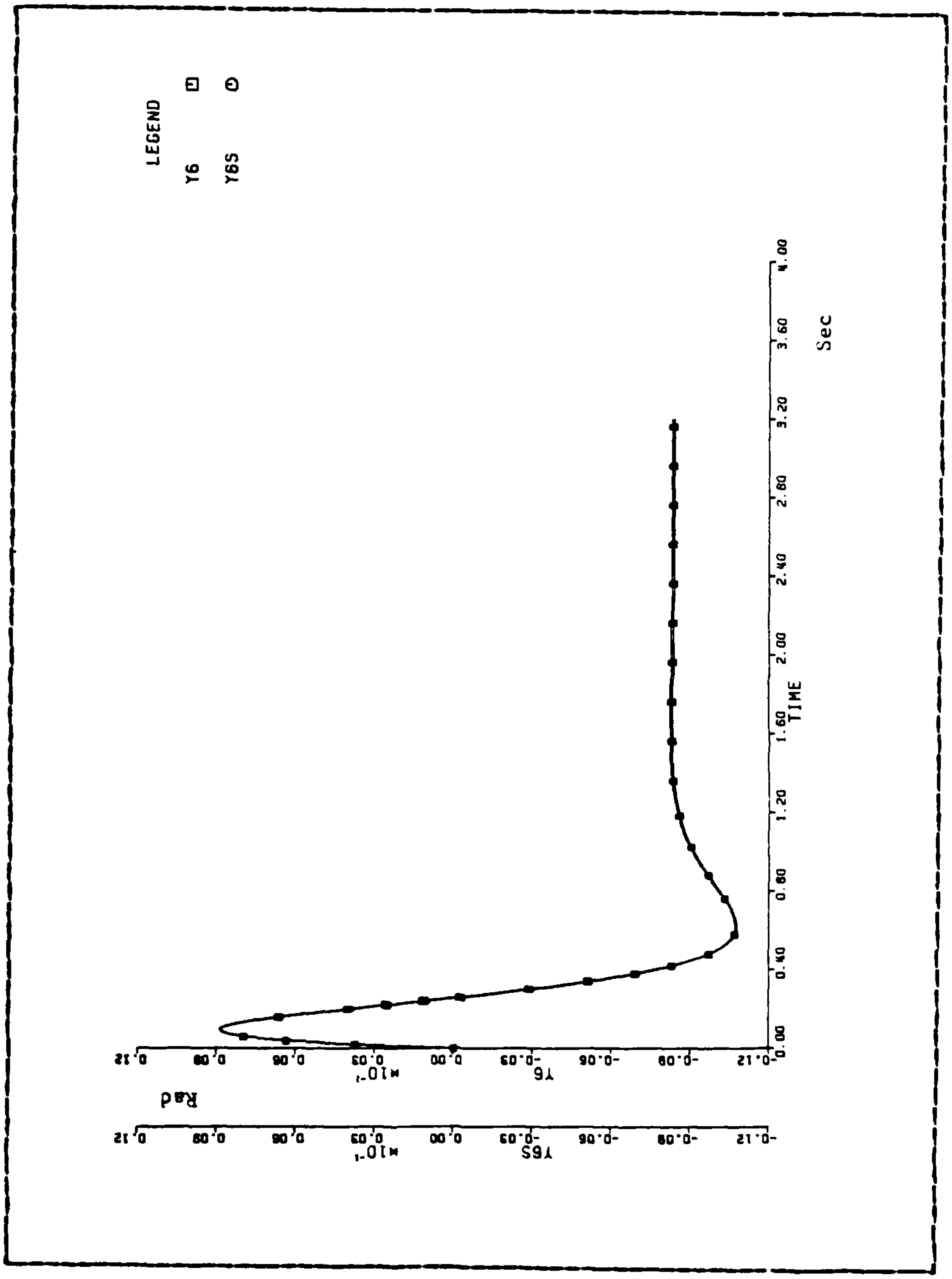

Figure 3.13 Actual and Nominal Output of X6 (10% variation).

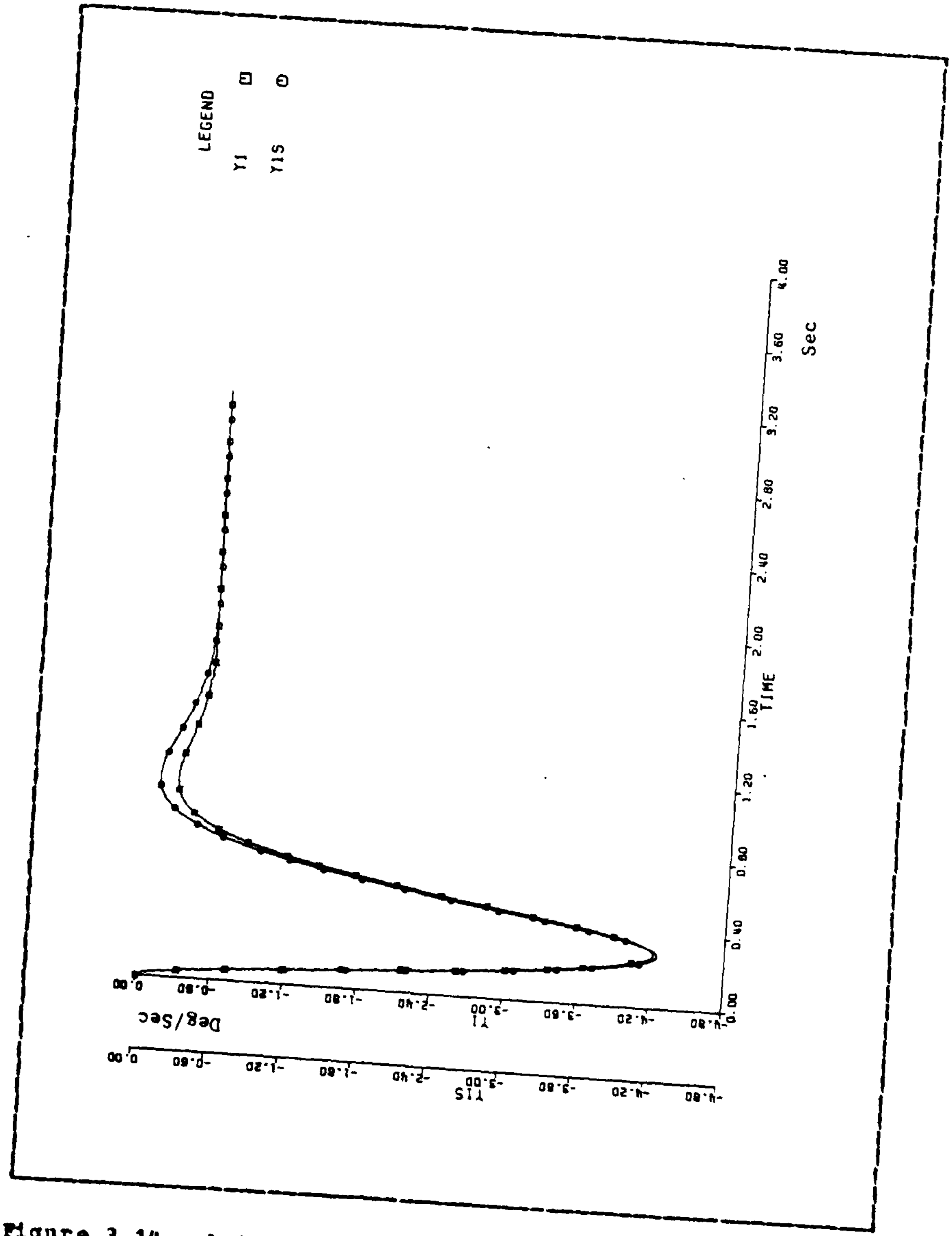

Figure 3.14 Actual and Nominal Output of X1 (30% variation).

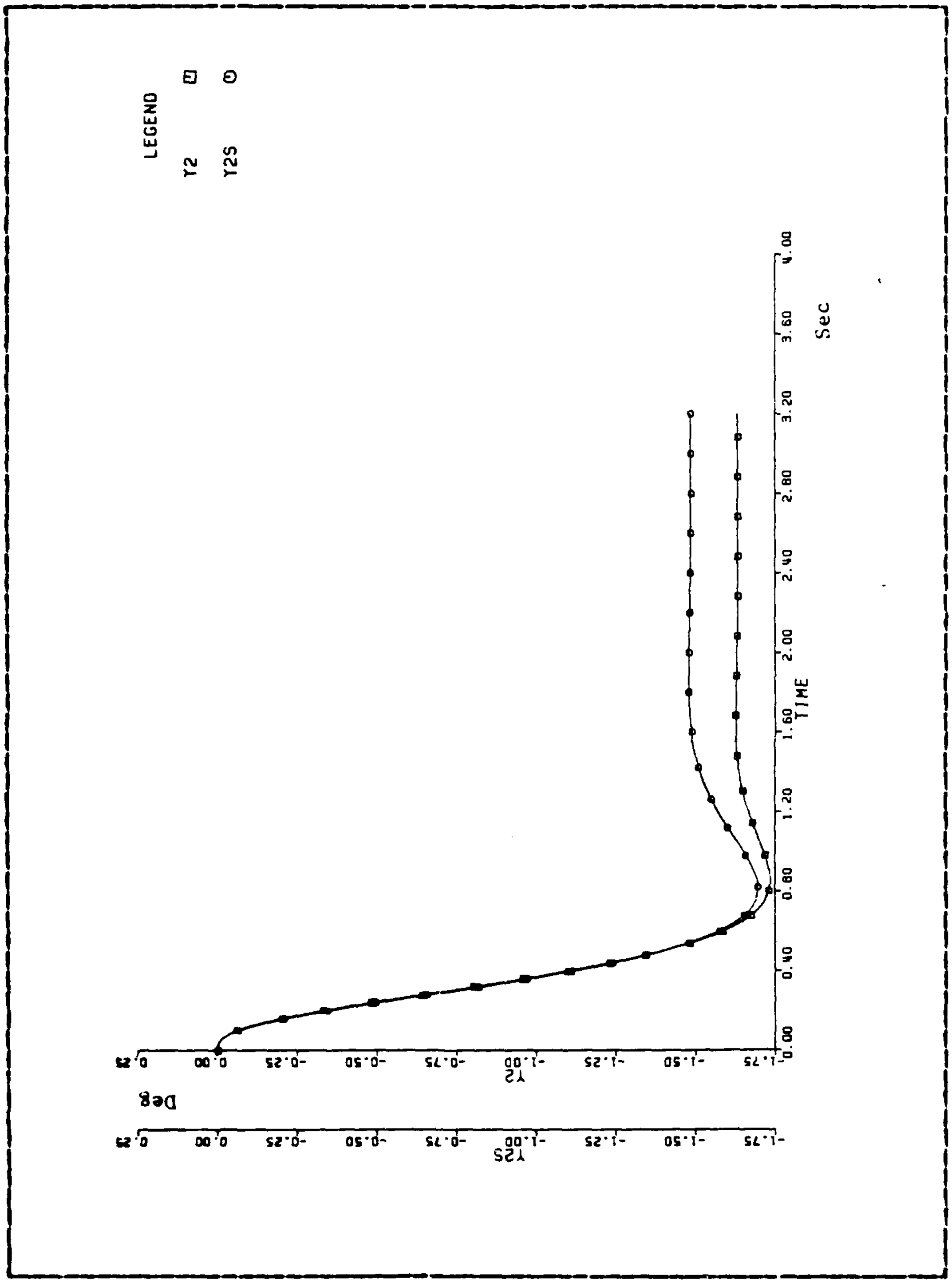

Figure 3.15   Actual and Nominal Output of Y2 (30% variation).

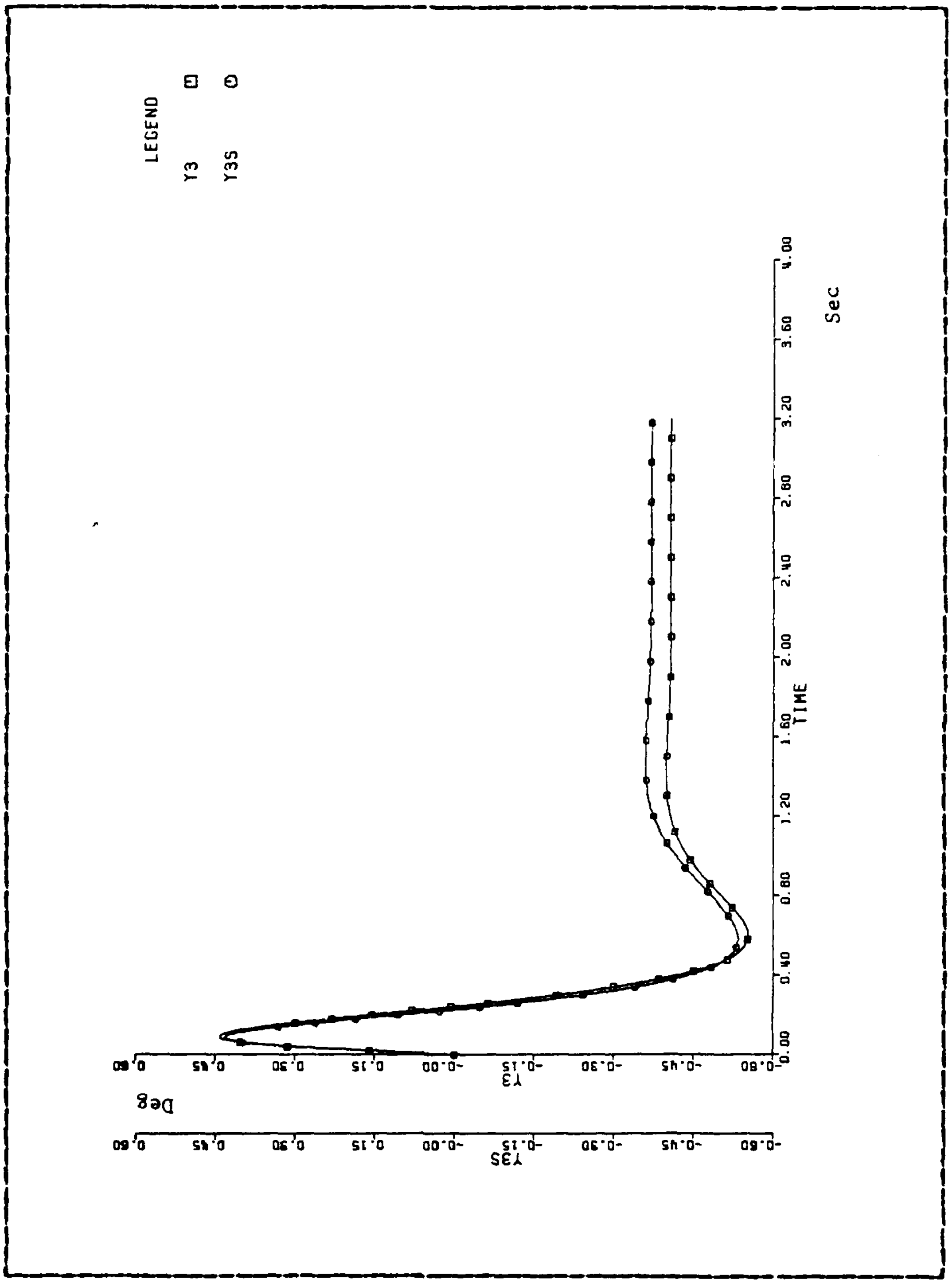

Figure 3.16   Actual and Nominal Output of X3 (30% variation).

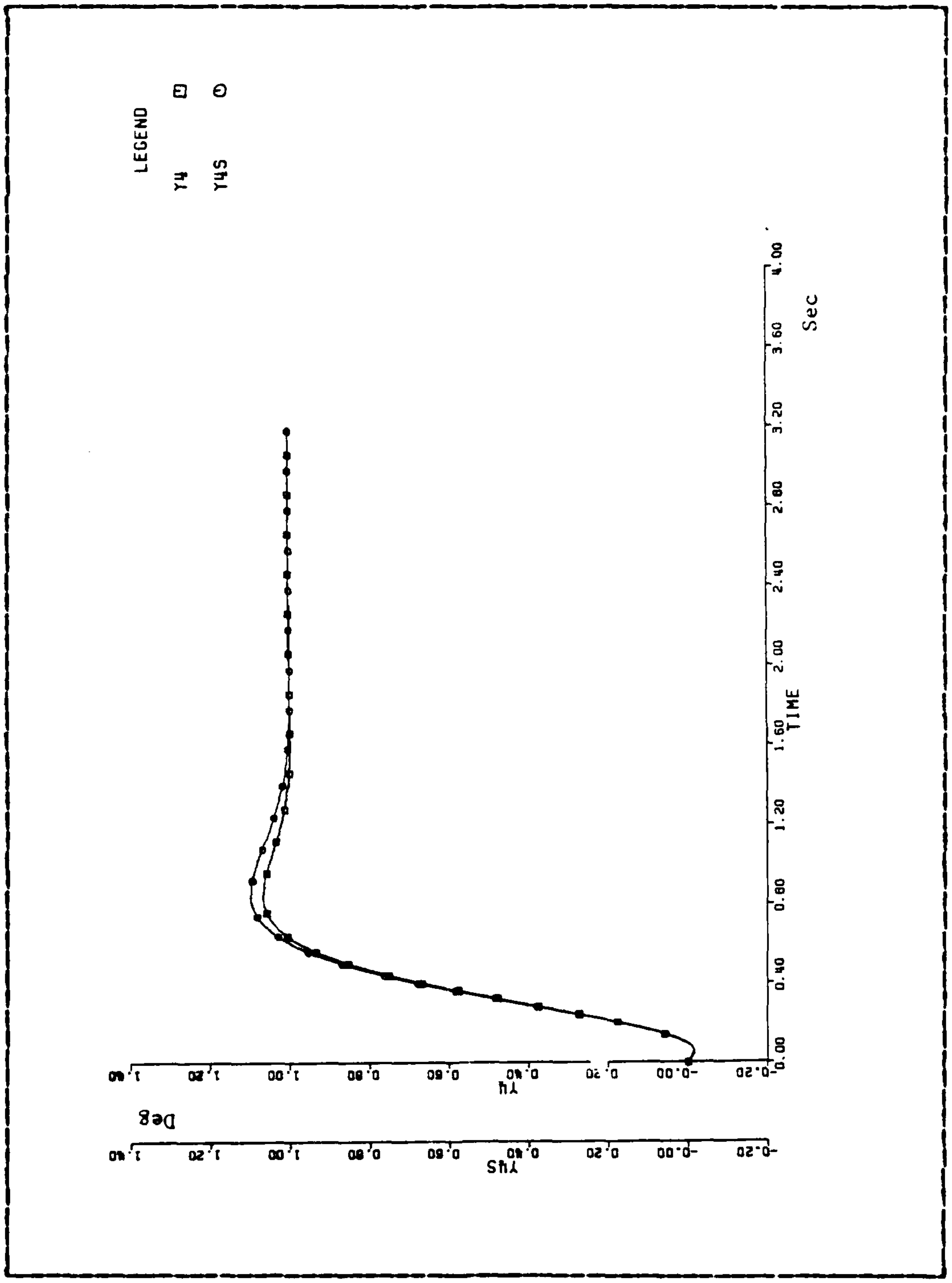

Figure 3.17   Actual and Nominal Output of X4 (30% variation).

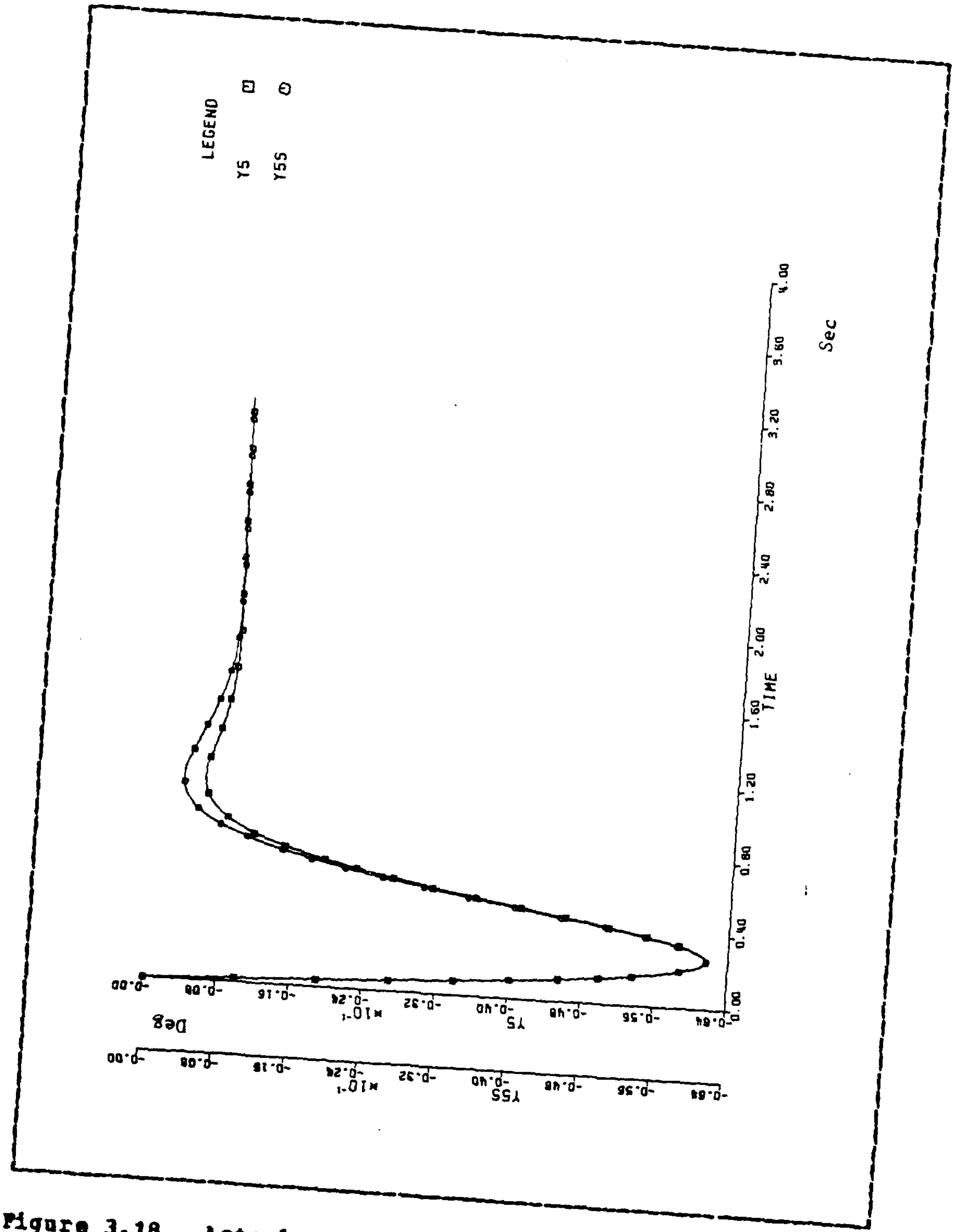

Figure 3.18 Actual and Nominal Output of Y5 (30% variation).

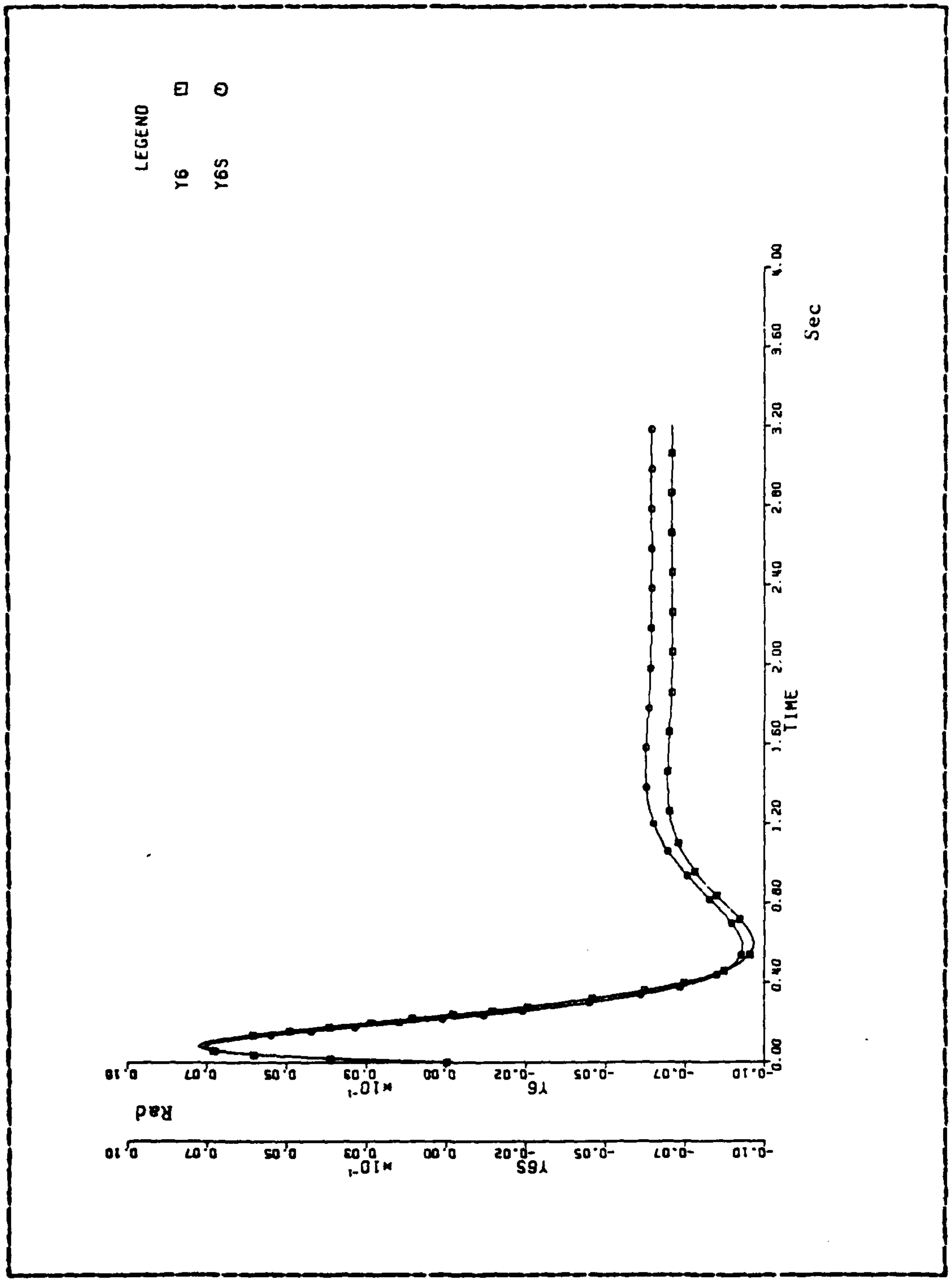

Figure 3.19   Actual and Nominal Output of X6 (30% variation).

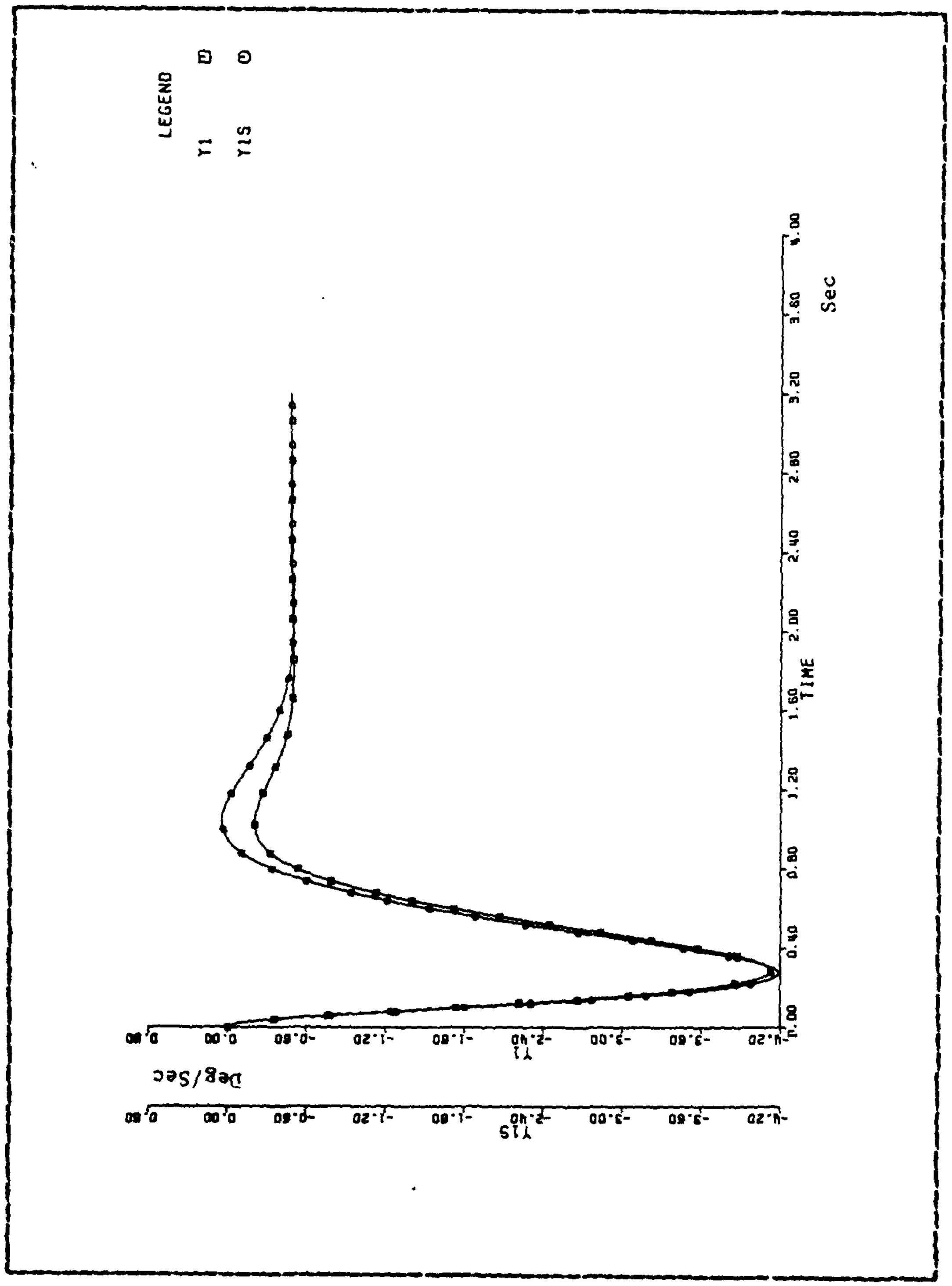

Figure 3.20 Actual and Nominal Output of X1 (40% variation).

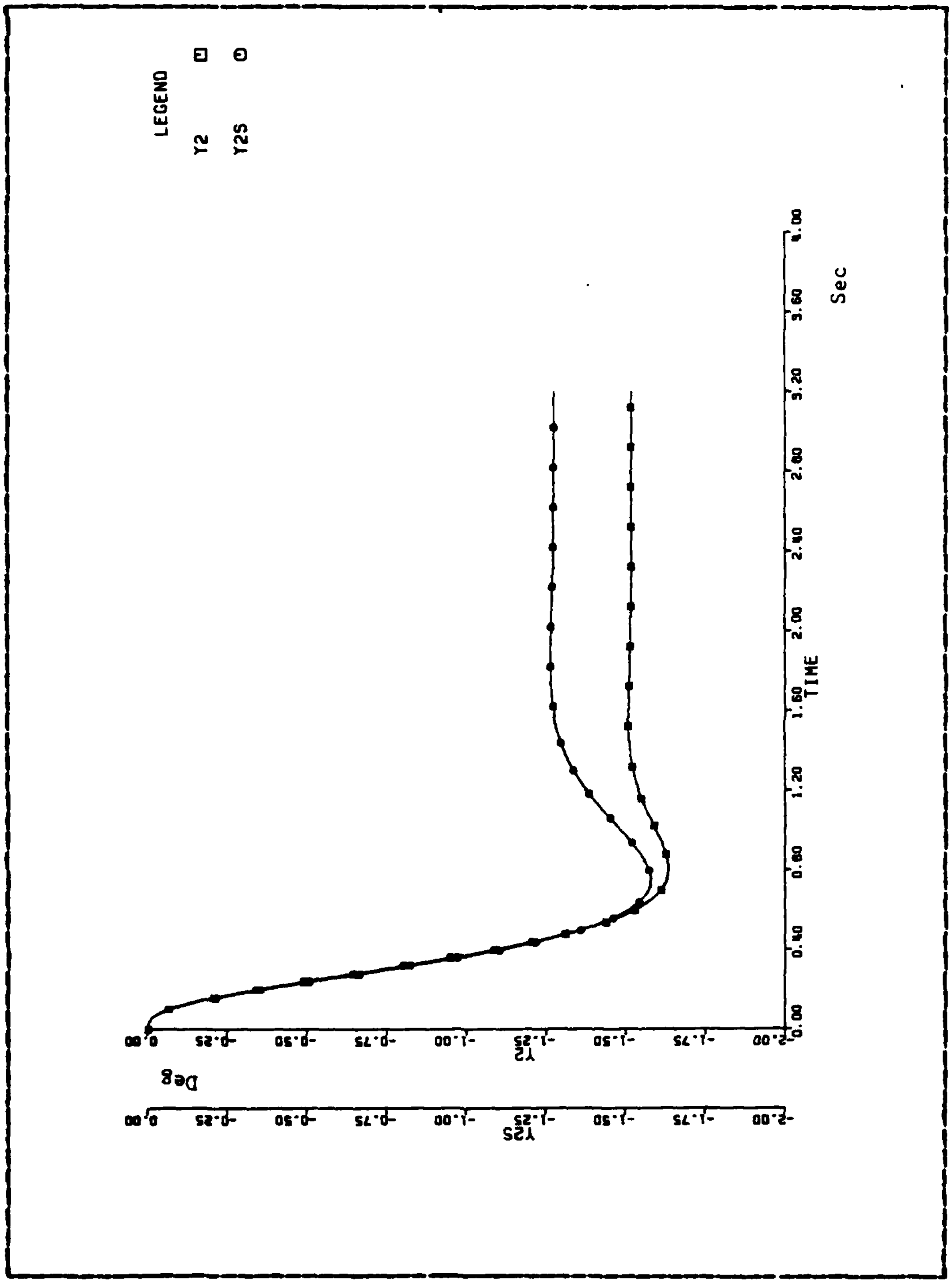

Figure 3.21 Actual and Nominal Output of X2 (40% variation).

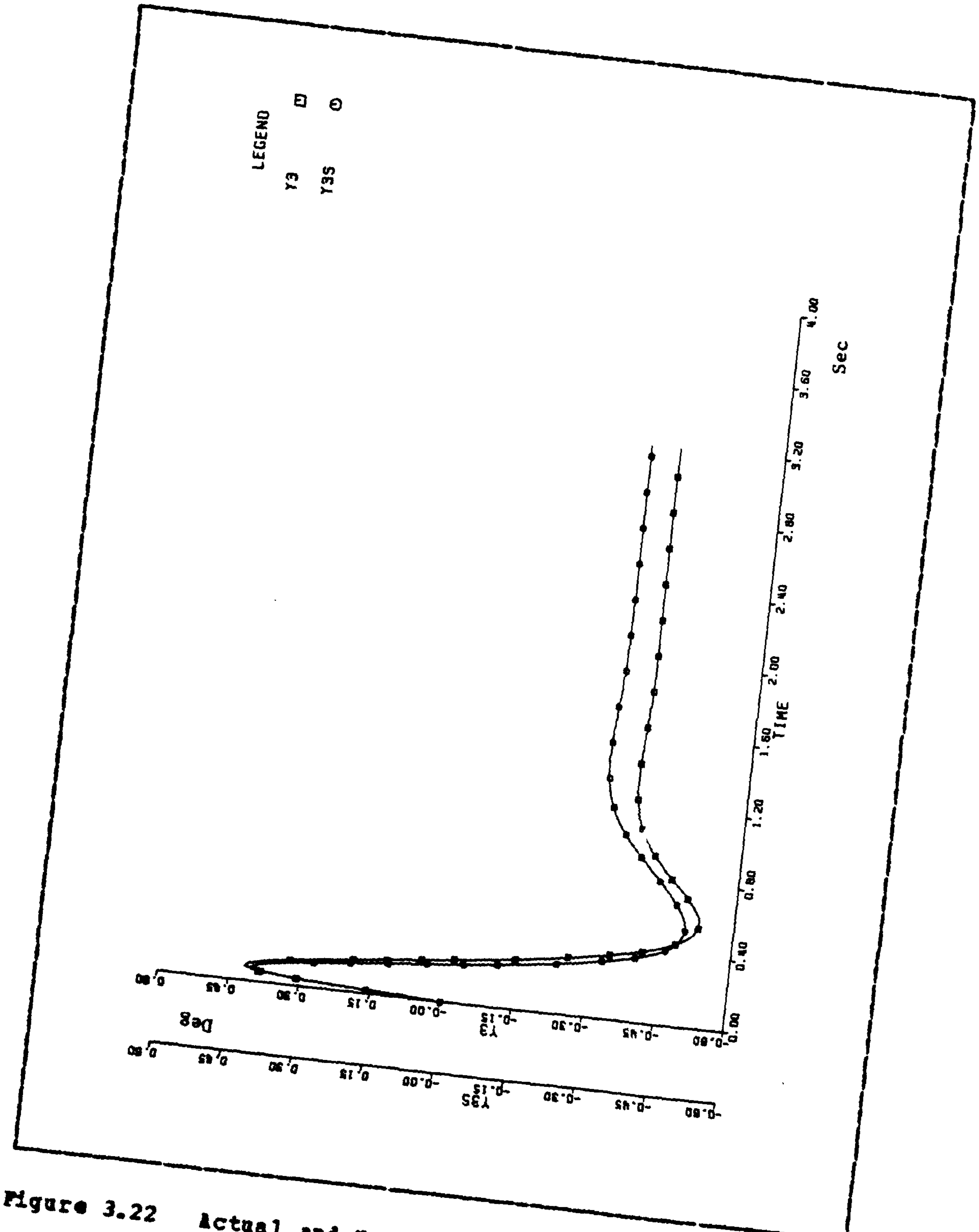

Figure 3.22 Actual and Nominal Output of X3 (40% variation).

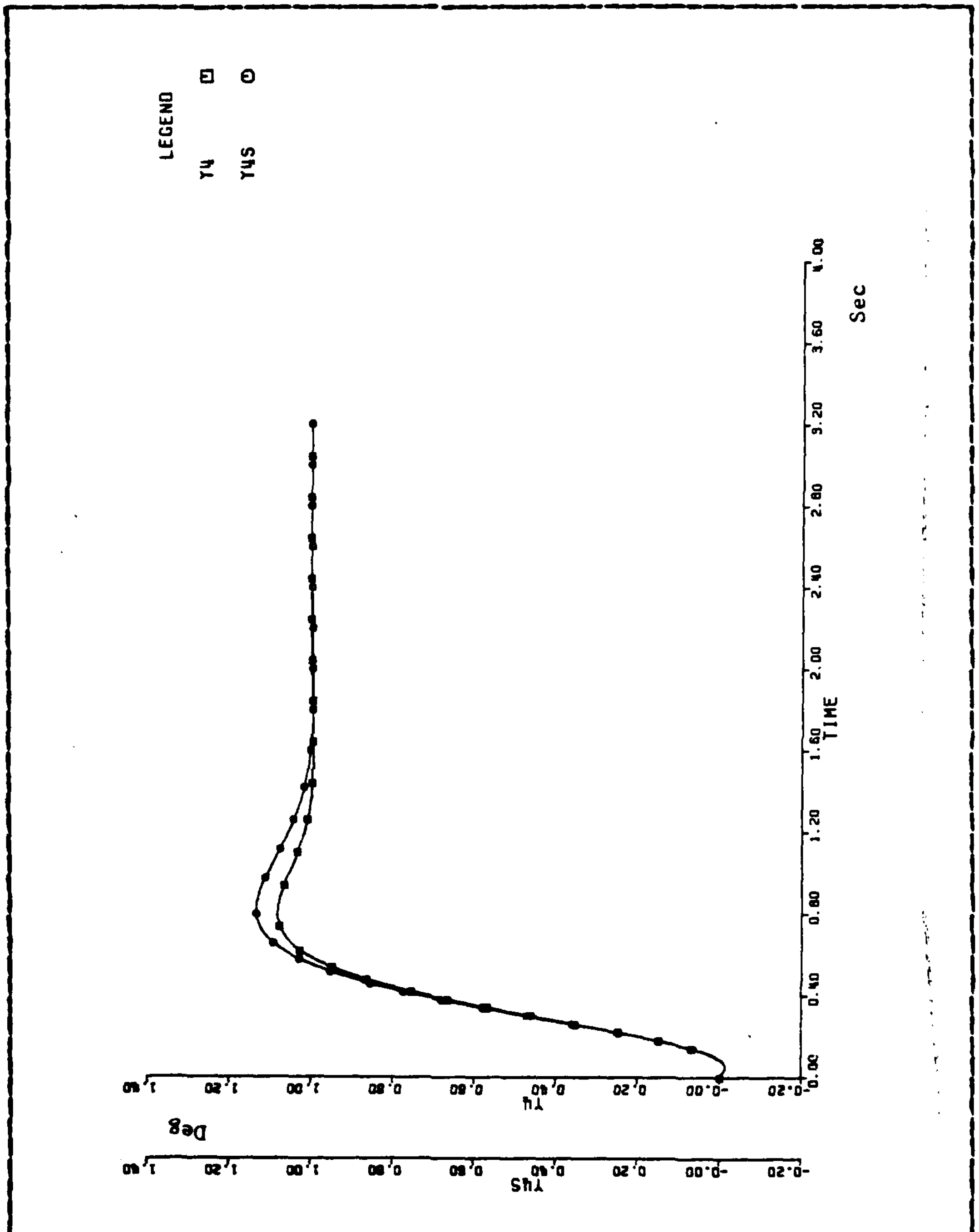

Figure 3.23 Actual and Nominal Output of X4 (40% variation).

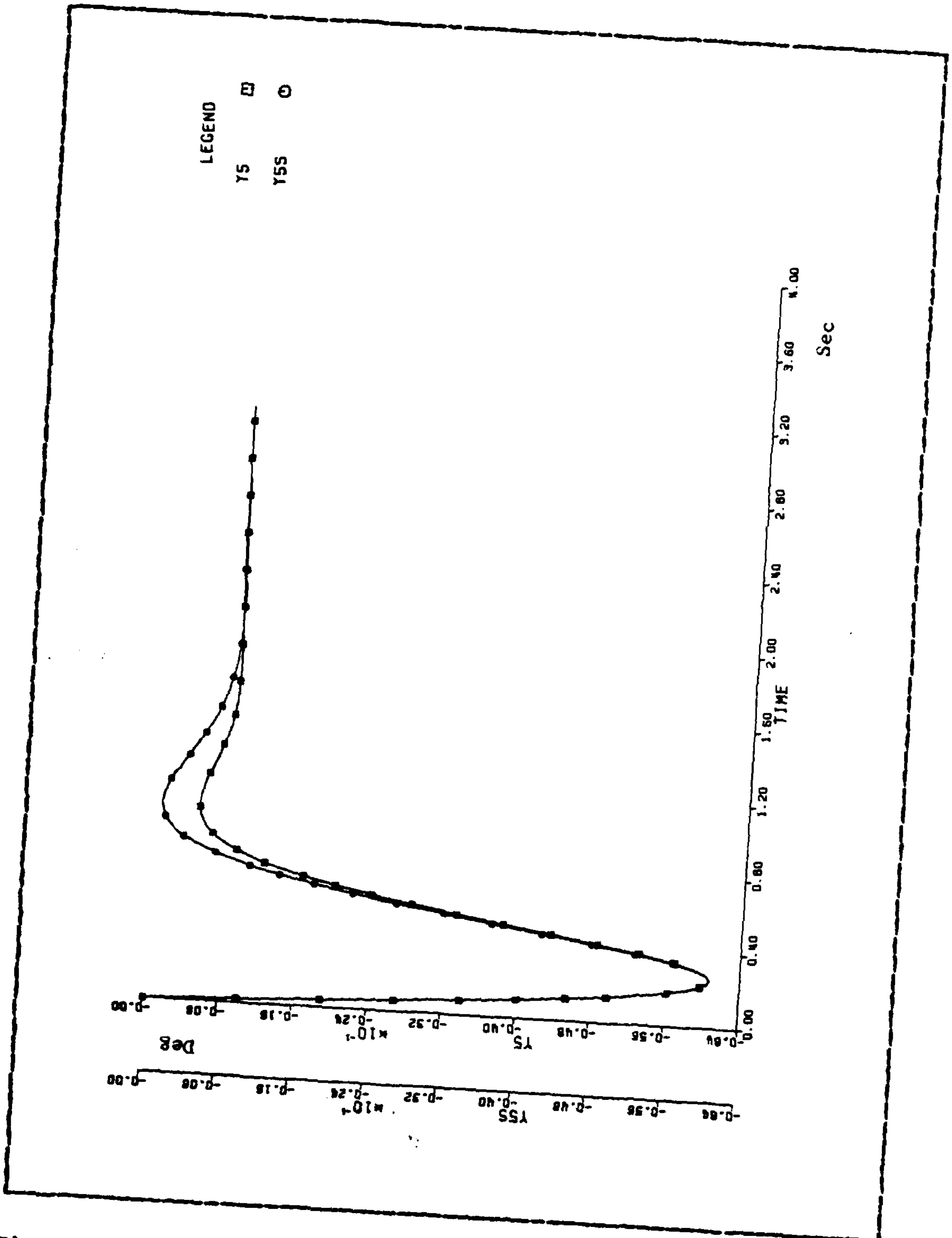

Figure 3.24 Actual and Nominal Output of X5 (40% variation).

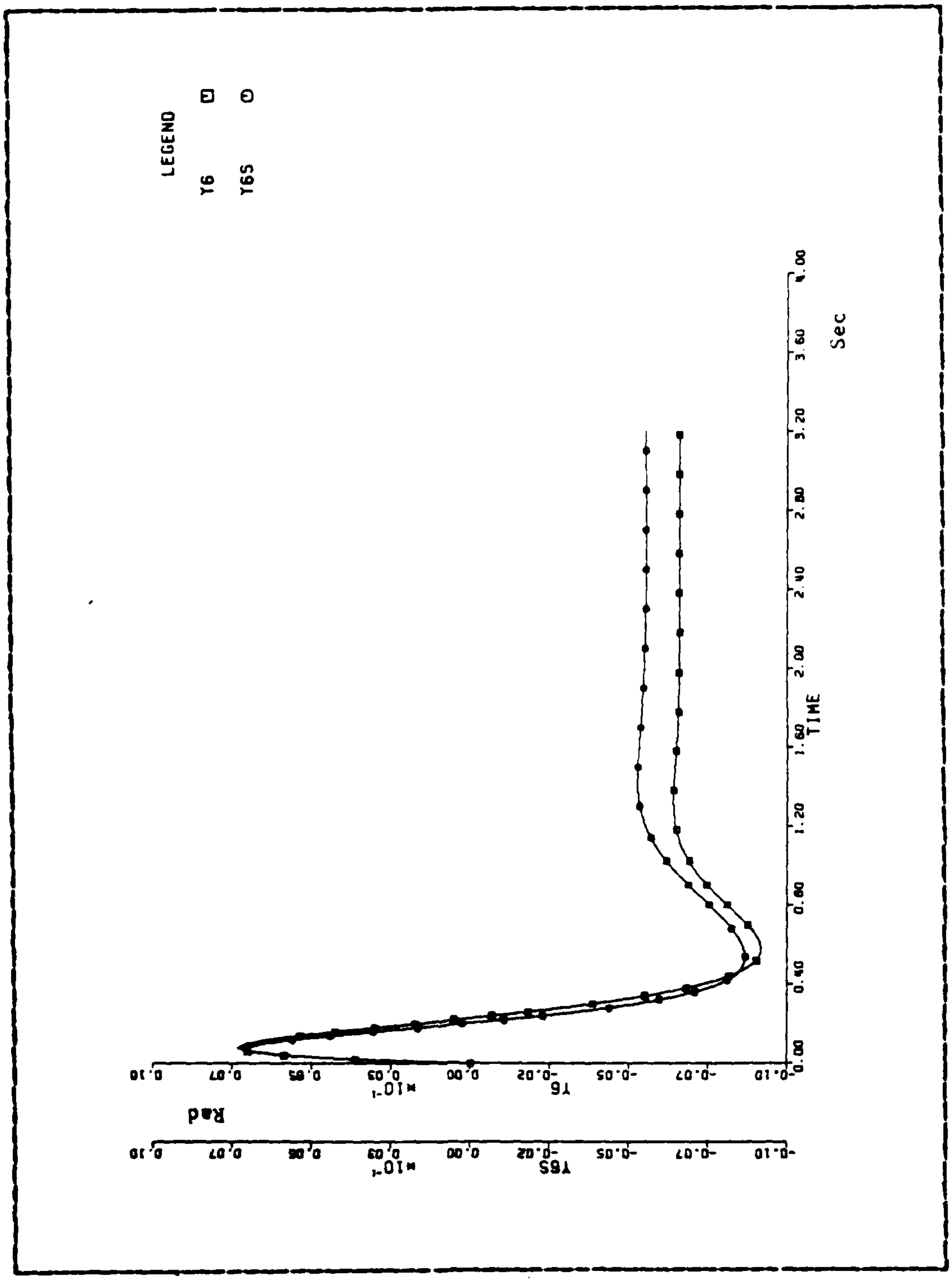

Figure 3.25 Actual and Nominal Output of X6 (40% variation).

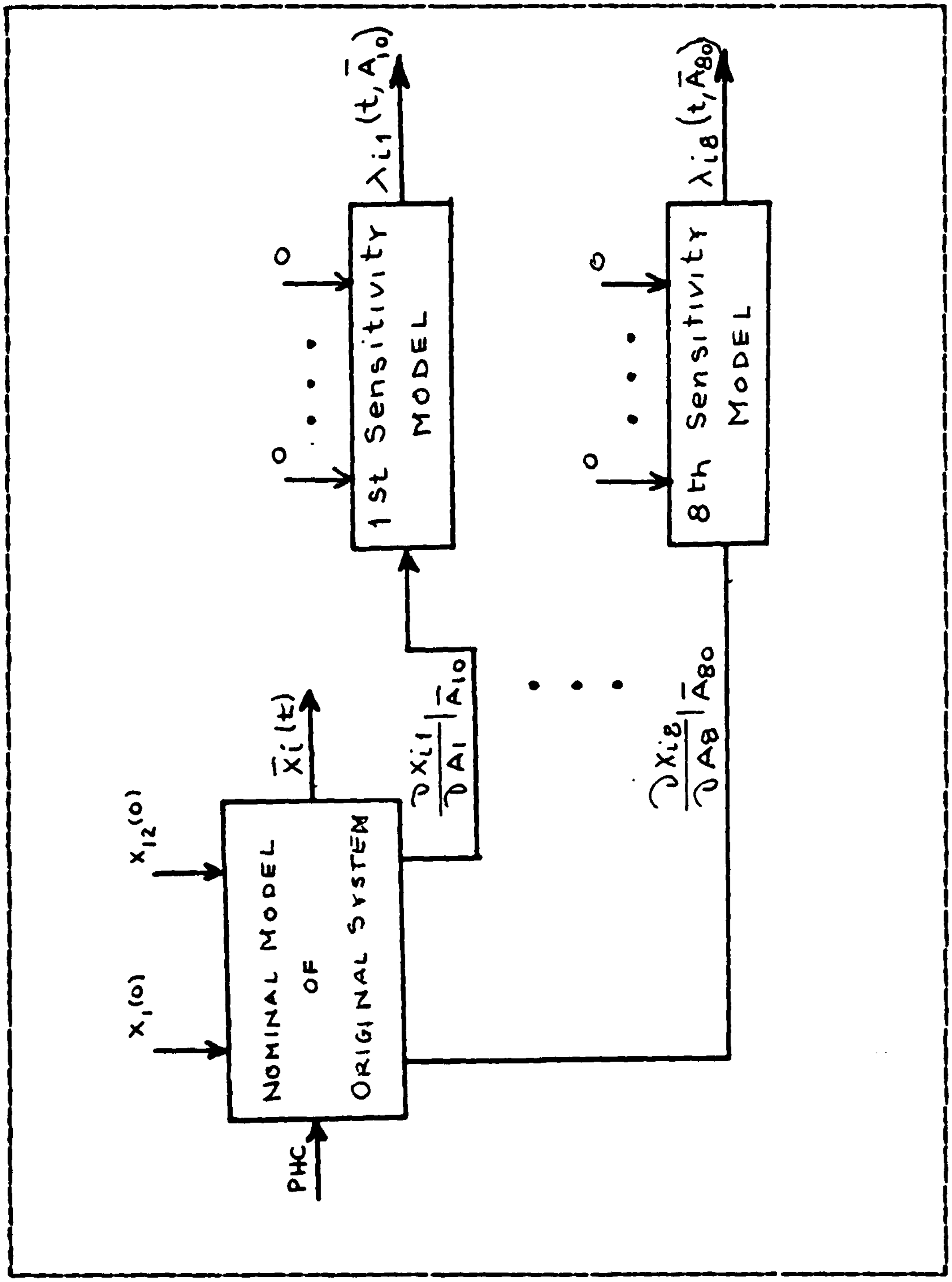

Figure 3.26 Roll-Yaw Nominal and Sensitivity Models.

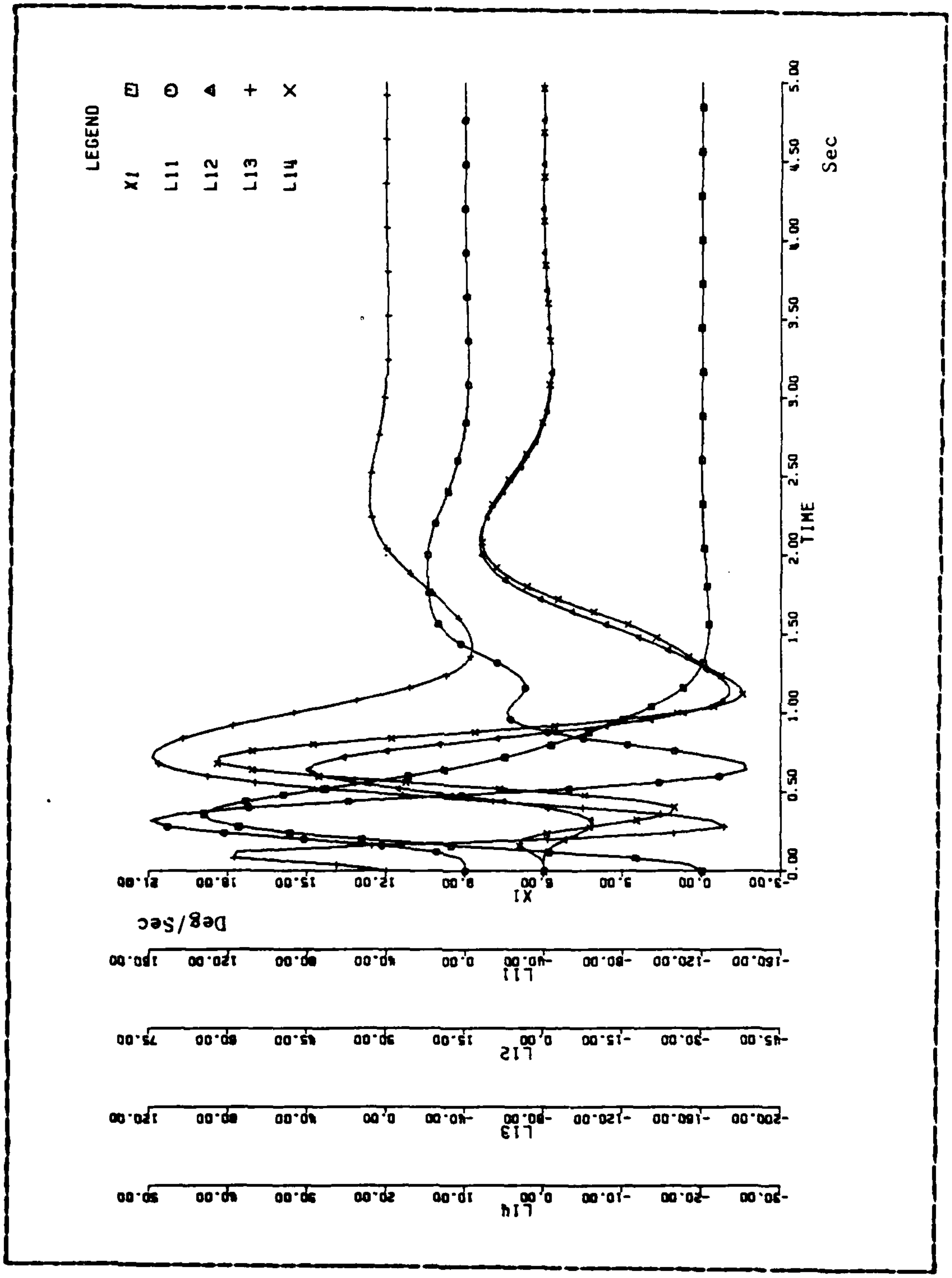

Figure 3.27 Sensitivity of X1 with Respect to $\lambda 1, \lambda 2, \lambda 3, \lambda 4$.

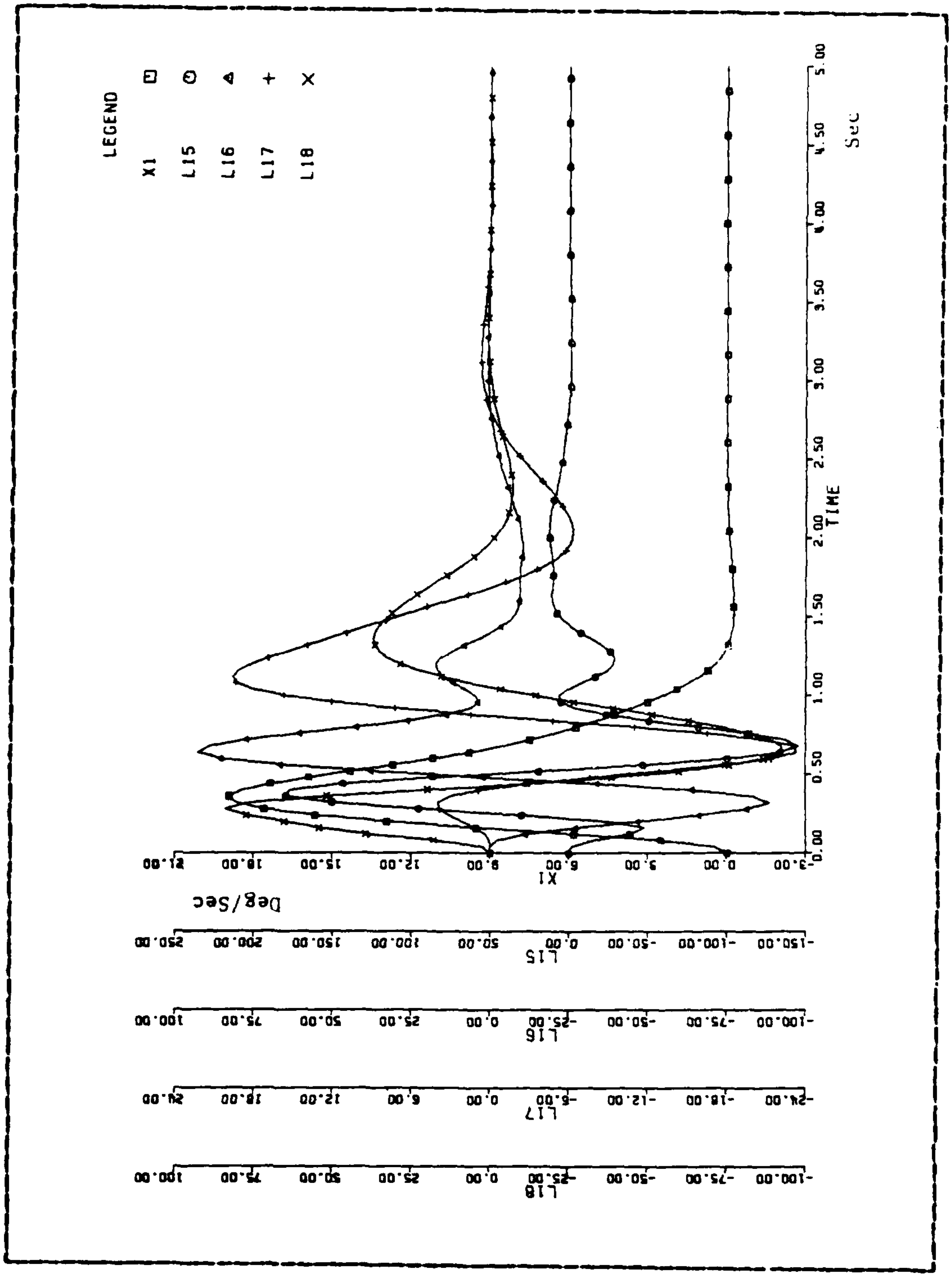

Figure 3.28 Sensitivity of X1 with Respect to A5,A6,A7,A8.

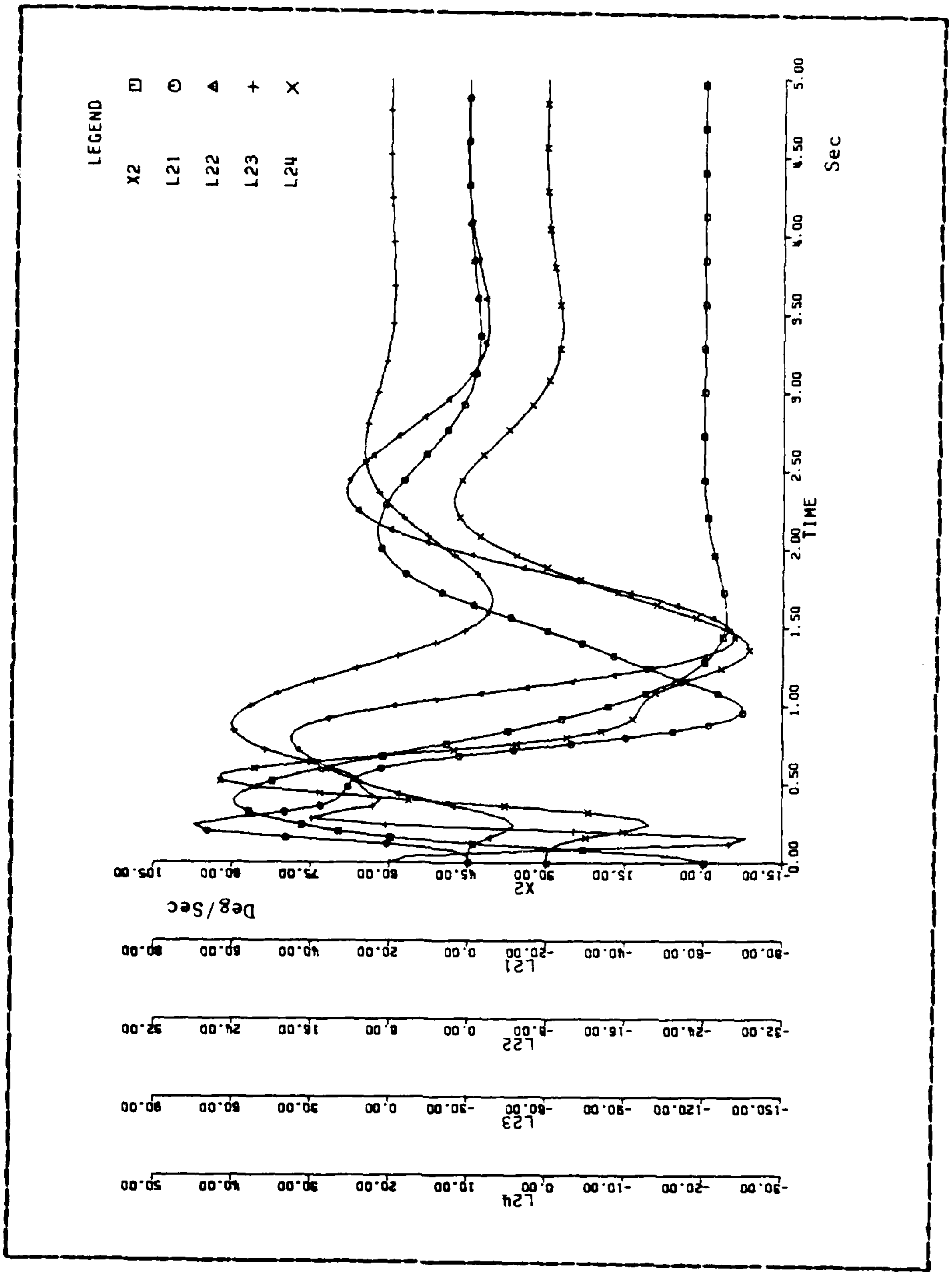

Figure 3.29 Sensitivity of X2 with Respect to A1,A2,A3,A4.

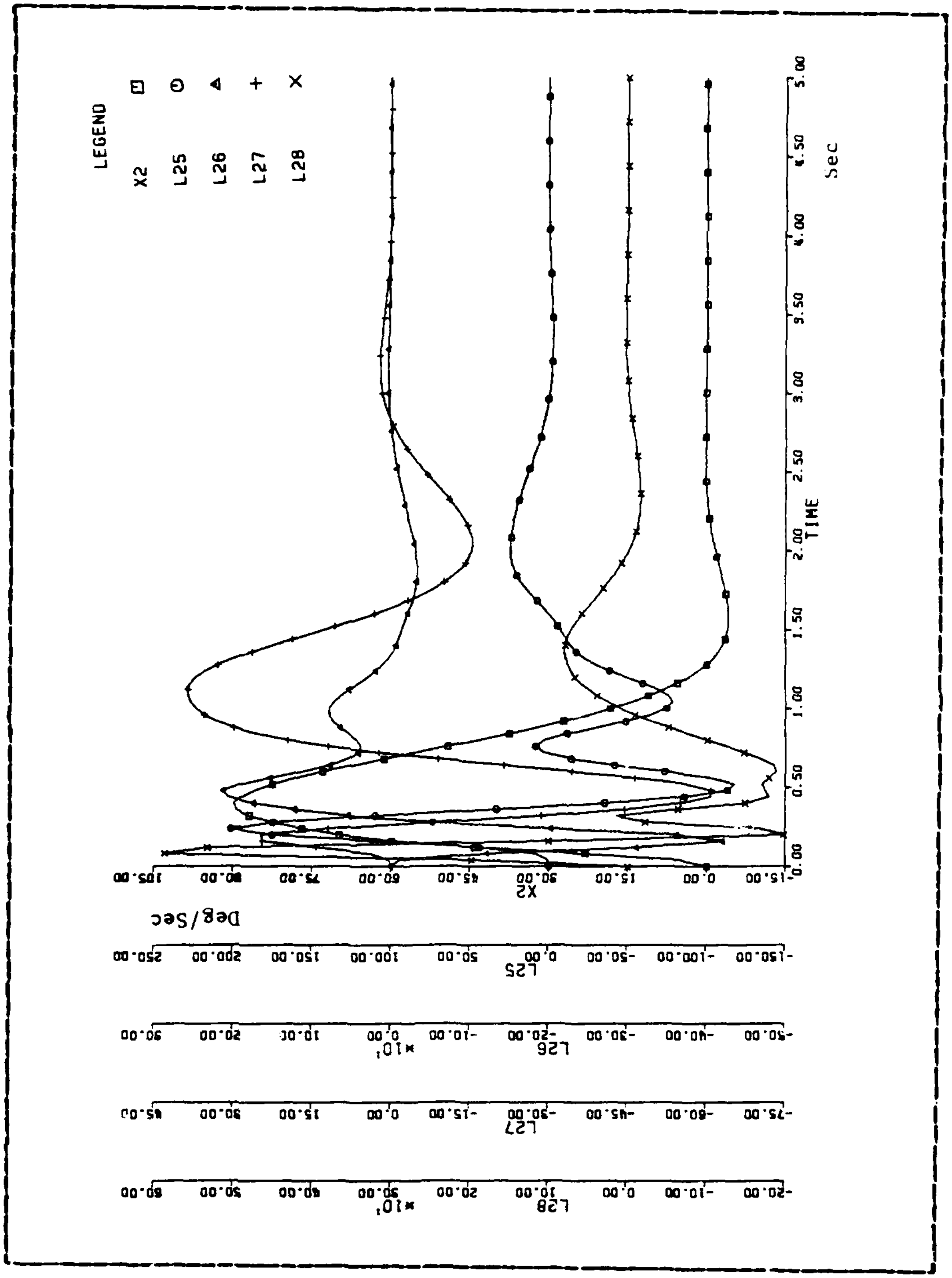

Figure 3.30 Sensitivity of X2 with Respect to A5,A6,A7,A8.

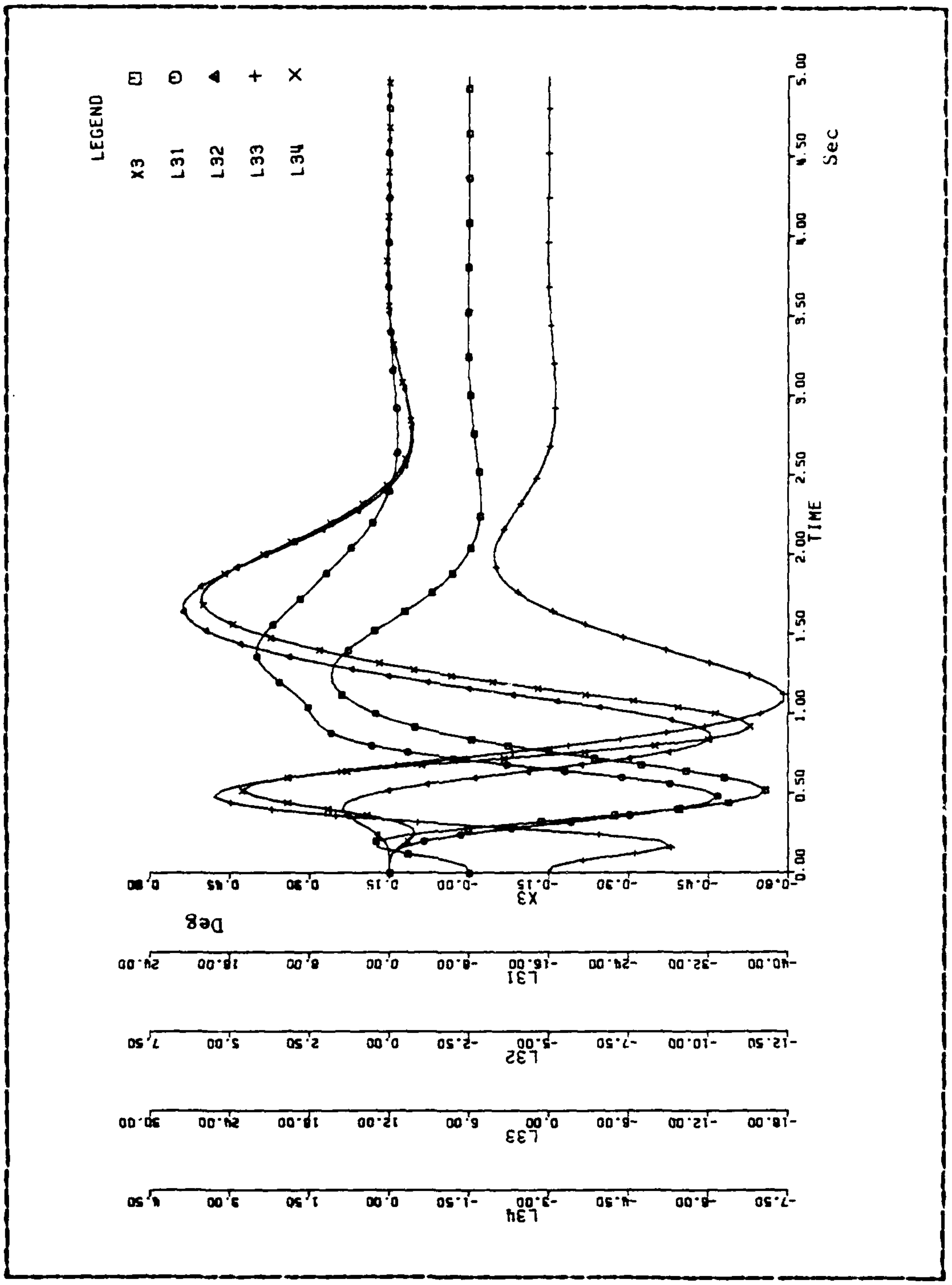

Figure 3.31 Sensitivity of X3 with Respect to A1,A2,A3,A4.

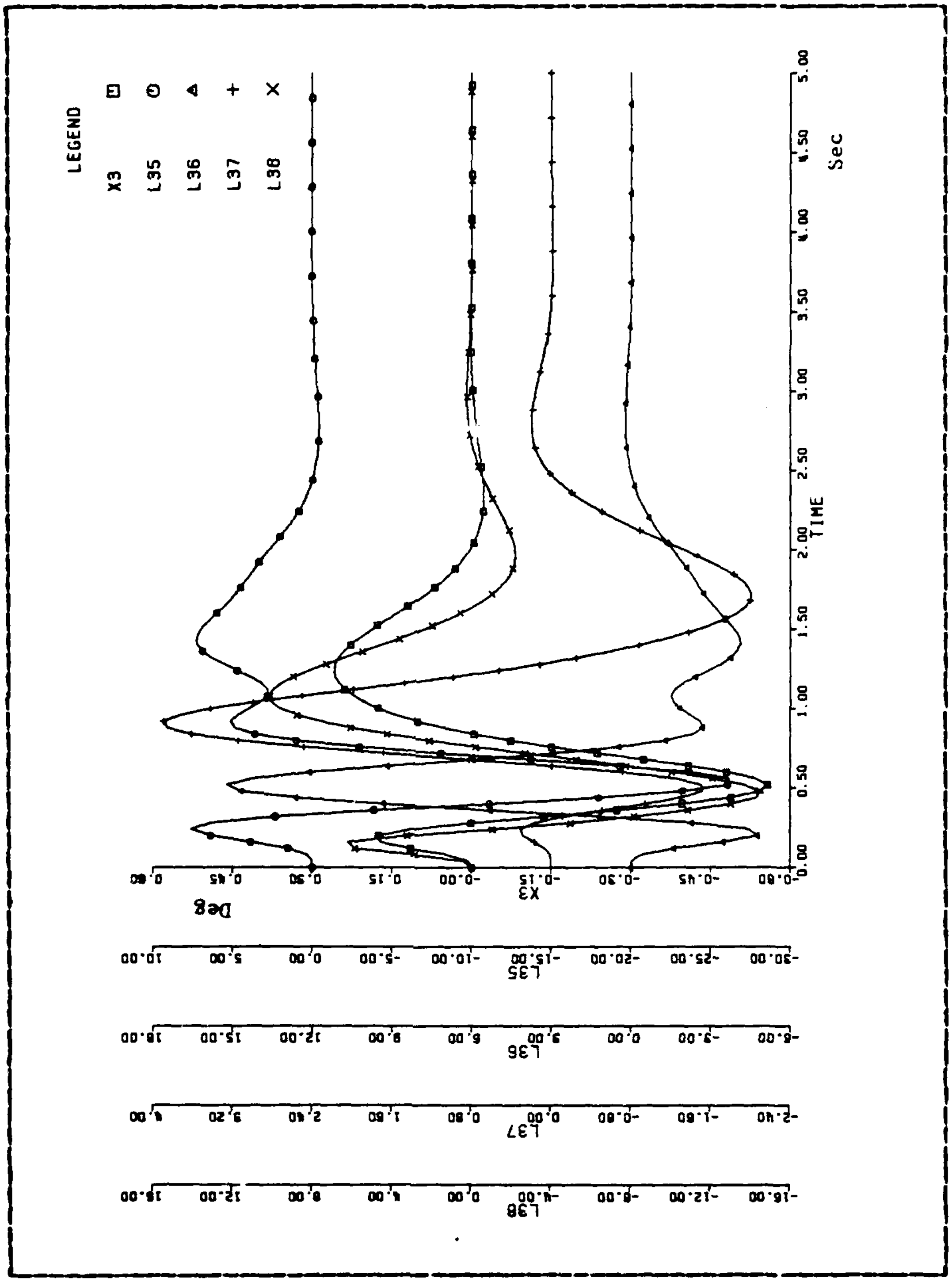

Figure 3.32   Sensitivity of X3 with Respect to A5,A6,A7,A8.

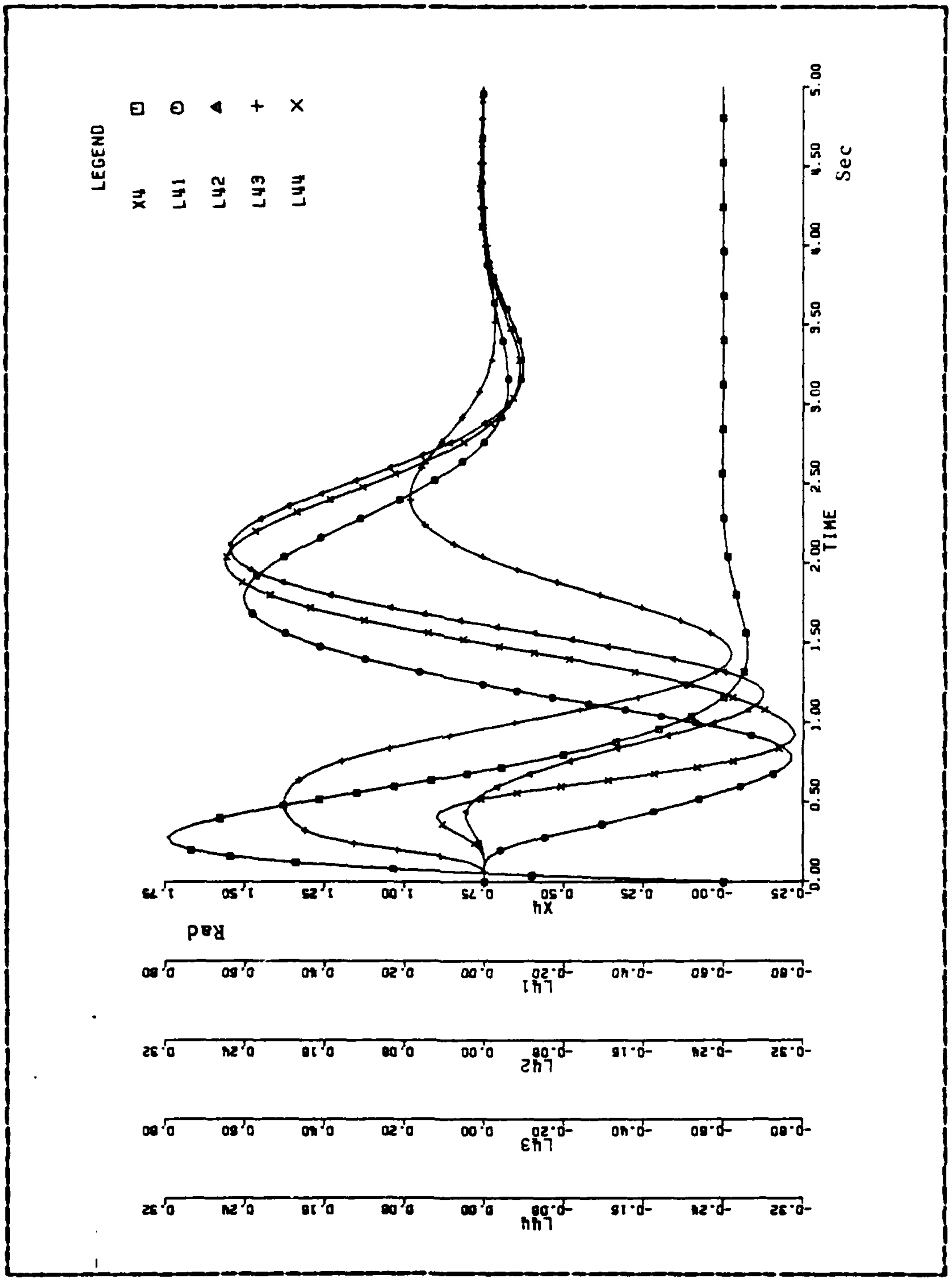

Figure 3.33 Sensitivity of X4 with Respect to A1,A2,A3,A4.

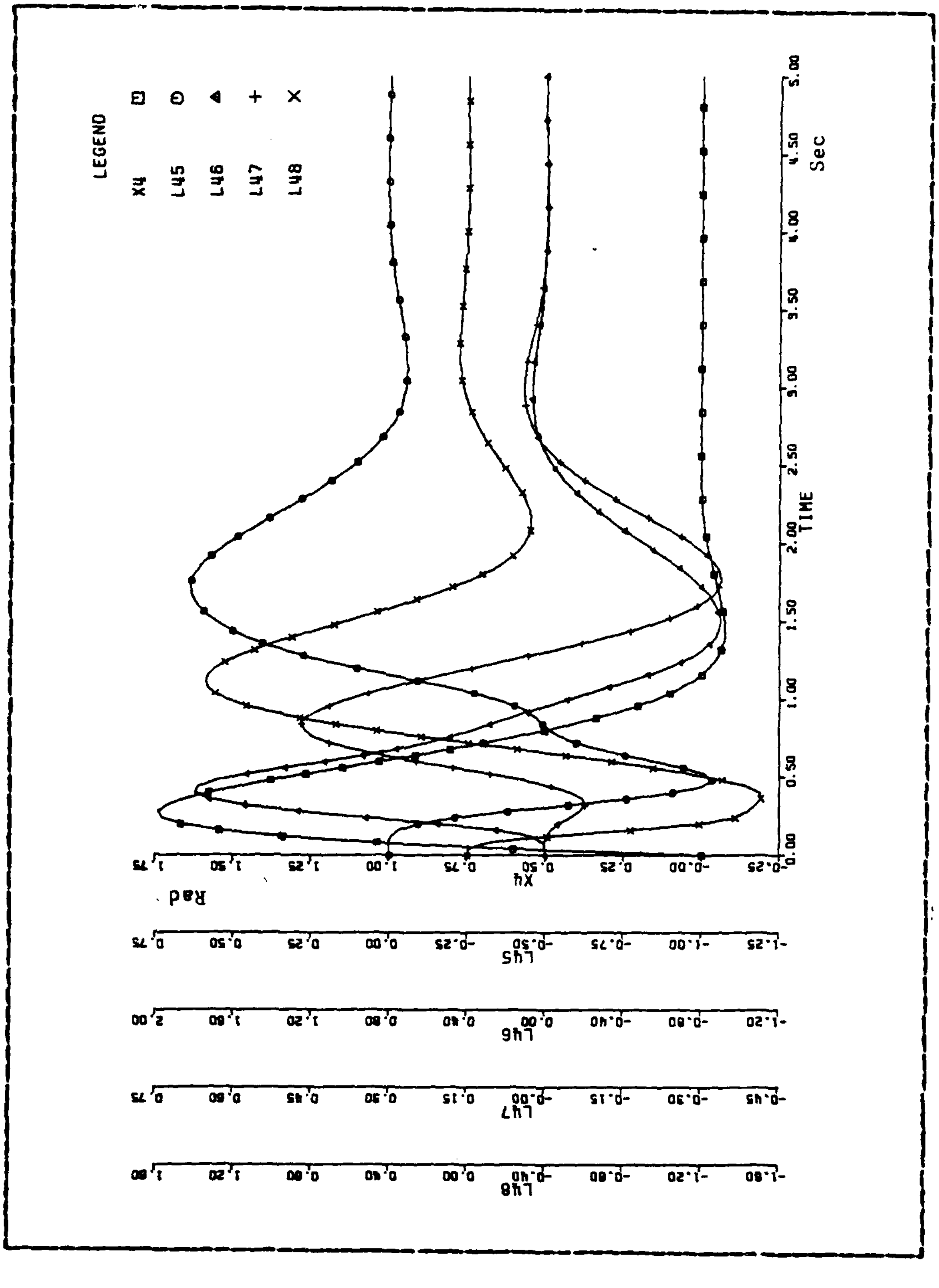

Figure 3.34 Sensitivity of X4 with Respect to A5,A6,A7,A8.

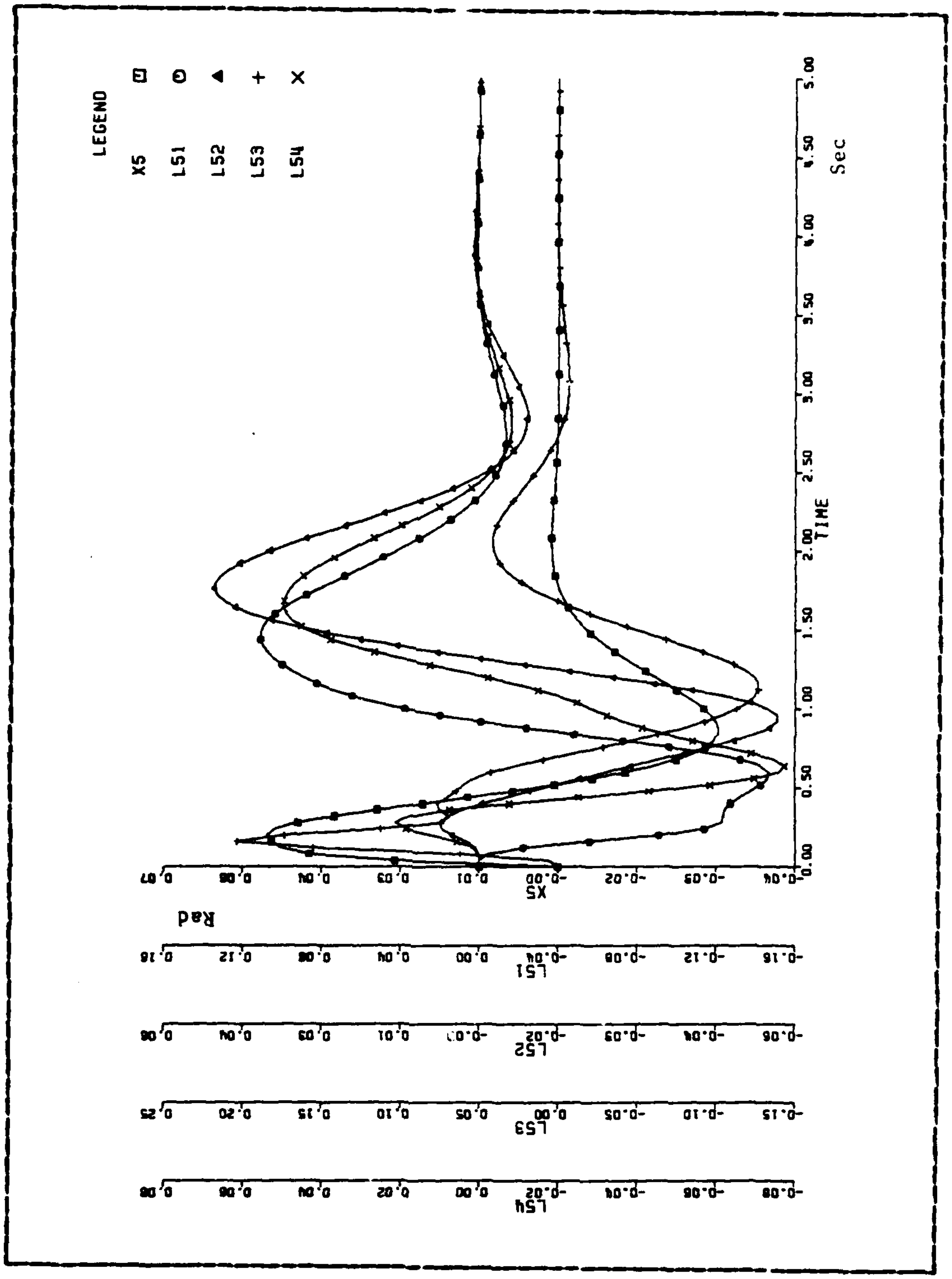

Figure 3.35 Sensitivity of X5 with Respect to λ1,λ2,λ3,λ4.

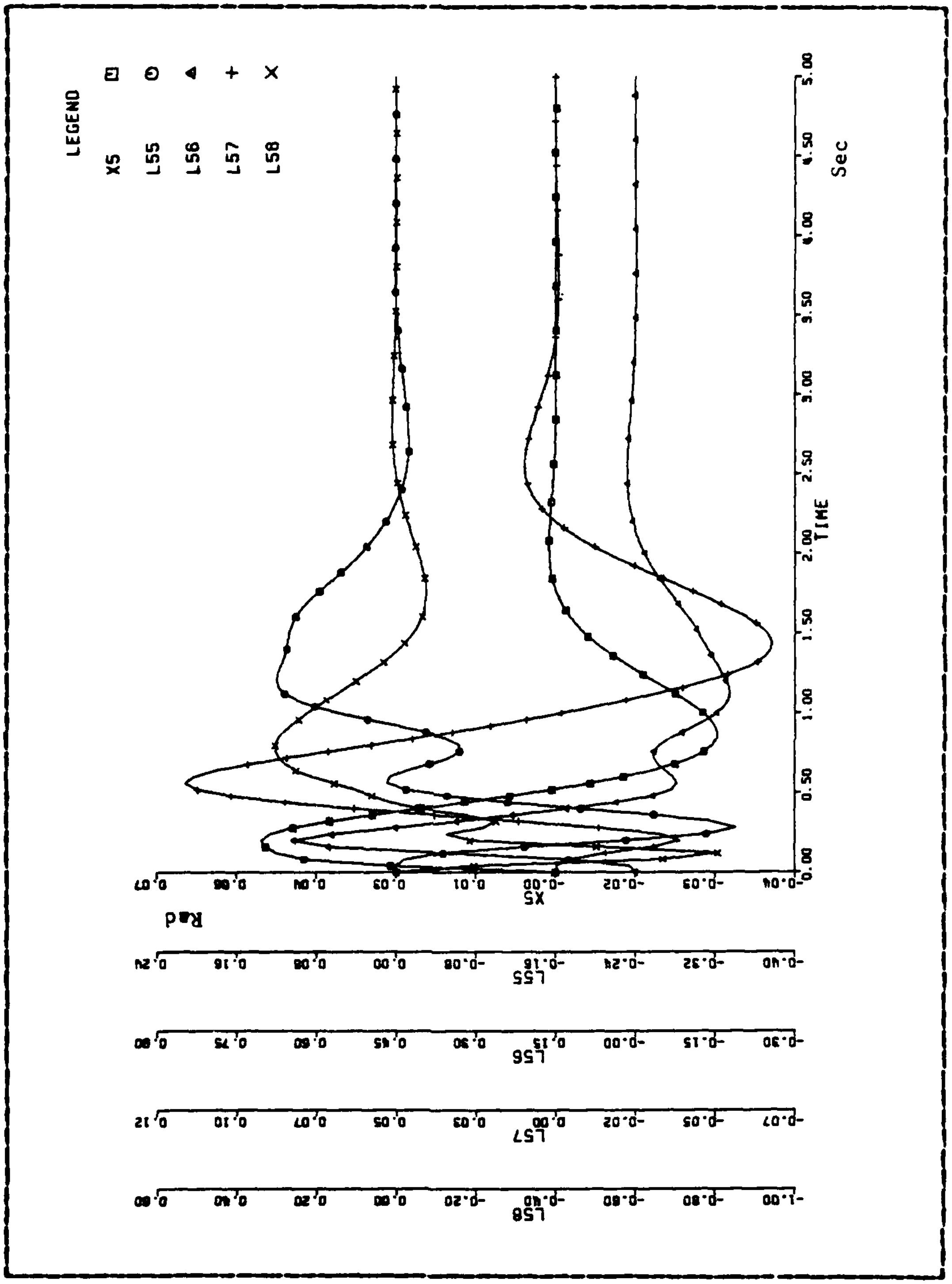

Figure 3.36   Sensitivity of X5 with Respect to A5,A6,A7,A8.

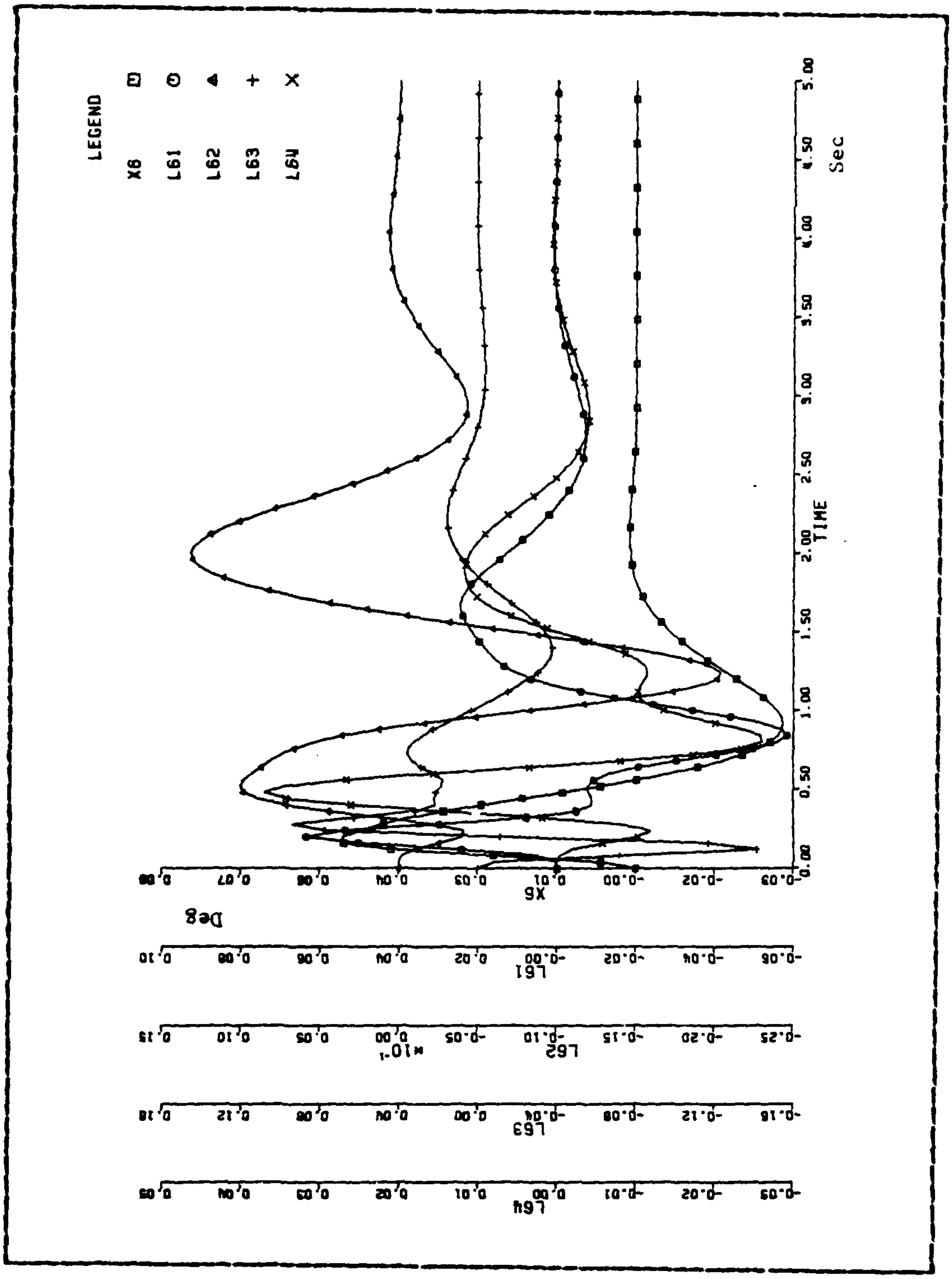

Figure 3.37 Sensitivity of X6 with Respect to A1,A2,A3,A4.

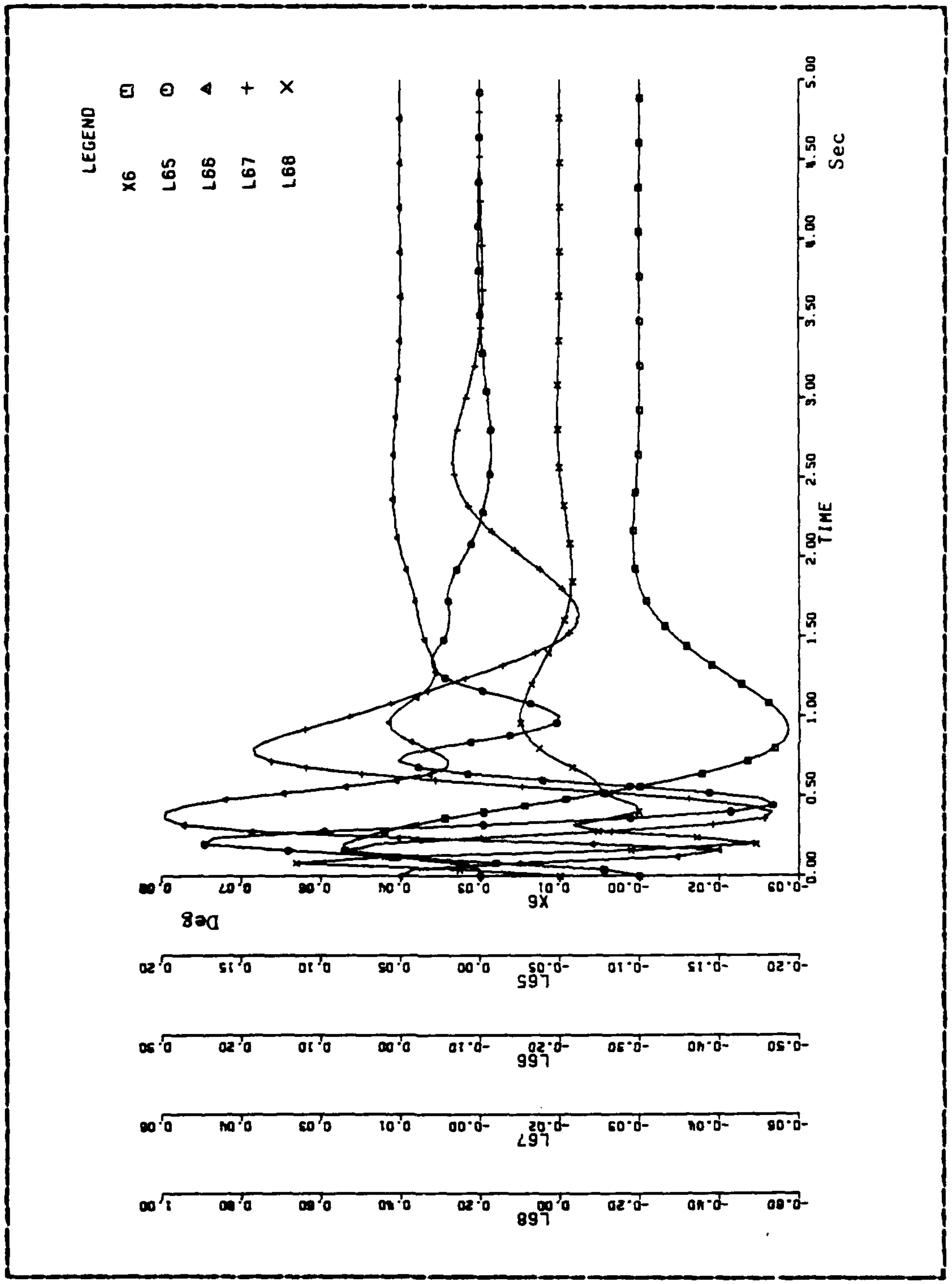

Figure 3.38 Sensitivity of X6 with Respect to A5,A6,A7,A8.

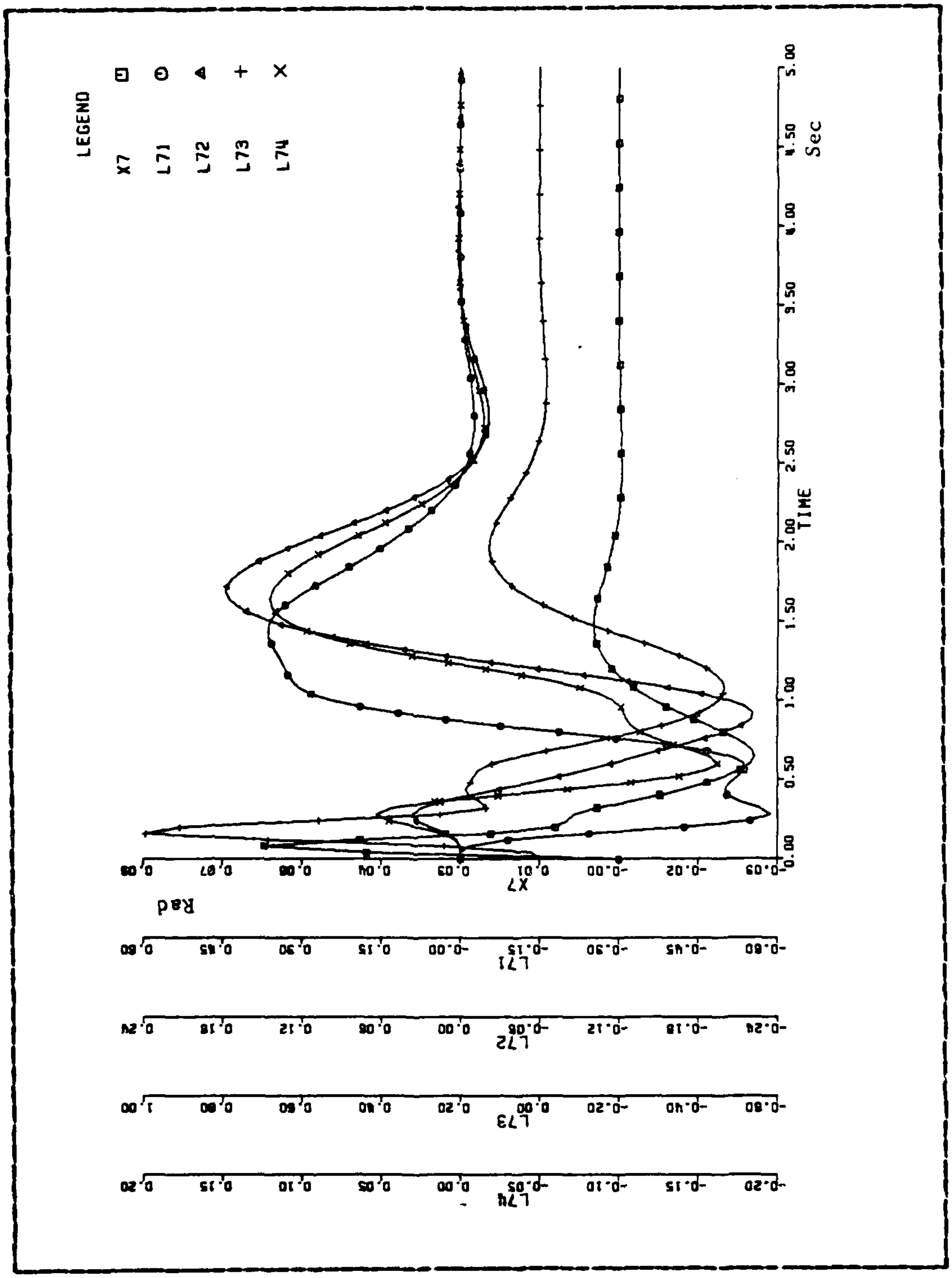

Figure 3.39 Sensitivity of X7 with Respect to A1,A2,A3,A4.

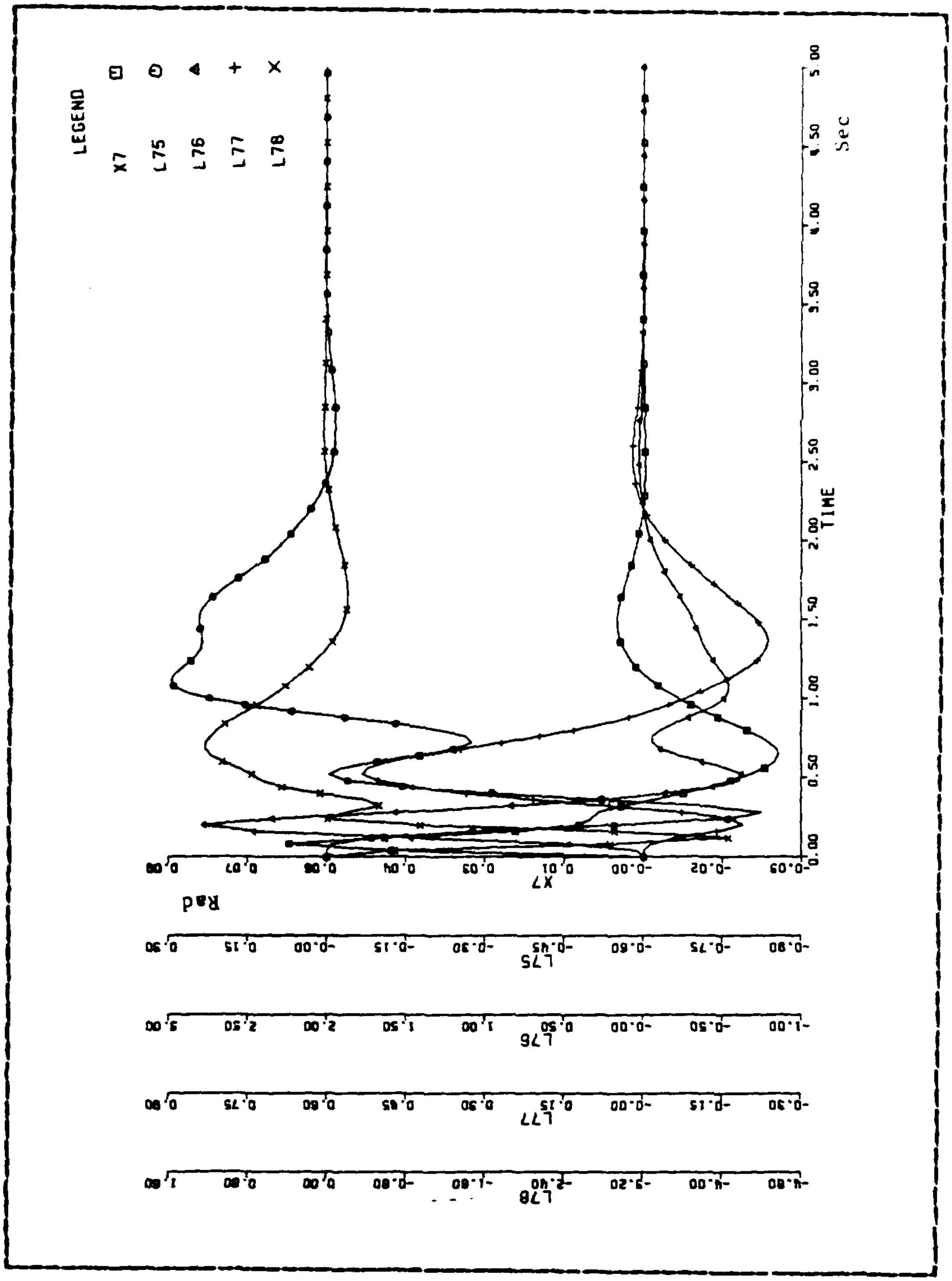

Figure 3.40 Sensitivity of X7 with Respect to A5,A6,A7,A8.

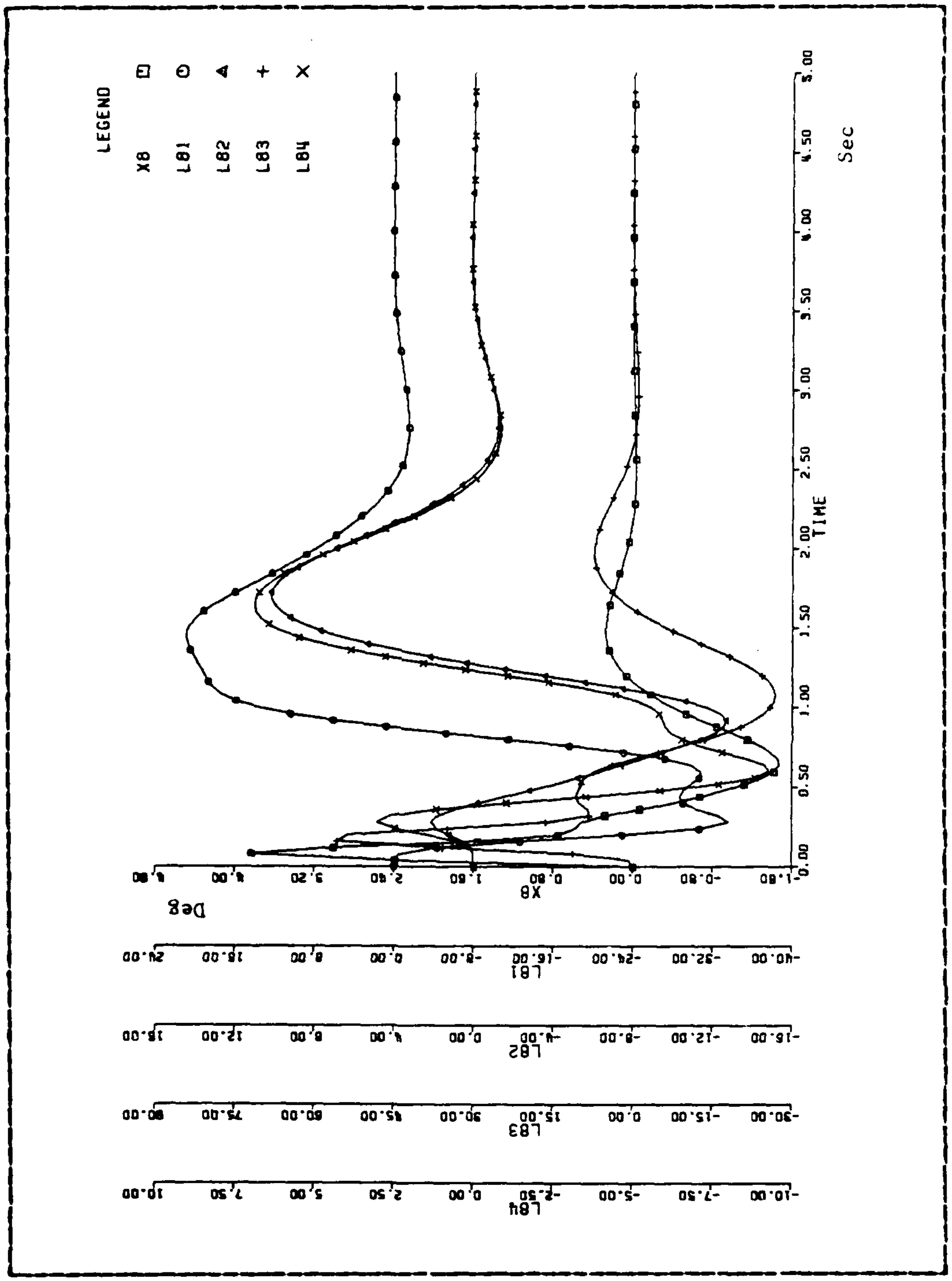

Figure 3.41 Sensitivity of X8 with Respect to A1,A2,A3,A4.

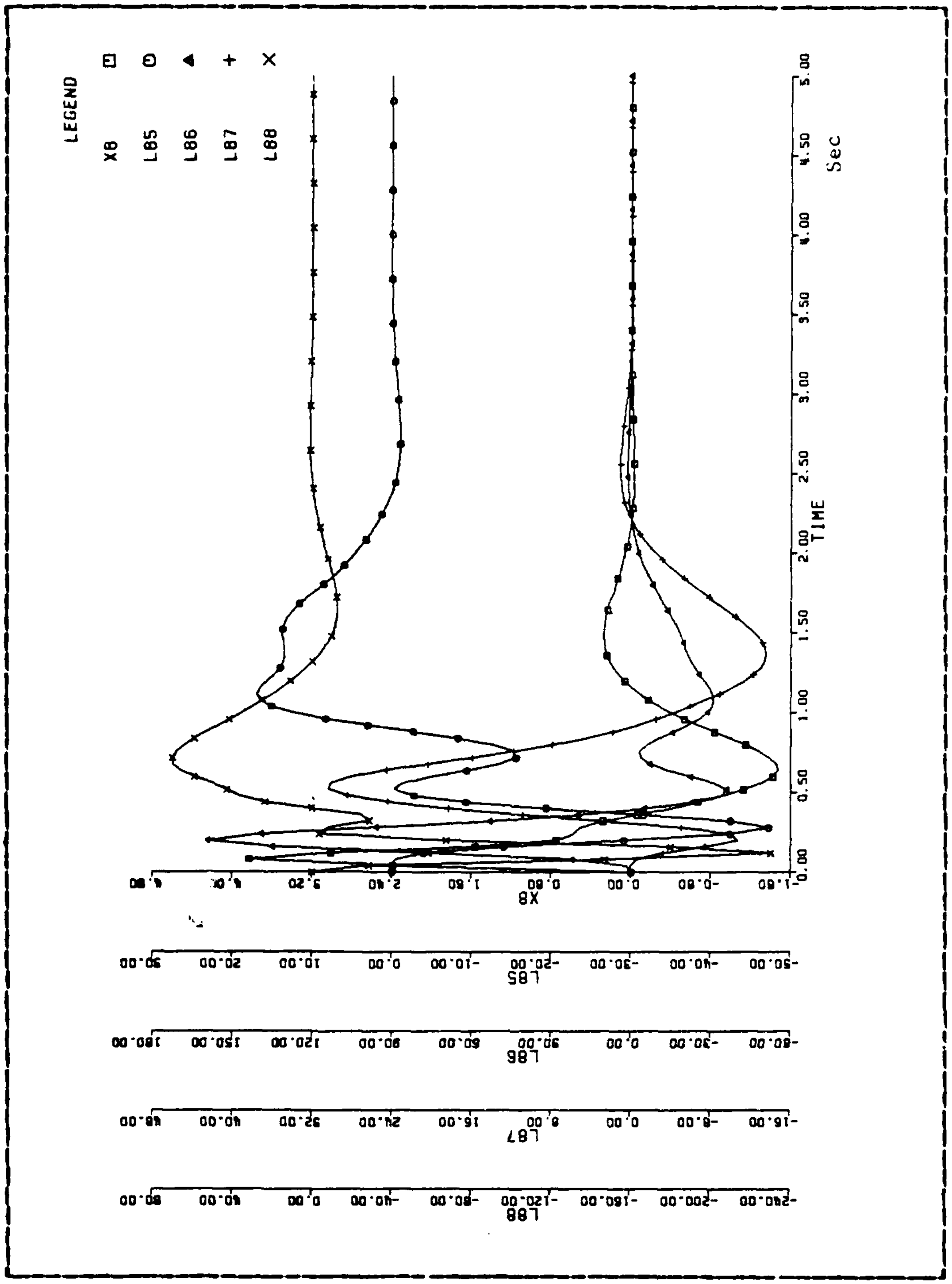

Figure 3.42   Sensitivity of X8 with Respect to A5,A6,A7,A8.

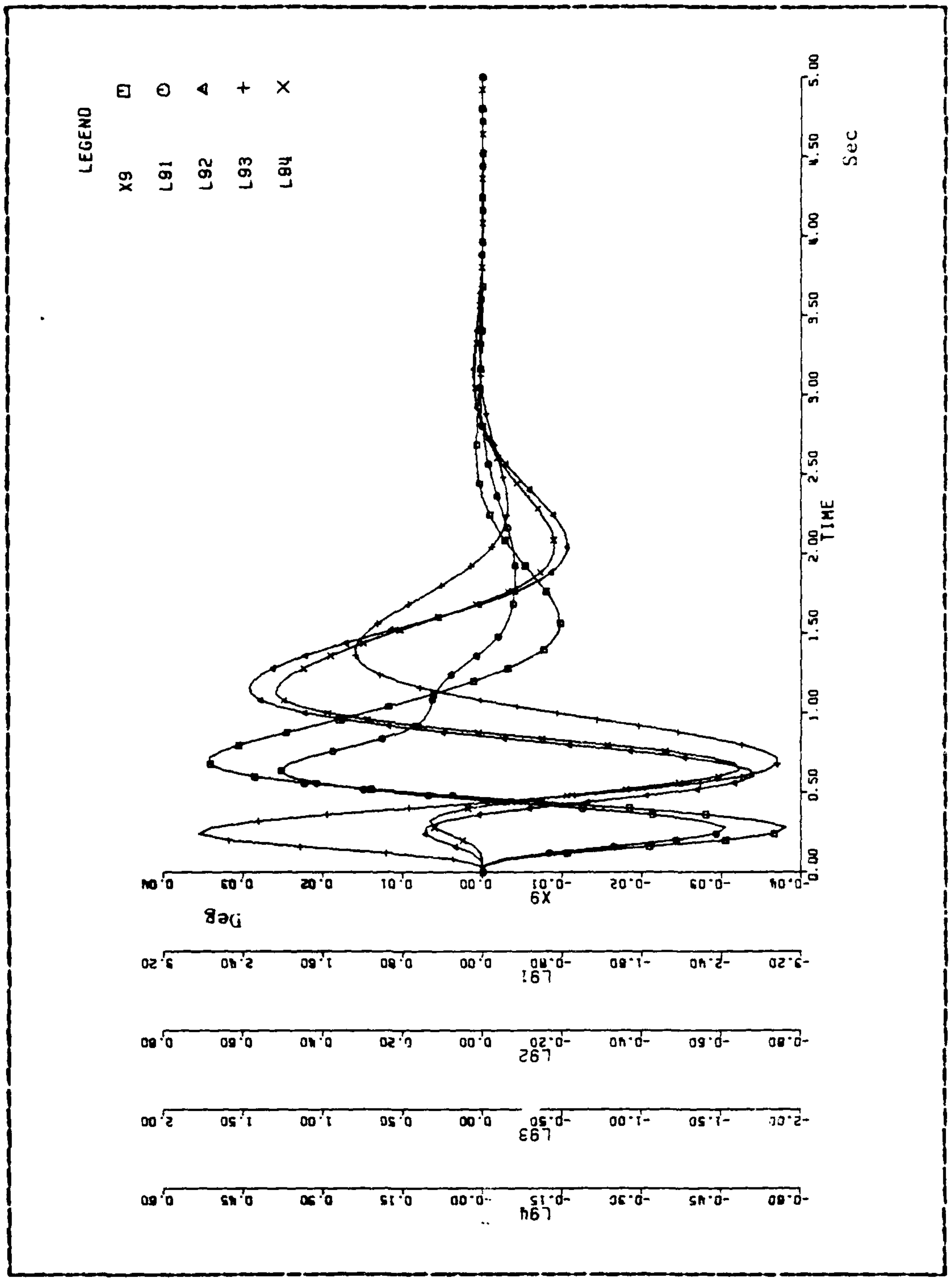

Figure 3.43 Sensitivity of X9 with Respect to λ1,λ2,λ3,λ4.

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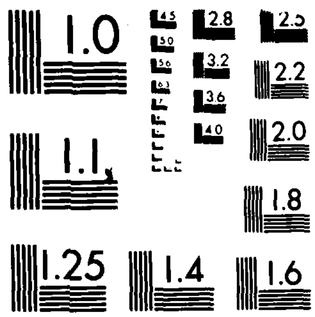

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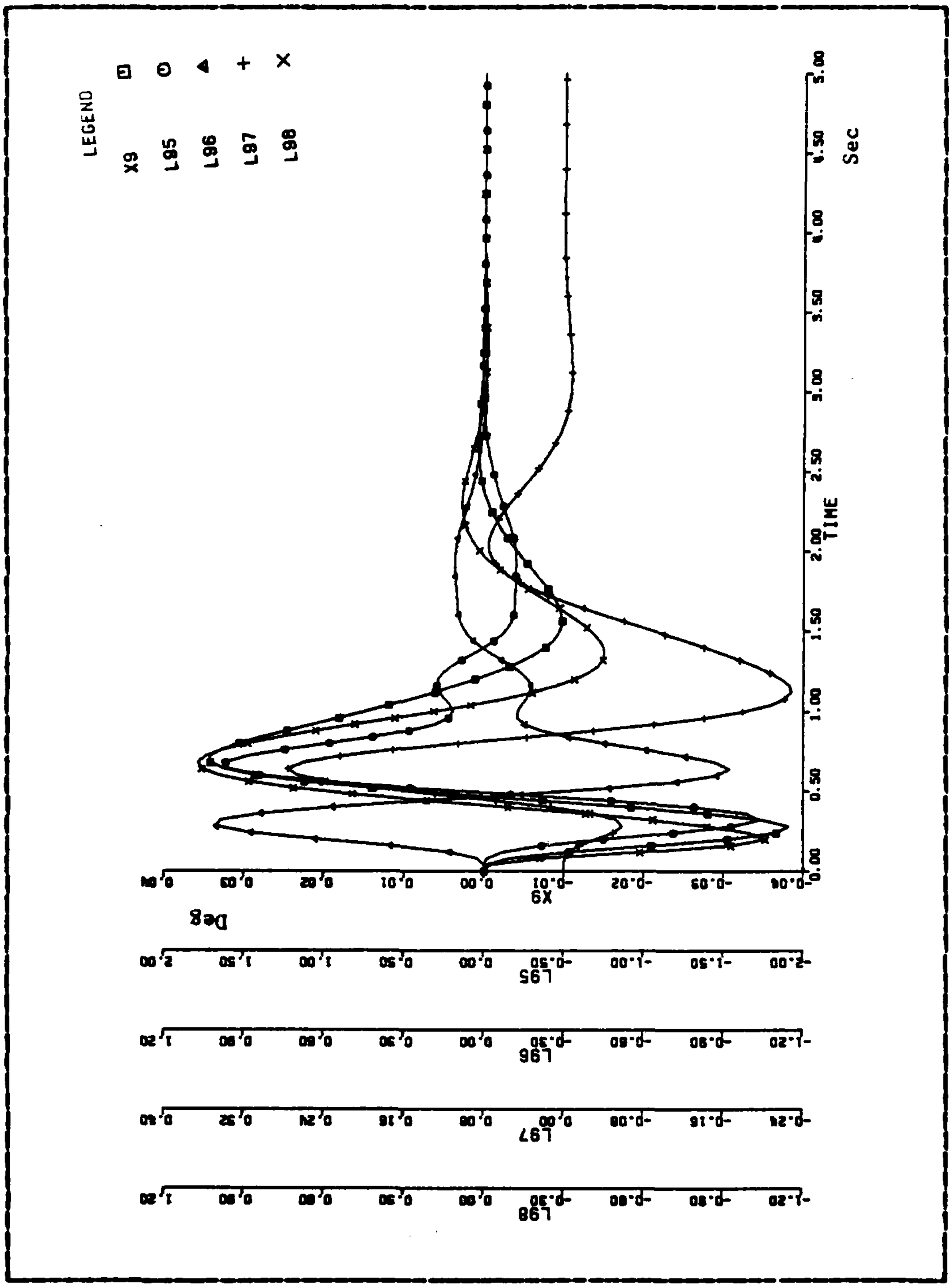

Figure 3.44 Sensitivity of X9 with Respect to λ5,λ6,λ7,λ8.

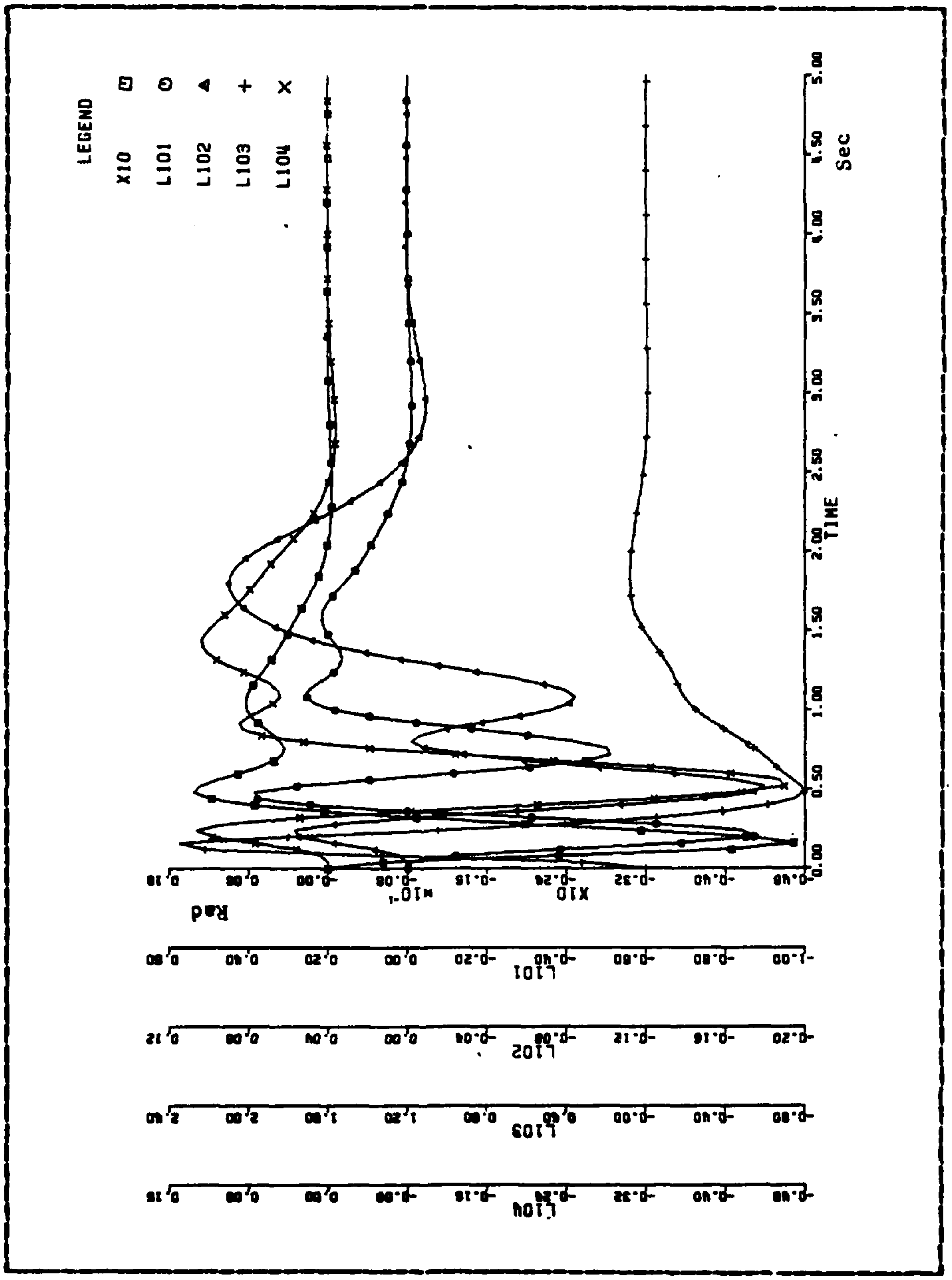

Figure 3.45   Sensitivity of X10 with Respect to λ1,λ2,λ3,λ4.

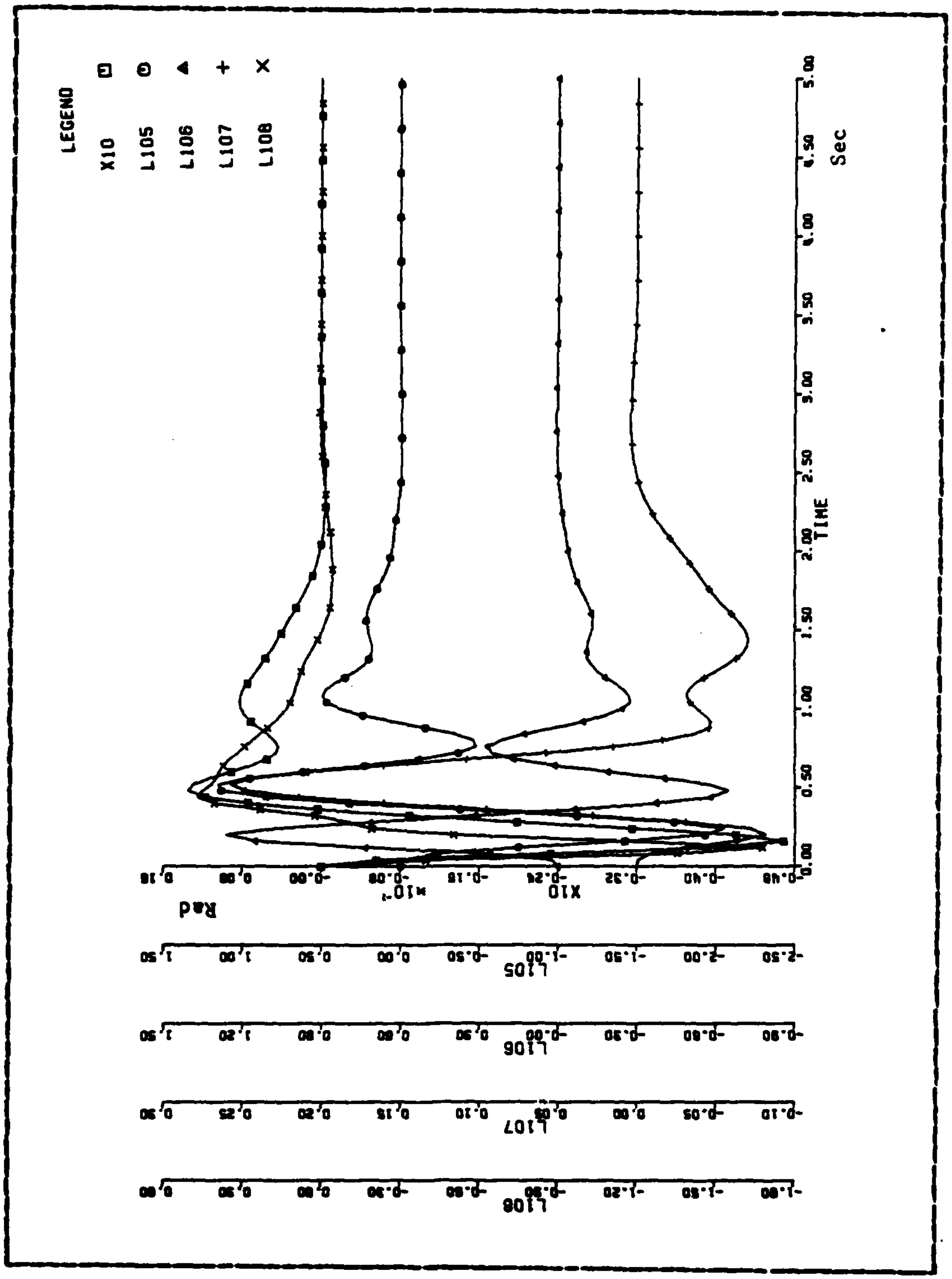

Figure 3.46 Sensitivity of X10 with Respect to A5,A6,A7,A8.

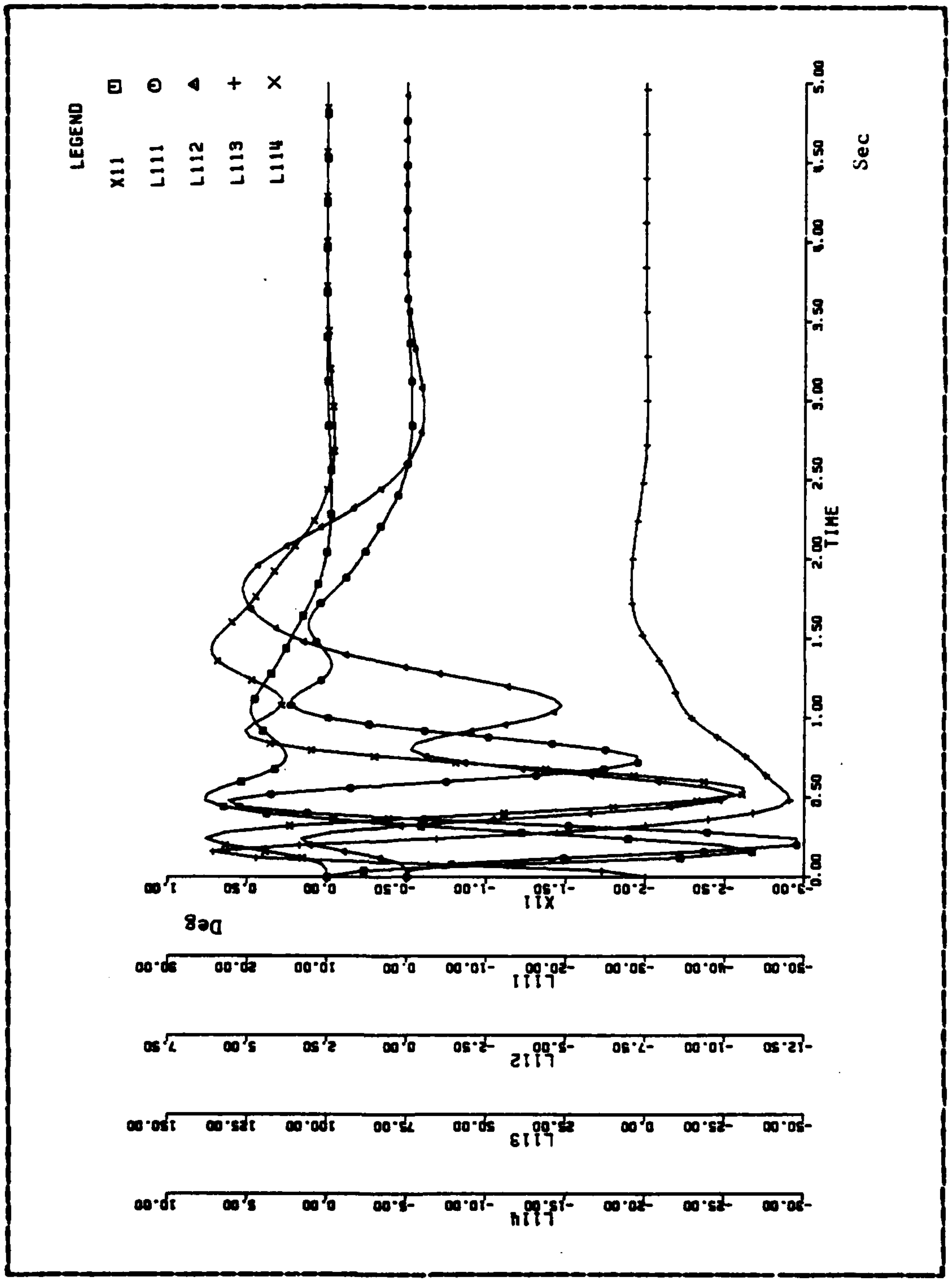

Figure 3.47 Sensitivity of X11 with Respect to A1,A2,A3,A4.

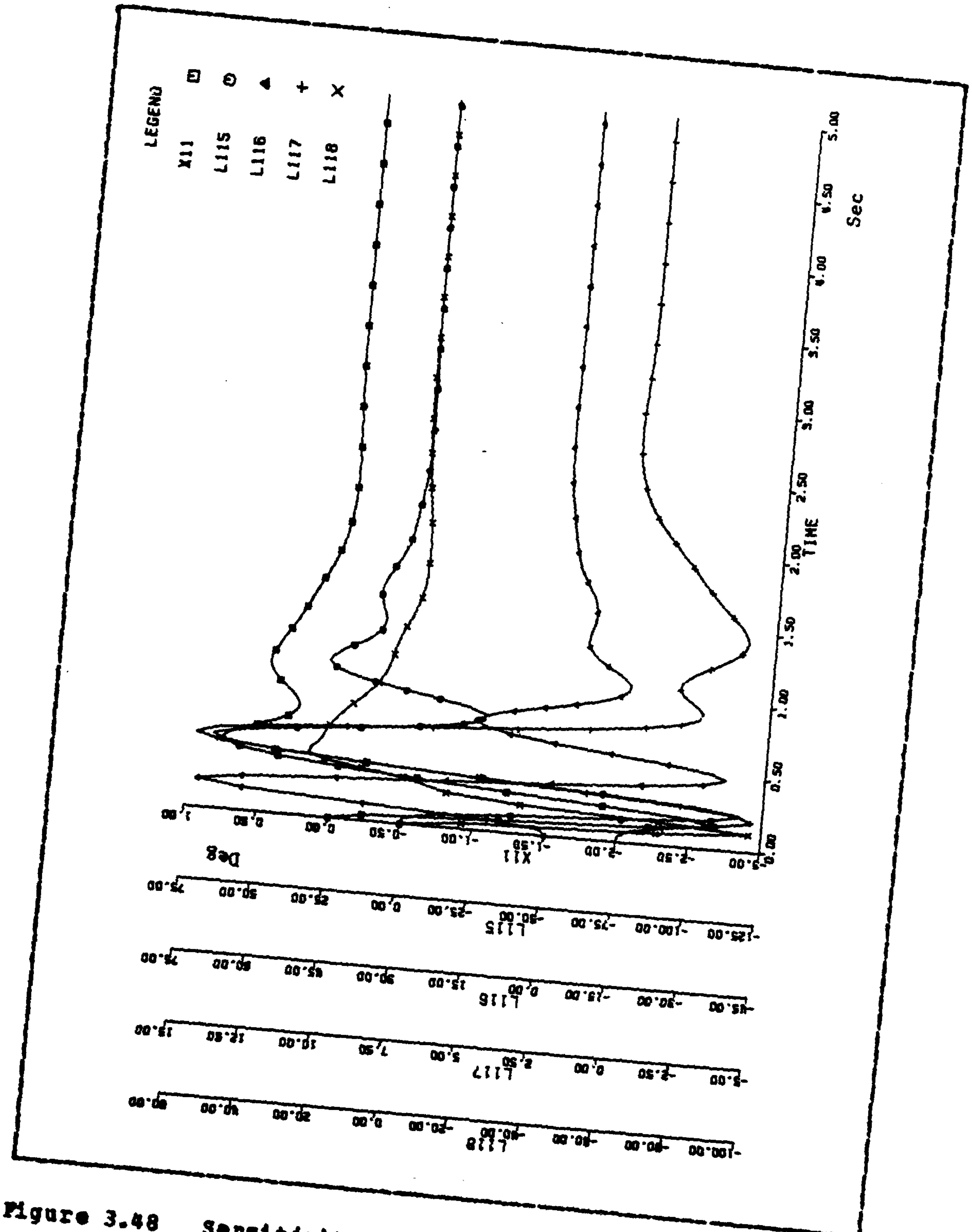

Figure 3.48 Sensitivity of X11 with Respect to A5,A6,A7,A8.

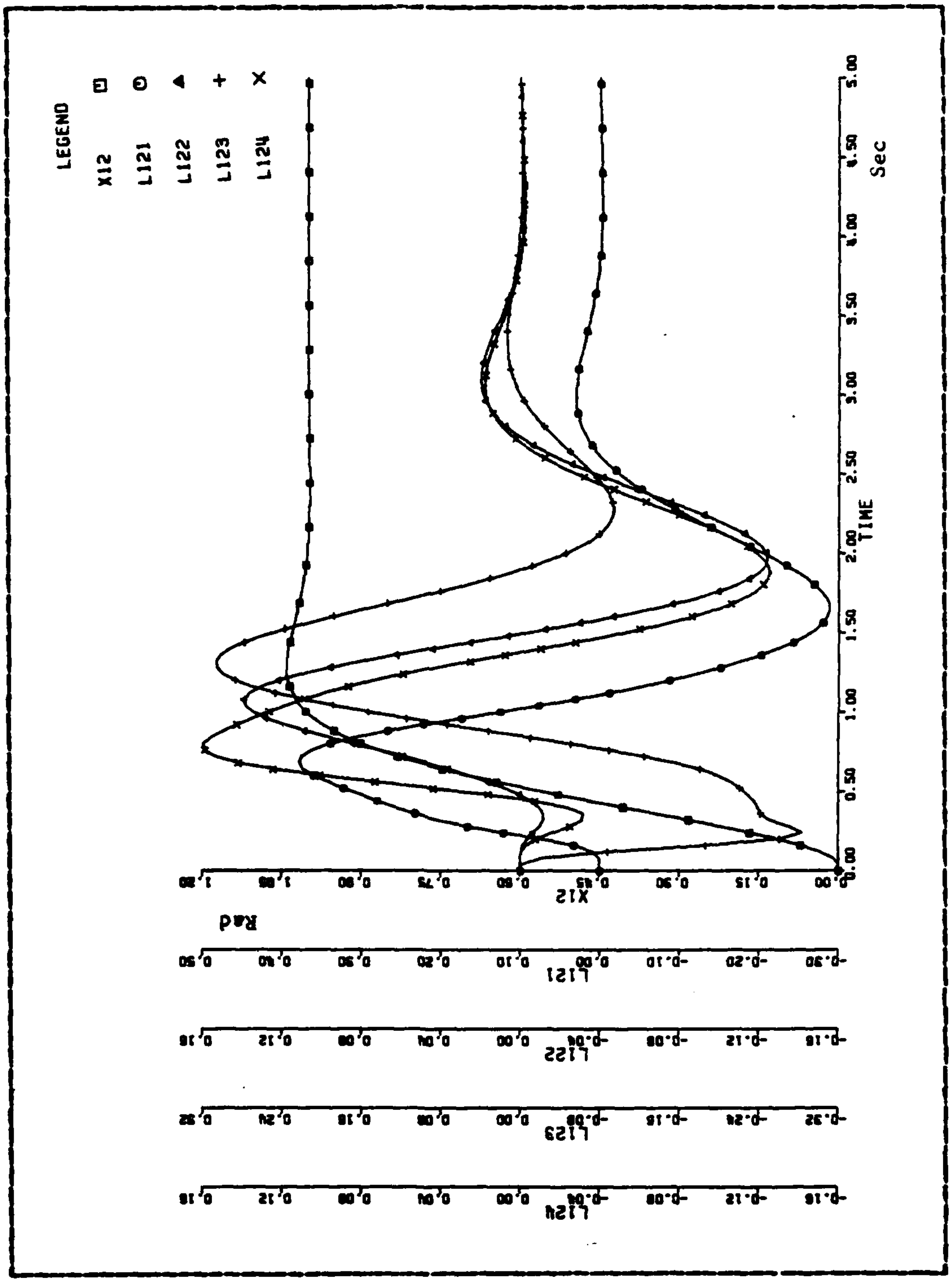

Figure 3.49 Sensitivity of X12 with Respect to λ1,λ2,λ3,λ4.

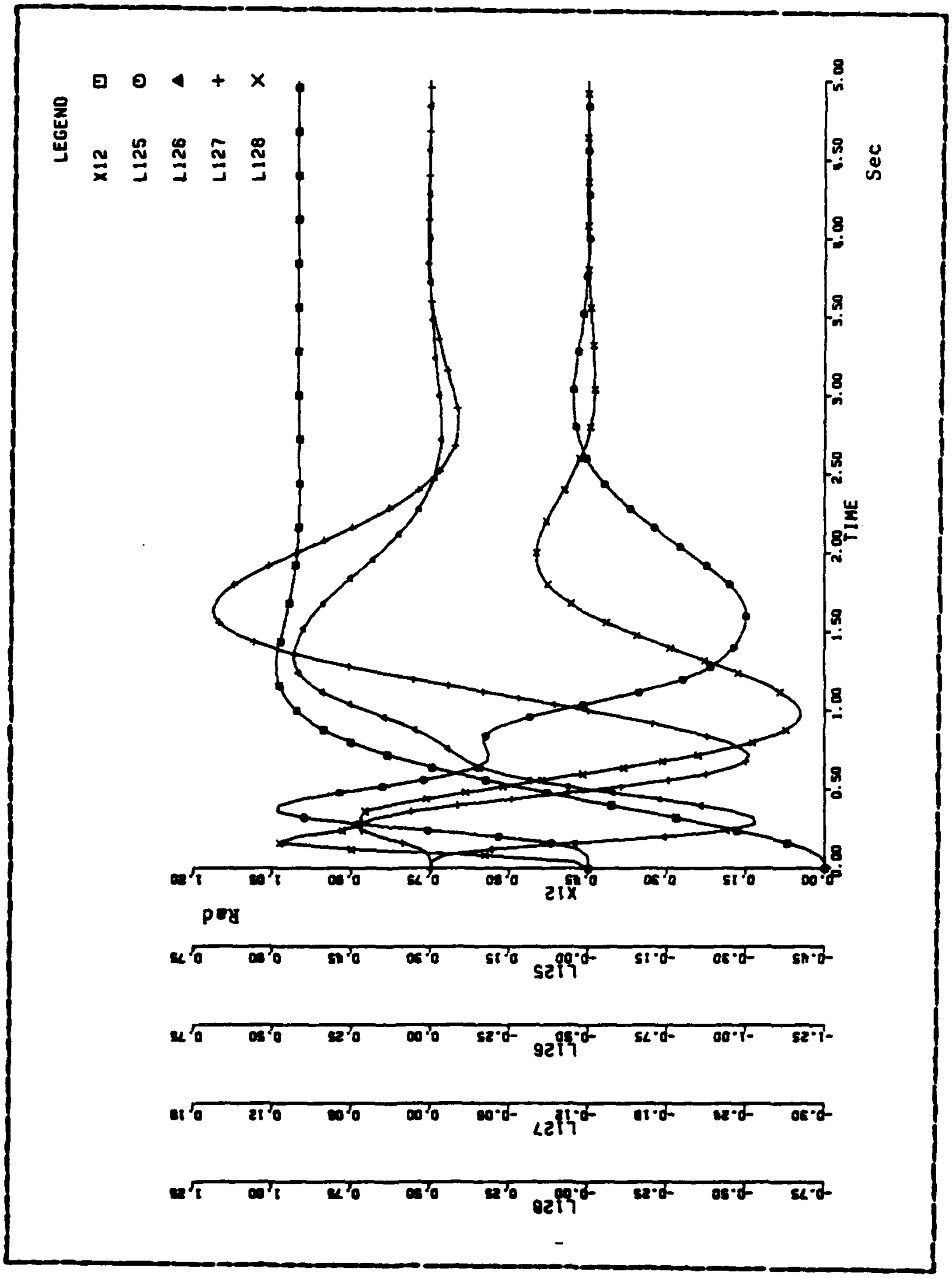

Figure 3.50 Sensitivity of X12 with Respect to A5,A6,A7,A8.

TABLE II

Influence of Parameters

X_1	λ_{11}	λ_{12}	λ_{13}	λ_{14}	λ_{15}	λ_{16}	λ_{17}	λ_{18}
RISE TIME	NE	NE	LE	LE	LE	LE	LE	LE
OVERSHOOT	SE	SE	SE	SE	SE	SE	SE	SE
STEADY STATE	NE	NE	NE	NE	NE	NE	NE	NE

X_2	λ_{21}	λ_{22}	λ_{23}	λ_{24}	λ_{25}	λ_{26}	λ_{27}	λ_{28}
RISE TIME	NE	NE	LE	NE	LE	LE	NE	LE
OVERSHOOT	SE	SE	SE	SE	SE	SE	SE	SE
STEADY STATE	NE	NE	NE	NE	NE	NE	NE	NE

X_3	λ_{31}	λ_{32}	λ_{33}	λ_{34}	λ_{35}	λ_{36}	λ_{37}	λ_{38}
RISE TIME	NE	NE	LE	NE	NE	NE	NE	LE
OVERSHOOT	SE	SE	SE	SE	SE	SE	SE	SE
STEADY STATE	NE	NE	NE	NE	NE	NE	NE	NE

X_4	λ_{41}	λ_{42}	λ_{43}	λ_{44}	λ_{45}	λ_{46}	λ_{47}	λ_{48}
RISE TIME	NE	NE	LE	NE	NE	NE	NE	NE
OVERSHOOT	LE	LE	LE	LE	LE	LE	LE	LE
STEADY STATE	NE	NE	NE	NE	NE	NE	NE	NE

X_5	λ_{51}	λ_{52}	λ_{53}	λ_{54}	λ_{55}	λ_{56}	λ_{57}	λ_{58}
RISE TIME	NE	NE	LE	NE	NE	NE	NE	LE
OVERSHOOT	LE	LE	LE	LE	LE	LE	LE	LE
STEADY STATE	NE	NE	NE	NE	NE	NE	NE	NE

X_6	λ_{61}	λ_{62}	λ_{63}	λ_{64}	λ_{65}	λ_{66}	λ_{67}	λ_{68}
RISE TIME	LE	NE	LE	NE	LE	LE	LE	LE
OVERSHOOT	LE	LE	LE	LE	LE	LE	LE	LE
STEADY STATE	NE	NE	NE	NE	NE	NE	NE	NE

TABLE III

Influence of Parameters (cont. Table II)

		λ_{71}	λ_{72}	λ_{73}	λ_{74}	λ_{75}	λ_{76}	λ_{77}	λ_{78}
X_7	RISE TIME	LE	NE	LE	NE	NE	LE	NE	LE
	OVERSHOOT	LE	LE	LE	LE	LE	LE	LE	LE
	STEADY STATE	NE	NE	NE	NE	NE	NE	NE	NE

		λ_{81}	λ_{82}	λ_{83}	λ_{84}	λ_{85}	λ_{86}	λ_{87}	λ_{88}
X_8	RISE TIME	NE	NE	NE	NE	NE	NE	NE	NE
	OVERSHOOT	SE	SE	SE	SE	SE	SE	SE	SE
	STEADY STATE	NE	NE	NE	NE	NE	NE	NE	NE

		λ_{91}	λ_{92}	λ_{93}	λ_{94}	λ_{95}	λ_{96}	λ_{97}	λ_{98}
X_9	RISE TIME	LE	LE	LE	LE	LE	LE	LE	LE
	OVERSHOOT	LE	LE	LE	LE	LE	LE	LE	LE
	STEADY STATE	NE	NE	NE	NE	NE	NE	NE	NE

		λ_{101}	λ_{102}	λ_{103}	λ_{104}	λ_{105}	λ_{106}	λ_{107}	λ_{108}
X_{10}	RISE TIME	LE	LE	LE	LE	LE	LE	LE	LE
	OVERSHOOT	LE	LE	LE	LE	LE	LE	LE	LE
	STEADY STATE	NE	NE	NE	NE	NE	NE	NE	NE

		λ_{111}	λ_{112}	λ_{113}	λ_{114}	λ_{115}	λ_{116}	λ_{117}	λ_{118}
X_{11}	RISE TIME	LE	NE	LE	NE	NE	NE	NE	LE
	OVERSHOOT	SE	SE	SE	SE	SE	SE	SE	SE
	STEADY STATE	NE	NE	NE	NE	NE	NE	NE	NE

		λ_{121}	λ_{122}	λ_{123}	λ_{124}	λ_{125}	λ_{126}	λ_{127}	λ_{128}
X_{12}	RISE TIME	NE	NE	LE	NE	NE	LE	NE	NE
	OVERSHOOT	SE	SE	SE	SE	SE	SE	SE	SE
	STEADY STATE	NE	NE	NE	NE	NE	NE	NE	NE

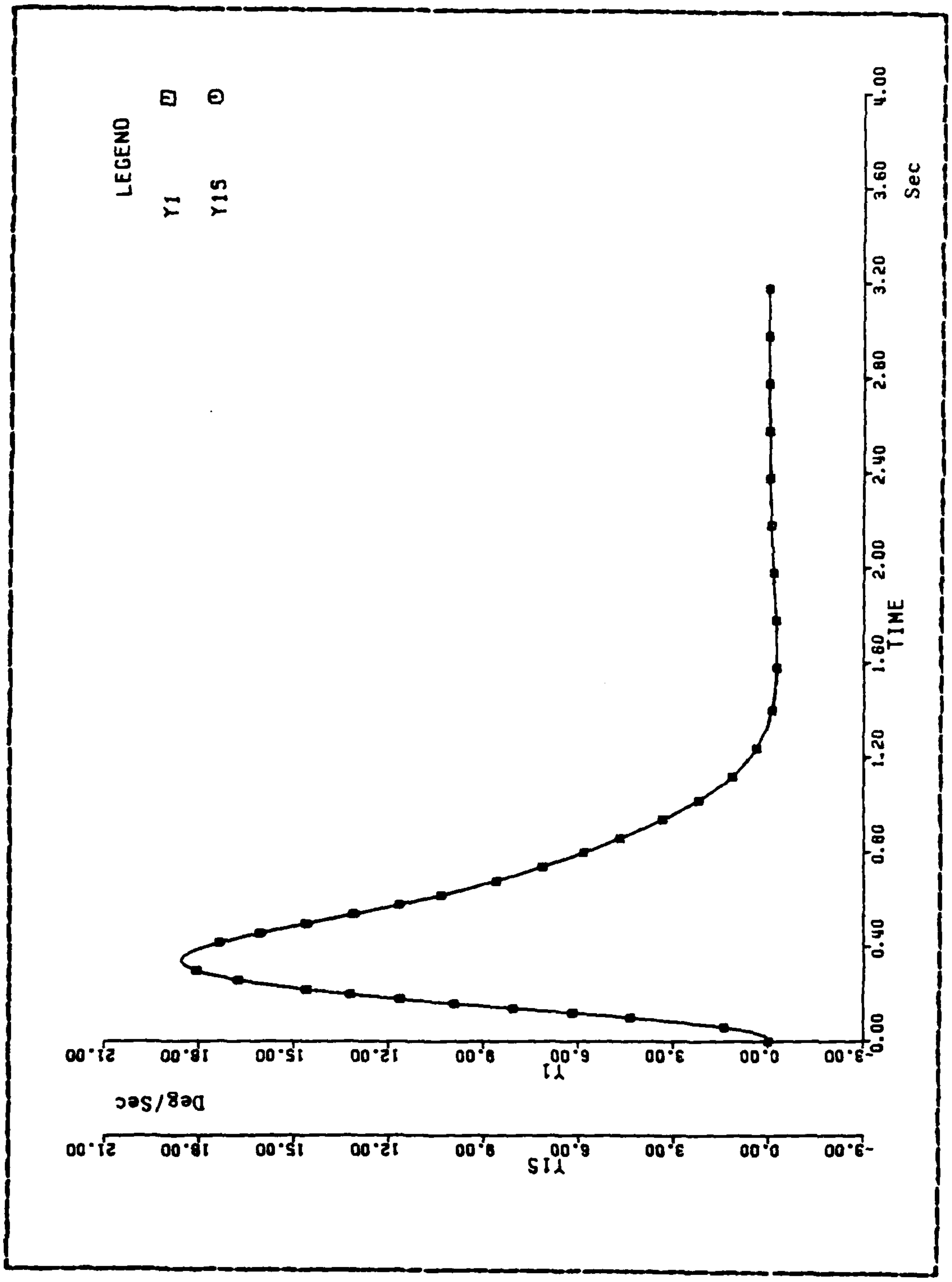

Figure 3.51 Actual and Nominal Output of X1(10% variation).

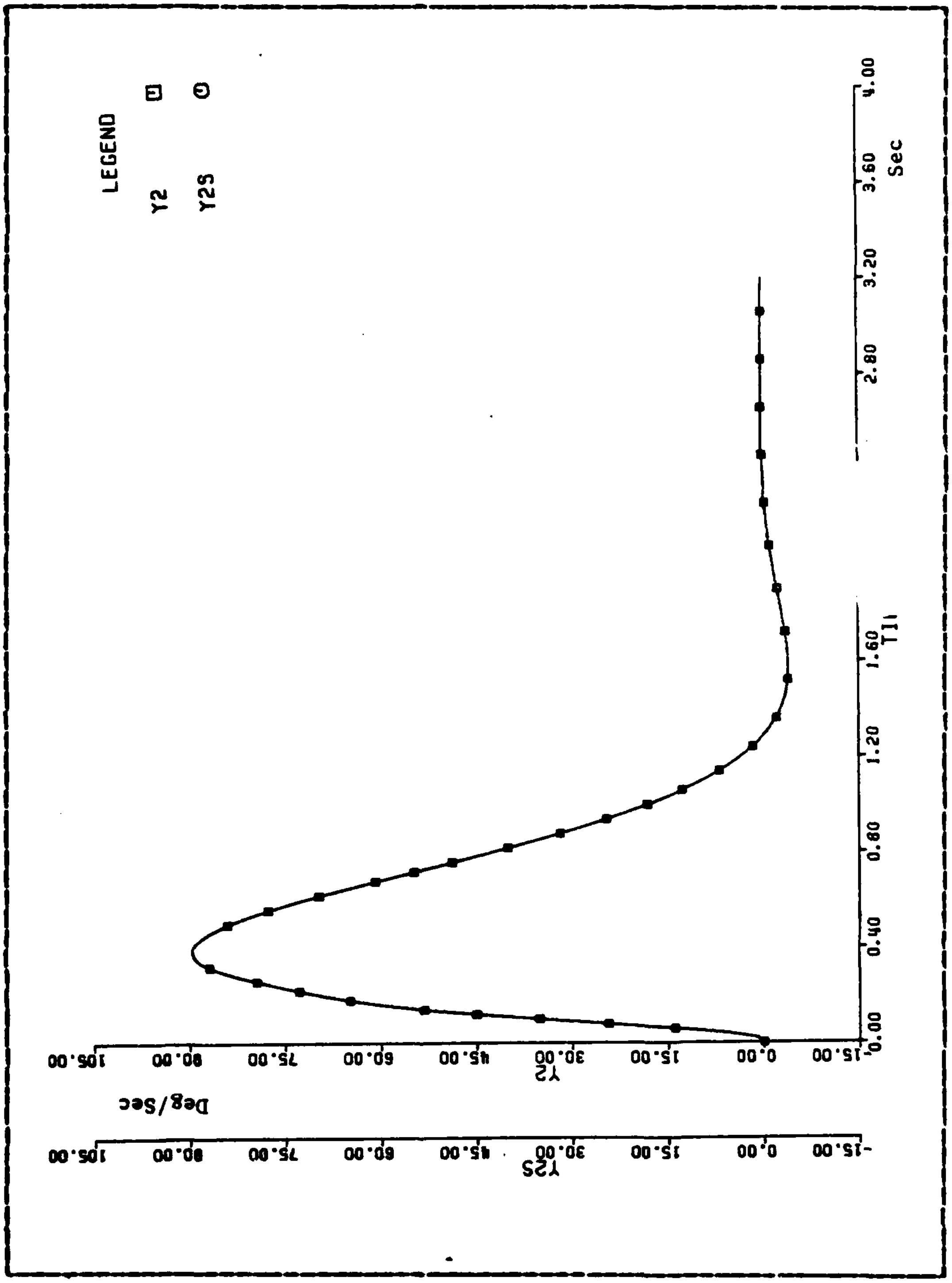

Figure 3.52   Actual and Nominal Output of X2(10% variation).

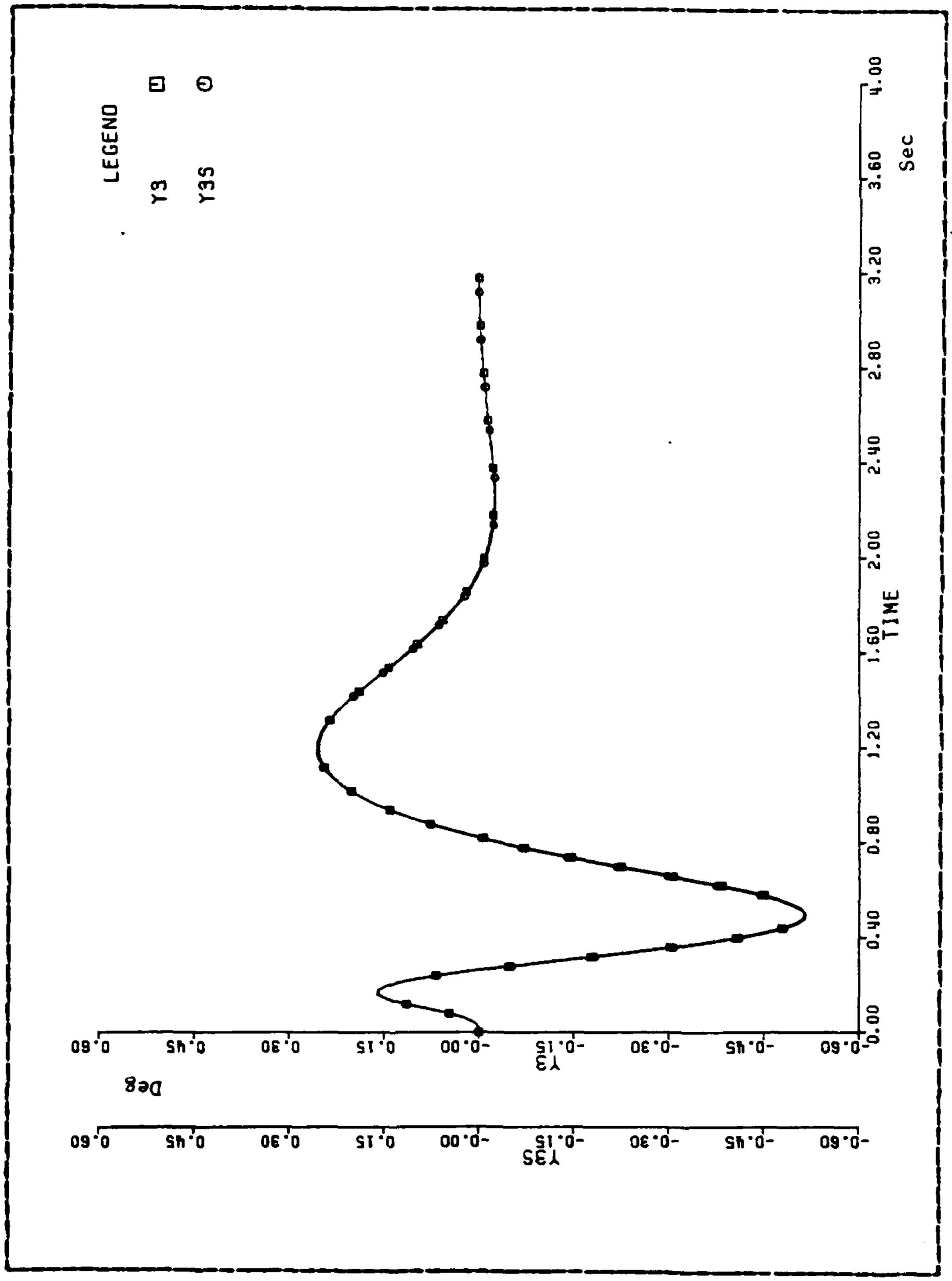

Figure 3.53   Actual and Nominal Output of X3(10% variation).

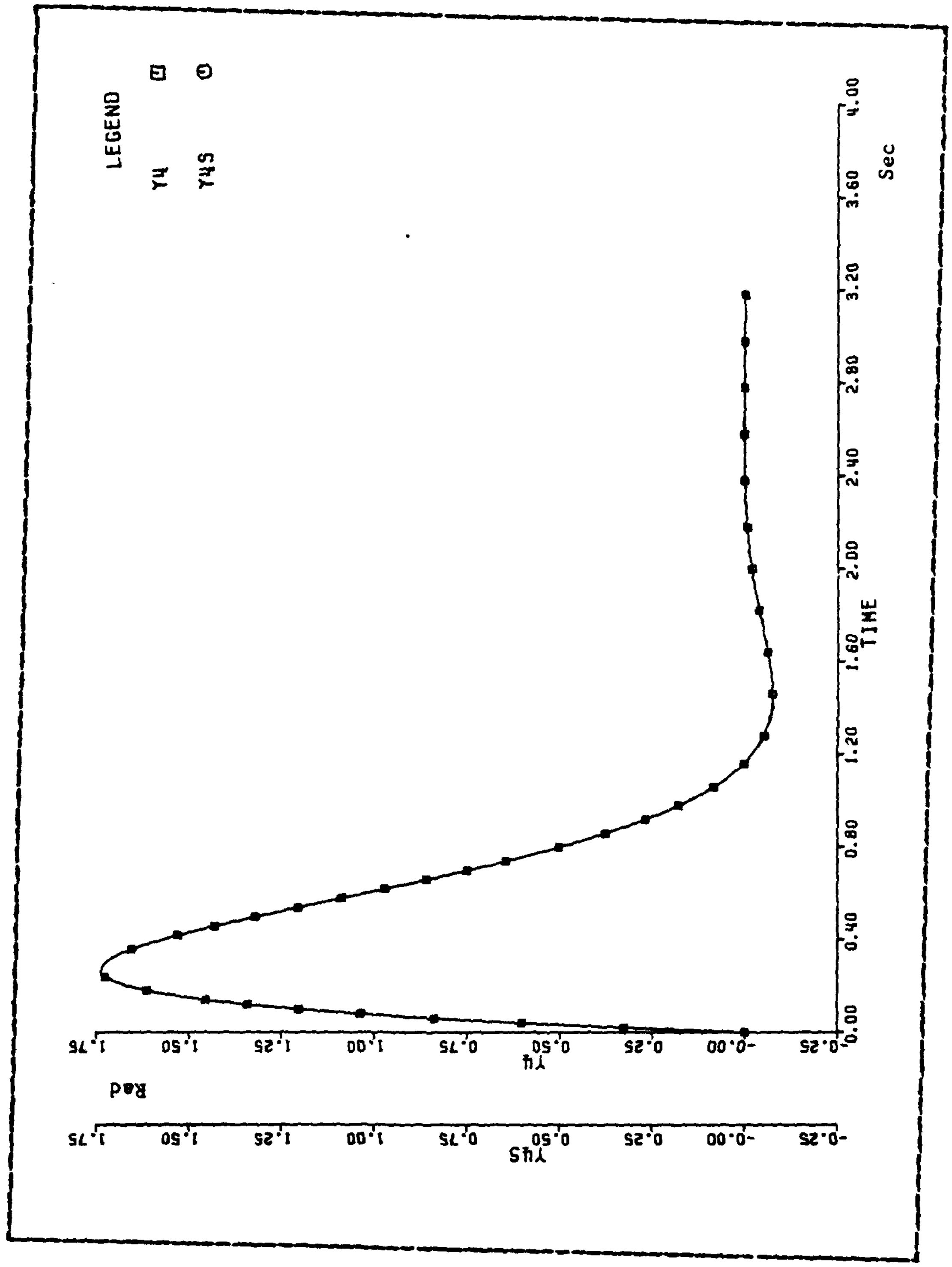

Figure 3.54 Actual and Nominal Output of X4 (10% variation).

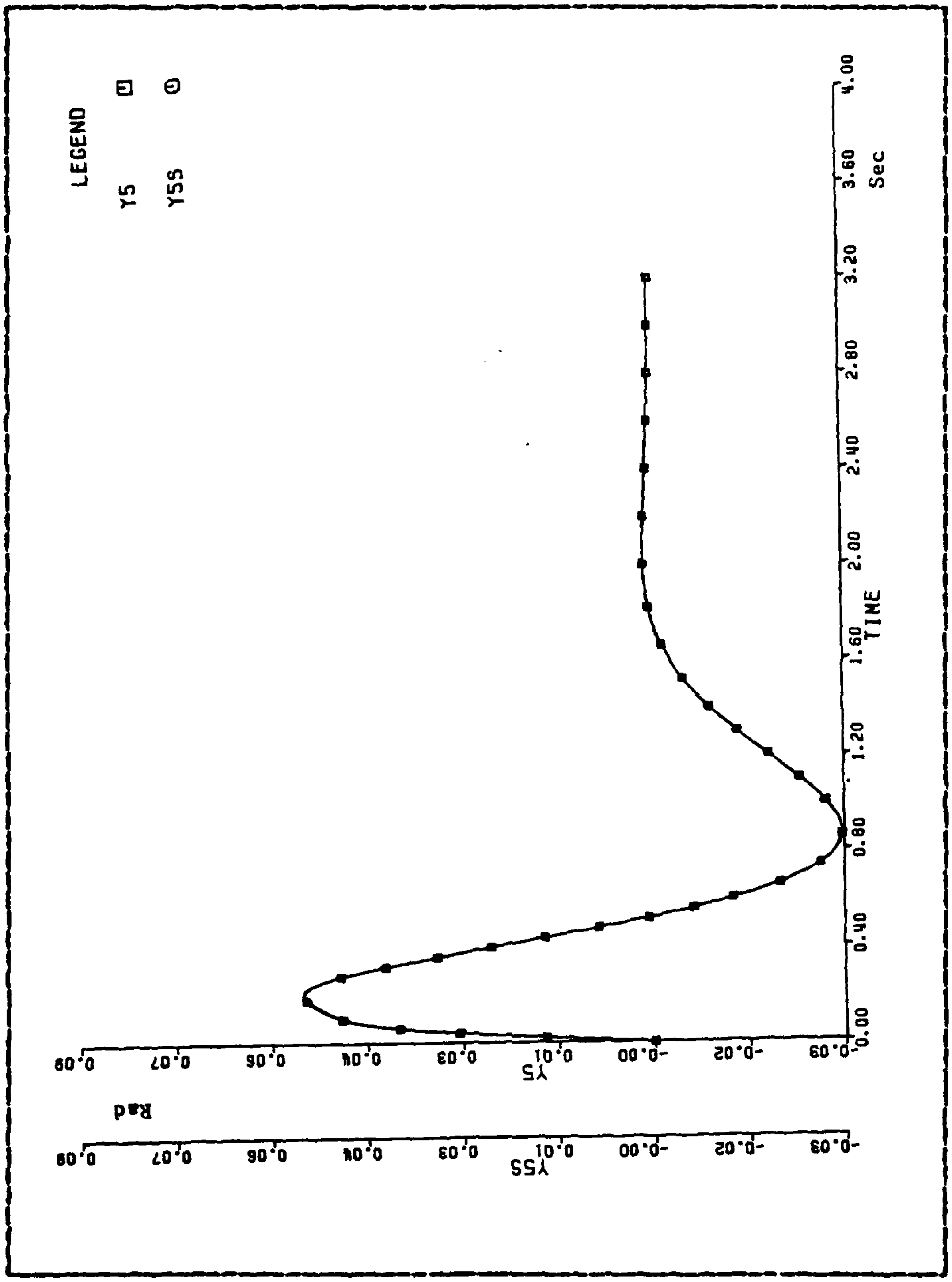

Figure 3.55 Actual and Nominal Output of X5 (10% variation).

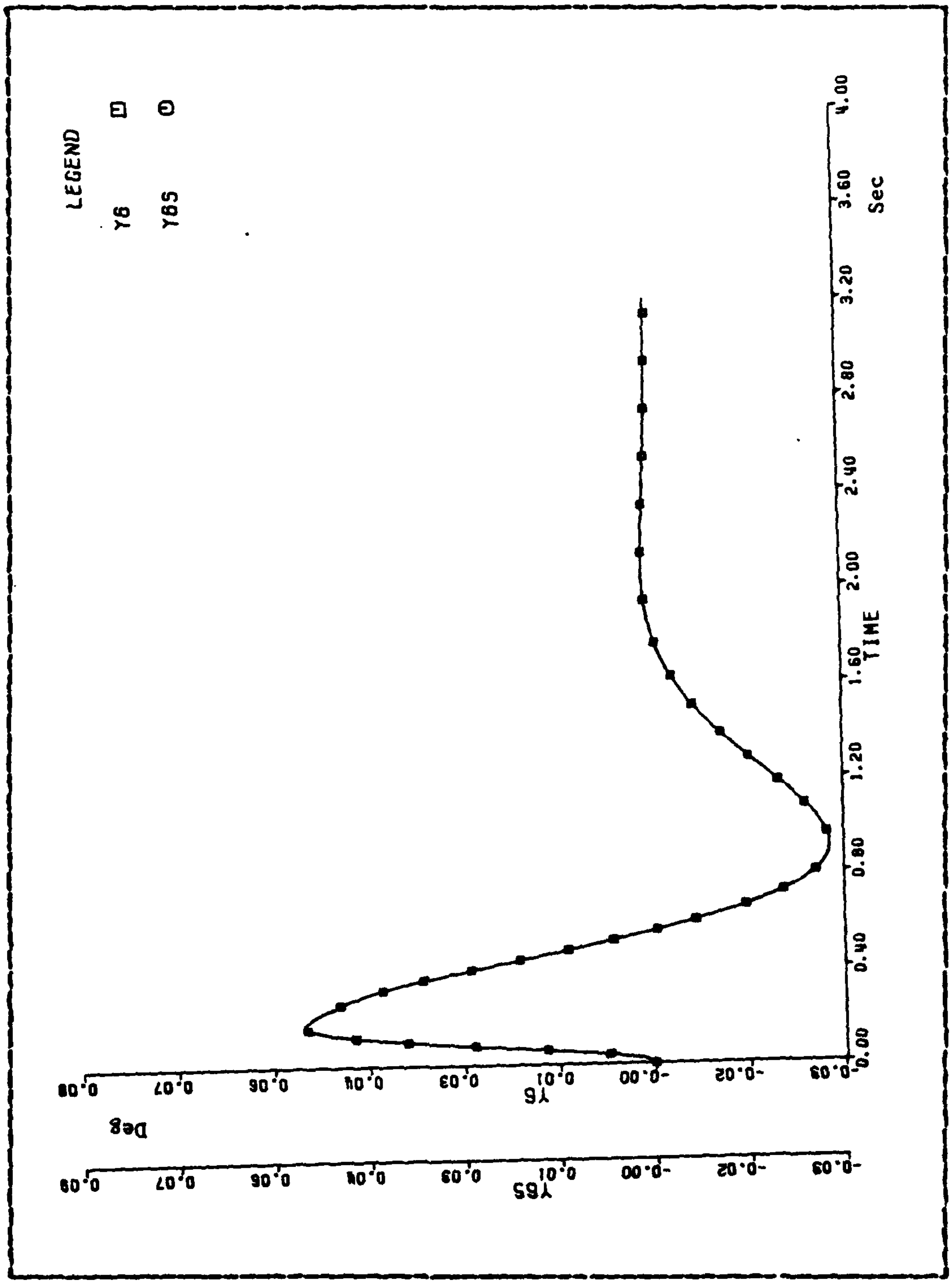

Figure 3.56   Actual and Nominal Output of X6 (10% variation).

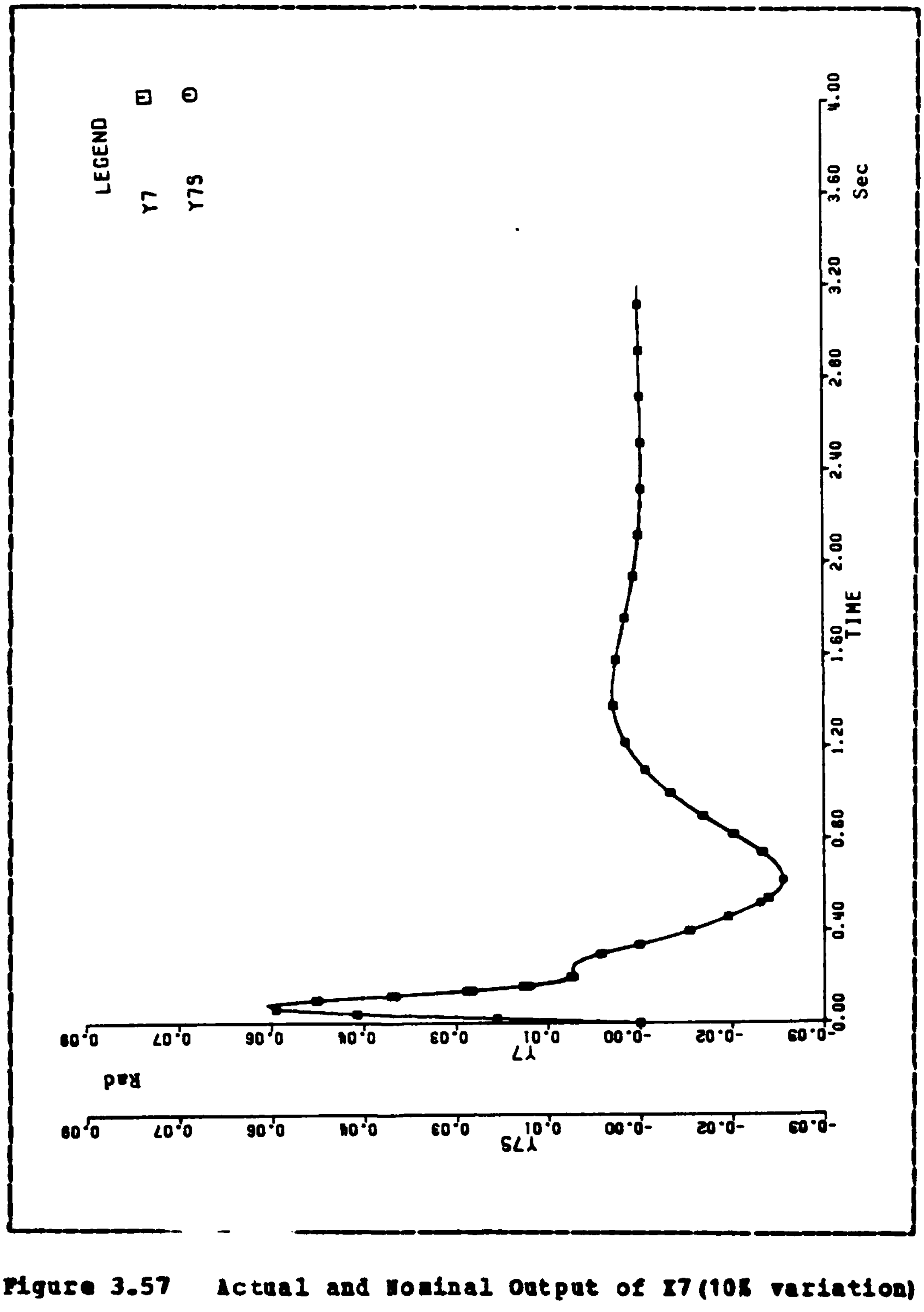

Figure 3.57 Actual and Nominal Output of X7(10% variation).

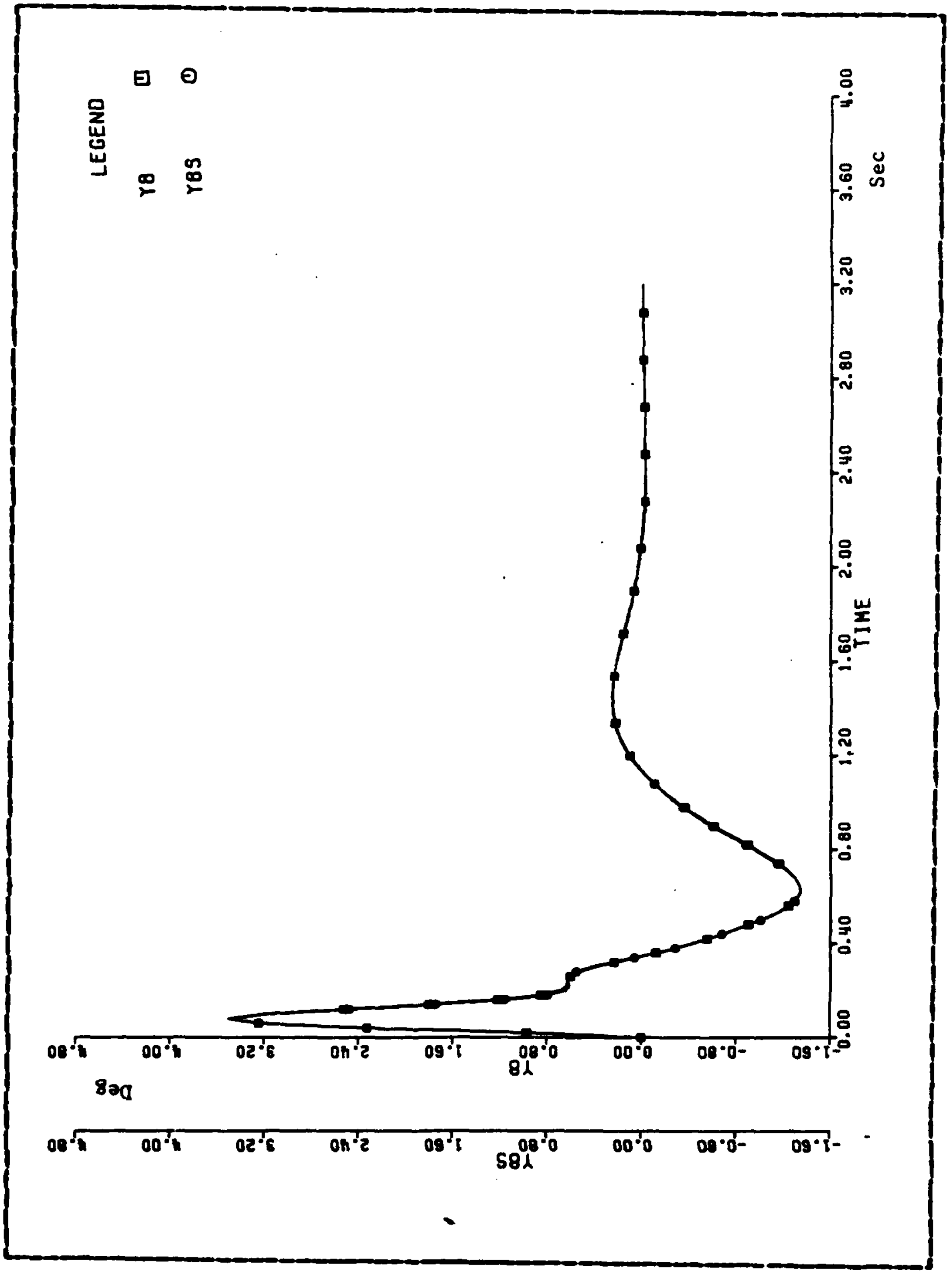

Figure 3.58 Actual and Nominal Output of X8 (10% variation).

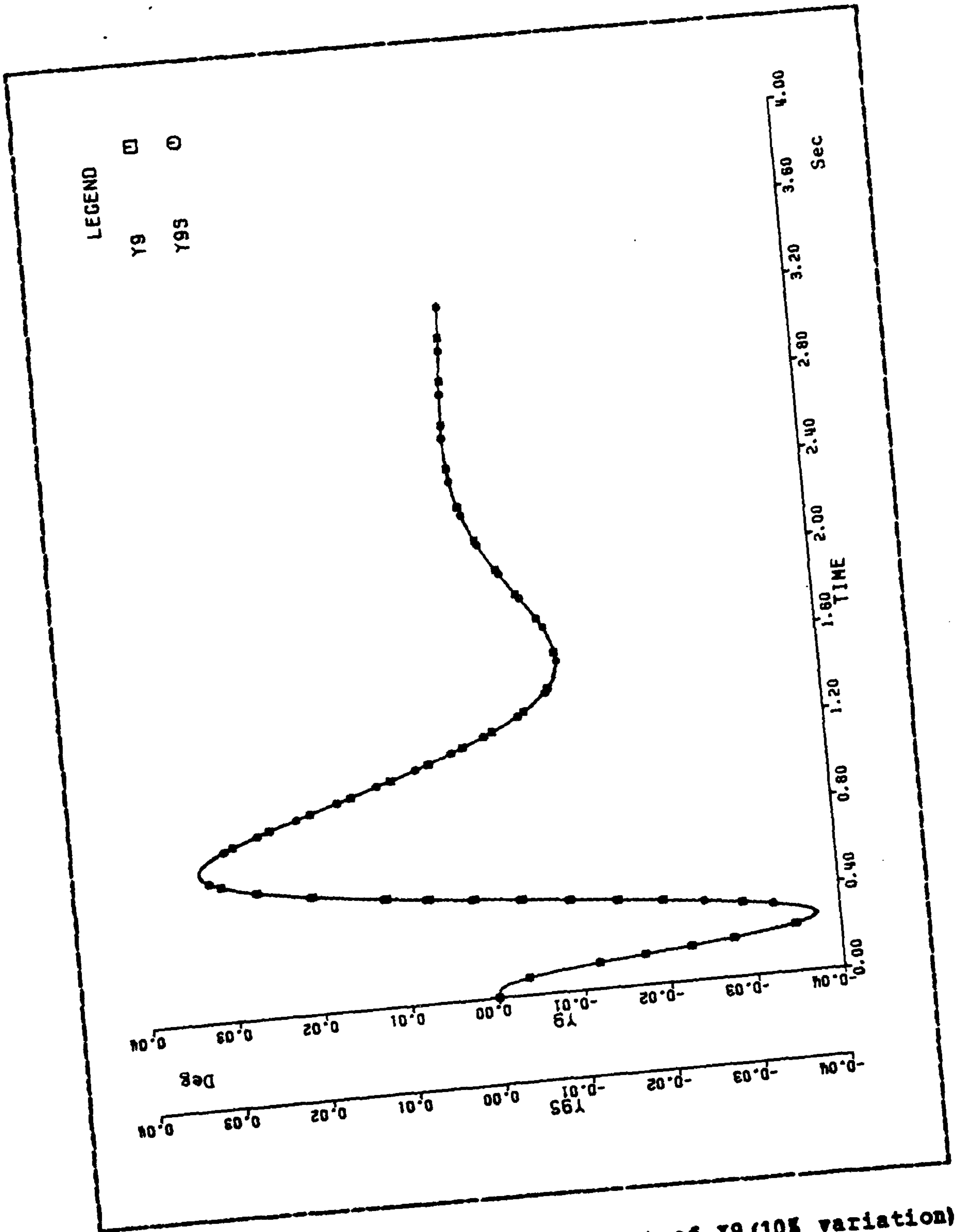

Figure 3.59 Actual and Nominal Output of X9 (10% variation).

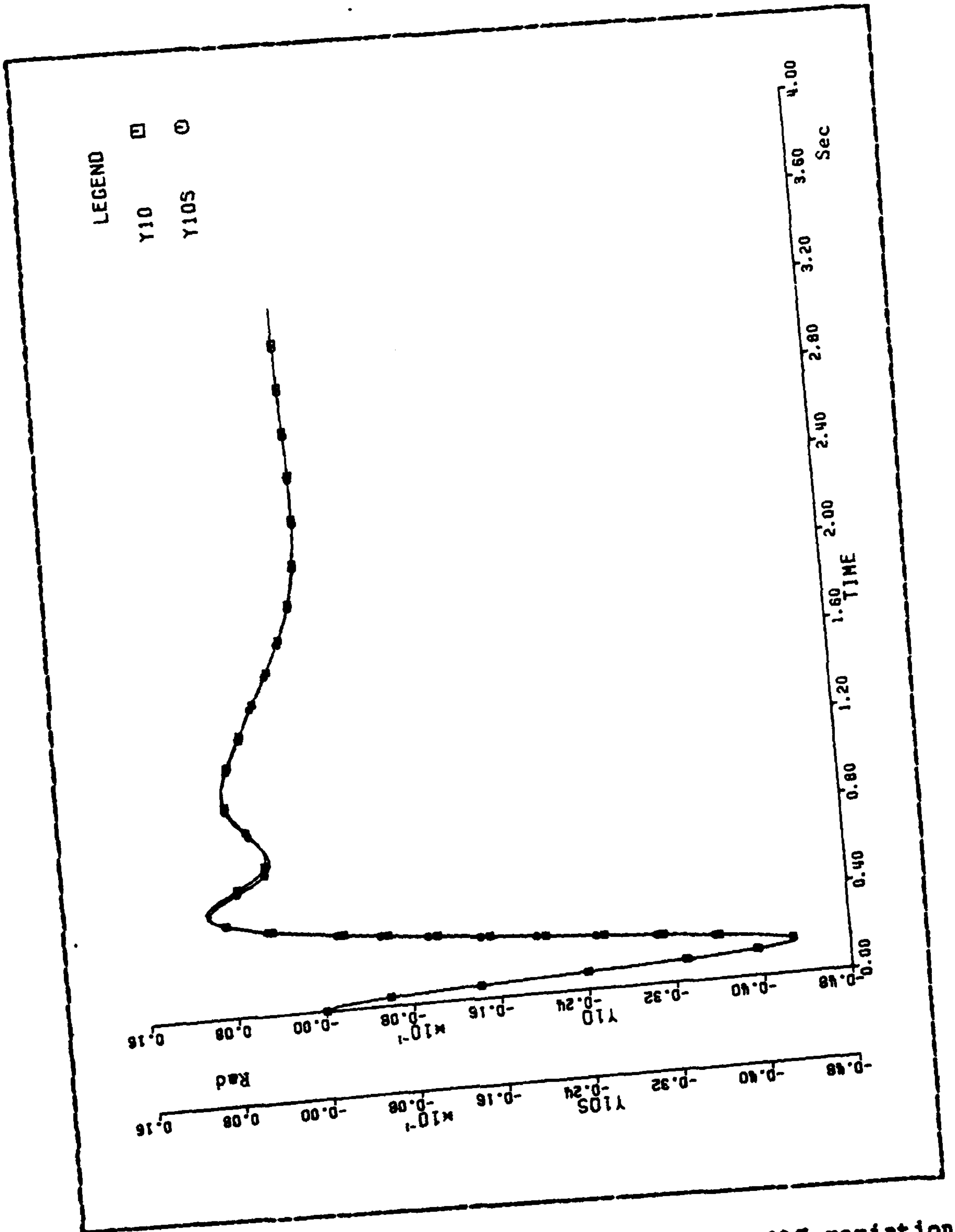

Figure 3.60   Actual and Nominal Output of X10(10% variation).

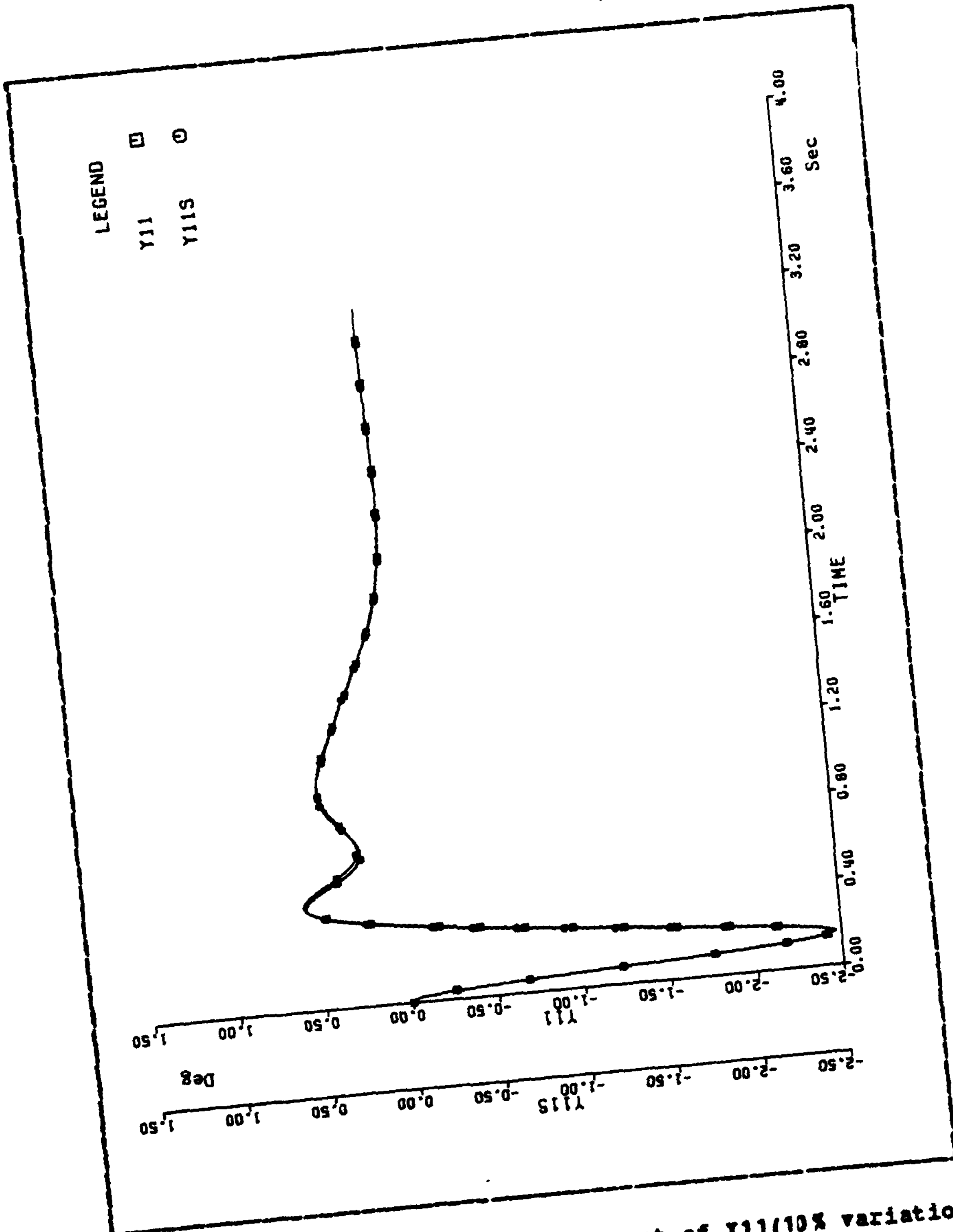

Figure 3.61 Actual and Nominal Output of X11(10% variation).

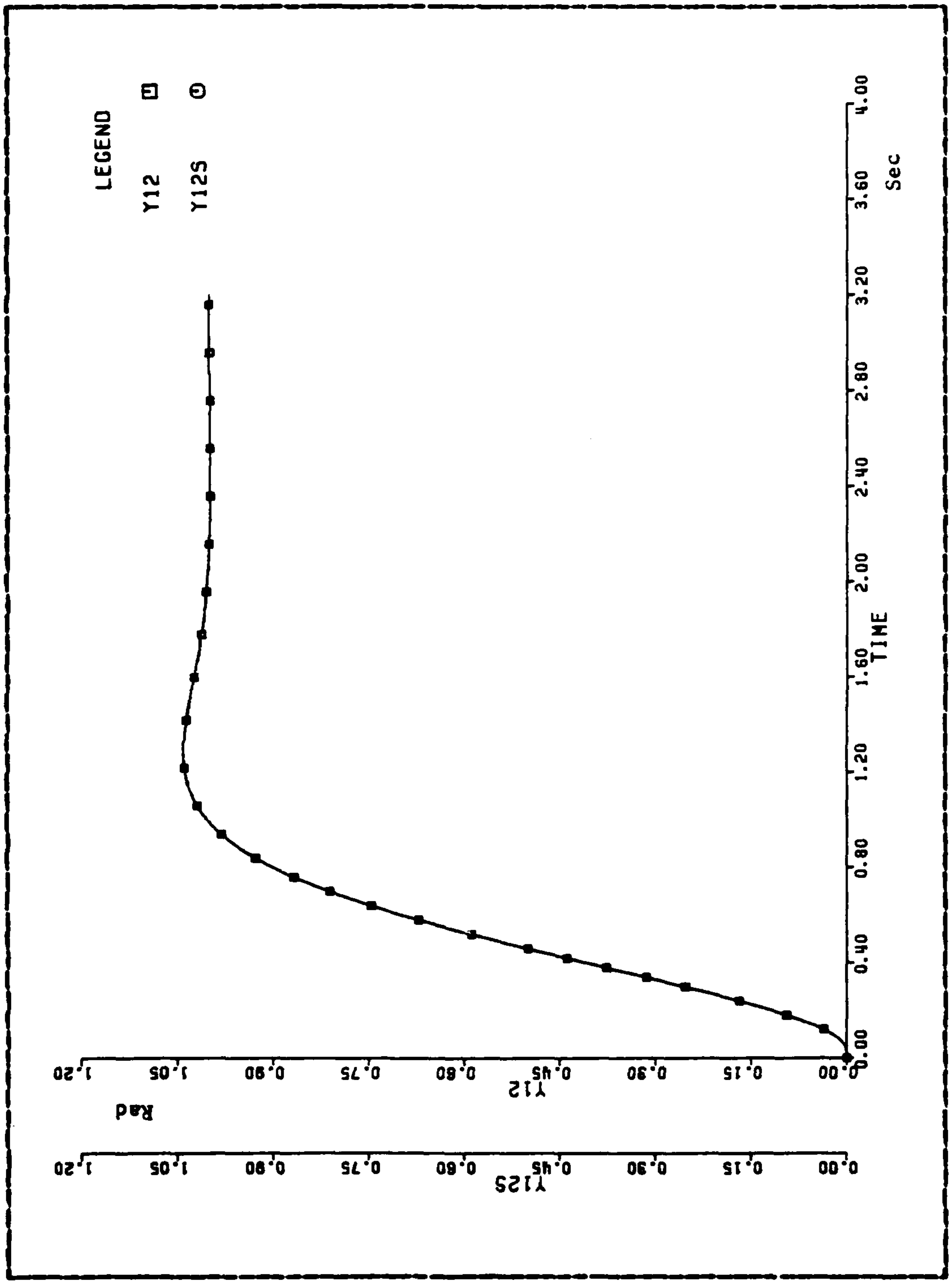

Figure 3.62   Actual and Nominal Output of X12(10% variation).

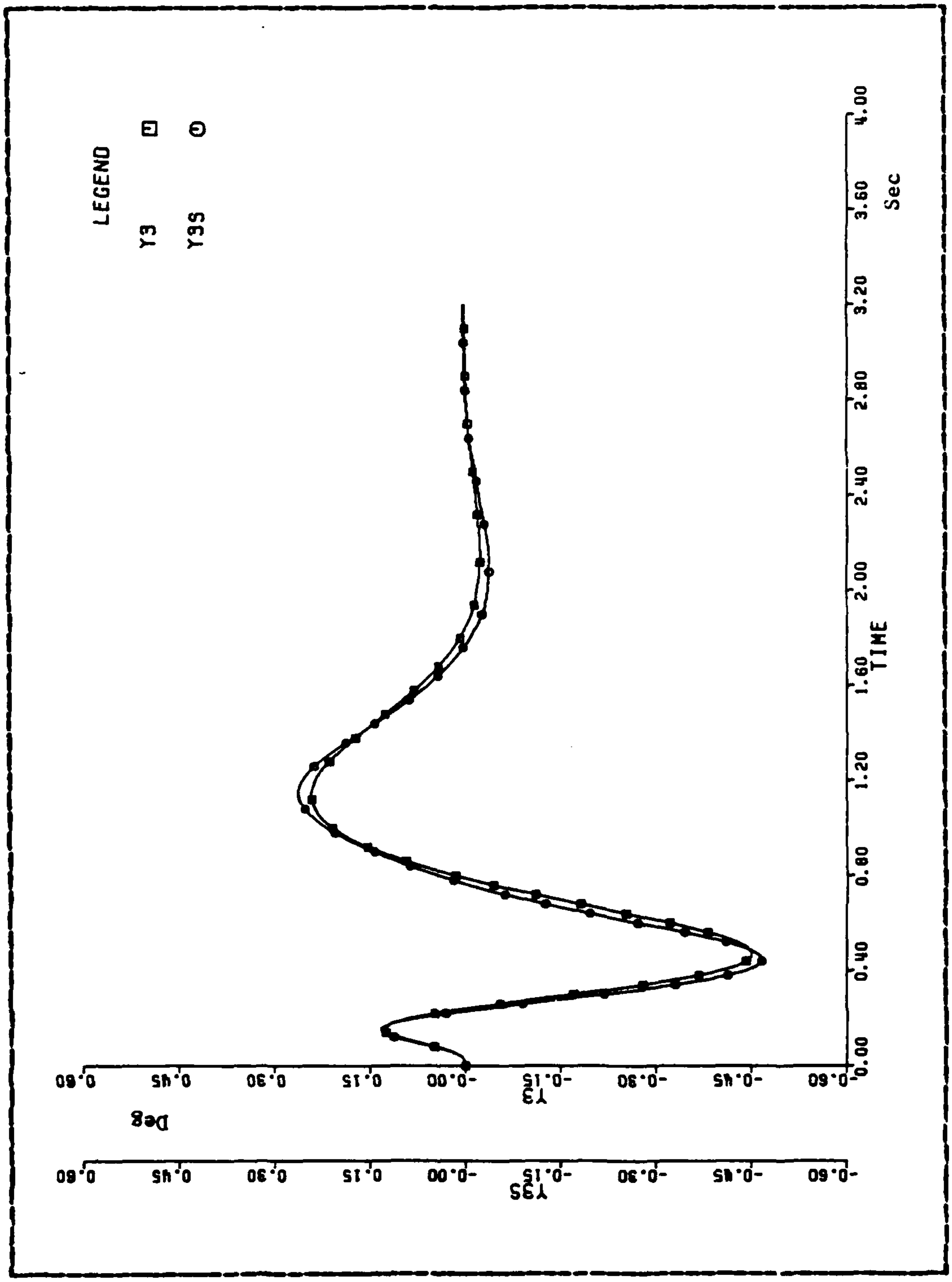

Figure 3.63   Actual and Nominal Output of X3(30% variation).

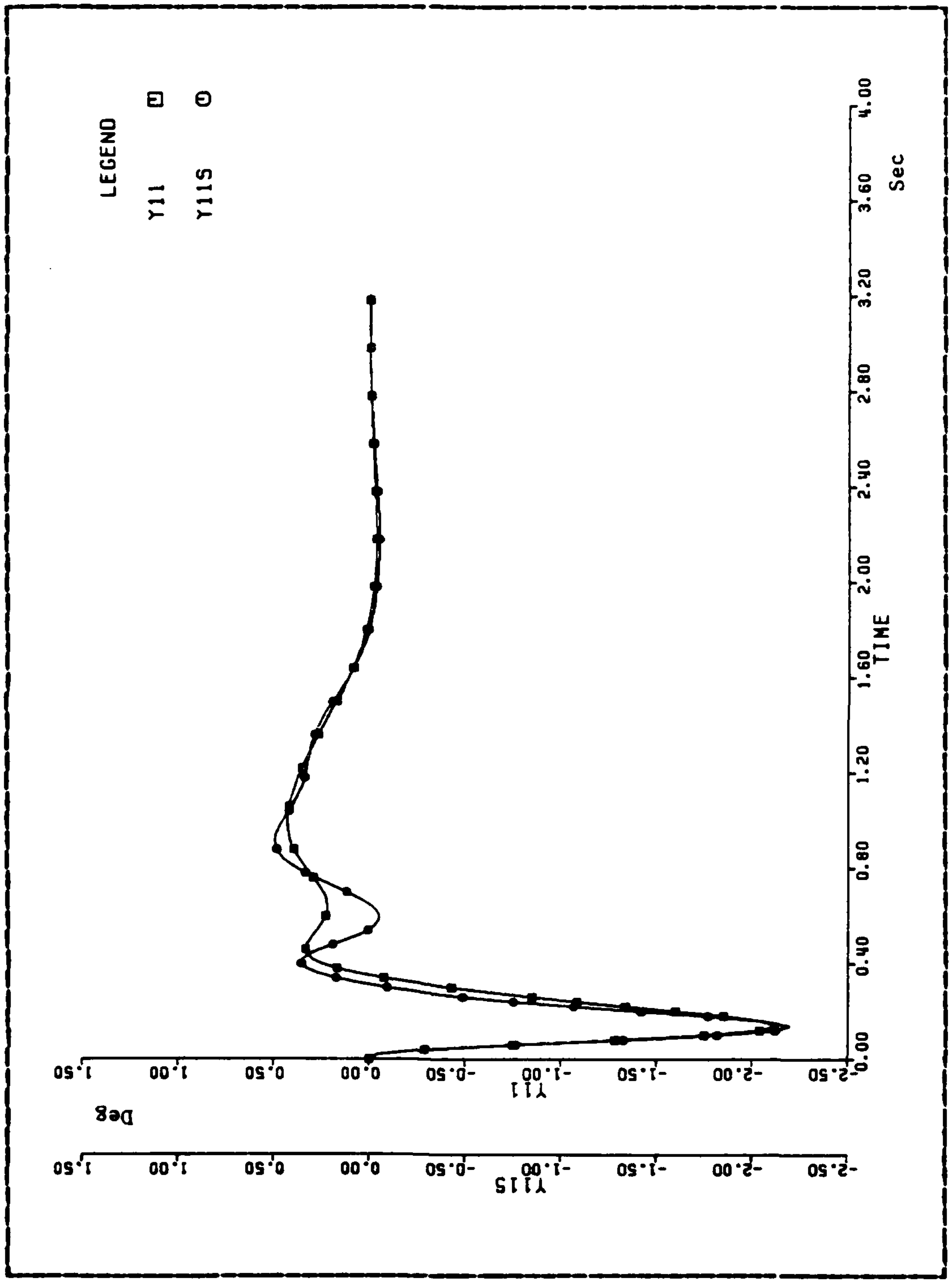

Figure 3.64 Actual and Nominal Output of X11(30% variation).

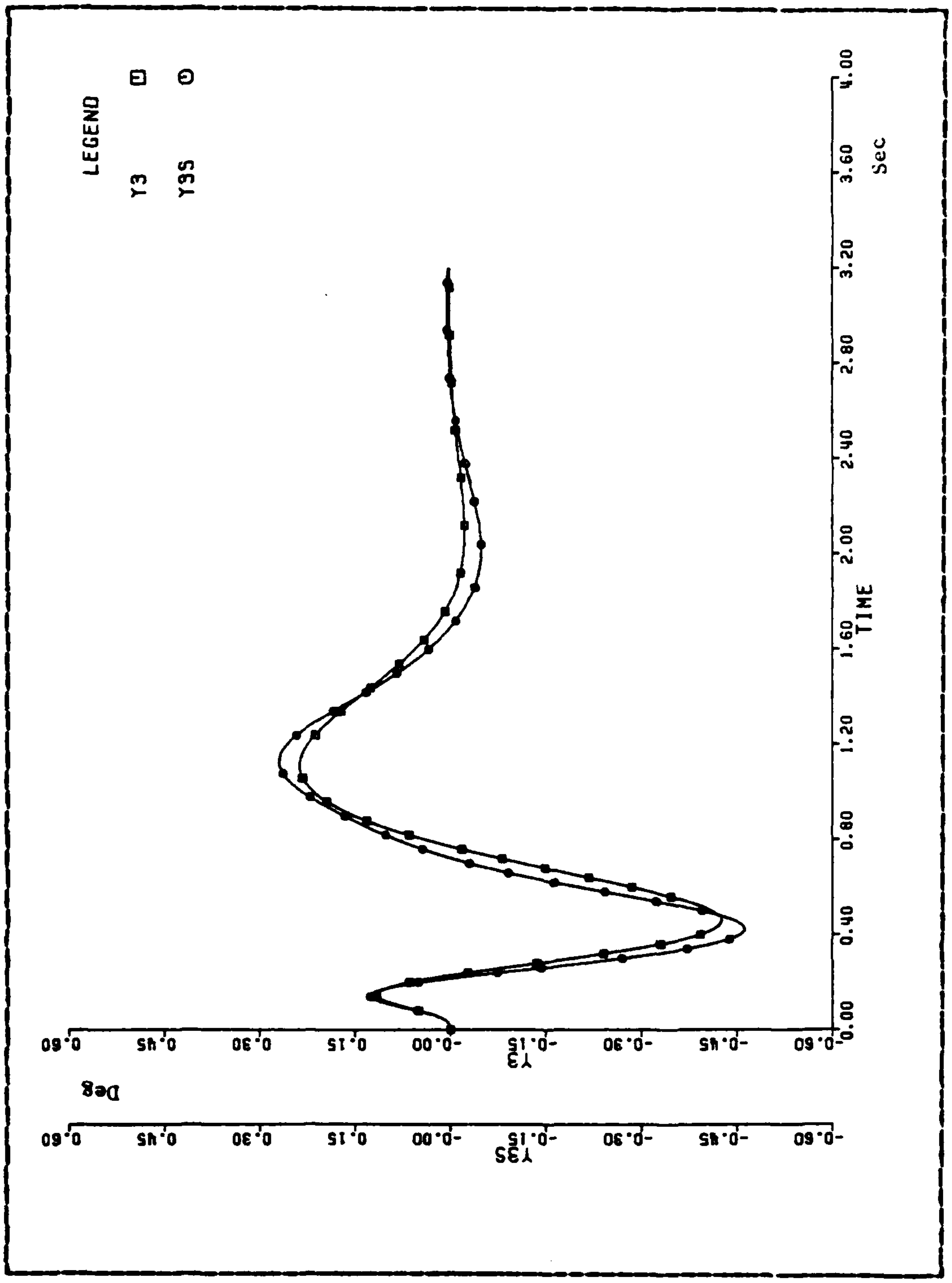

Figure 3.65 Actual and Nominal Output of X3 (40% variation).

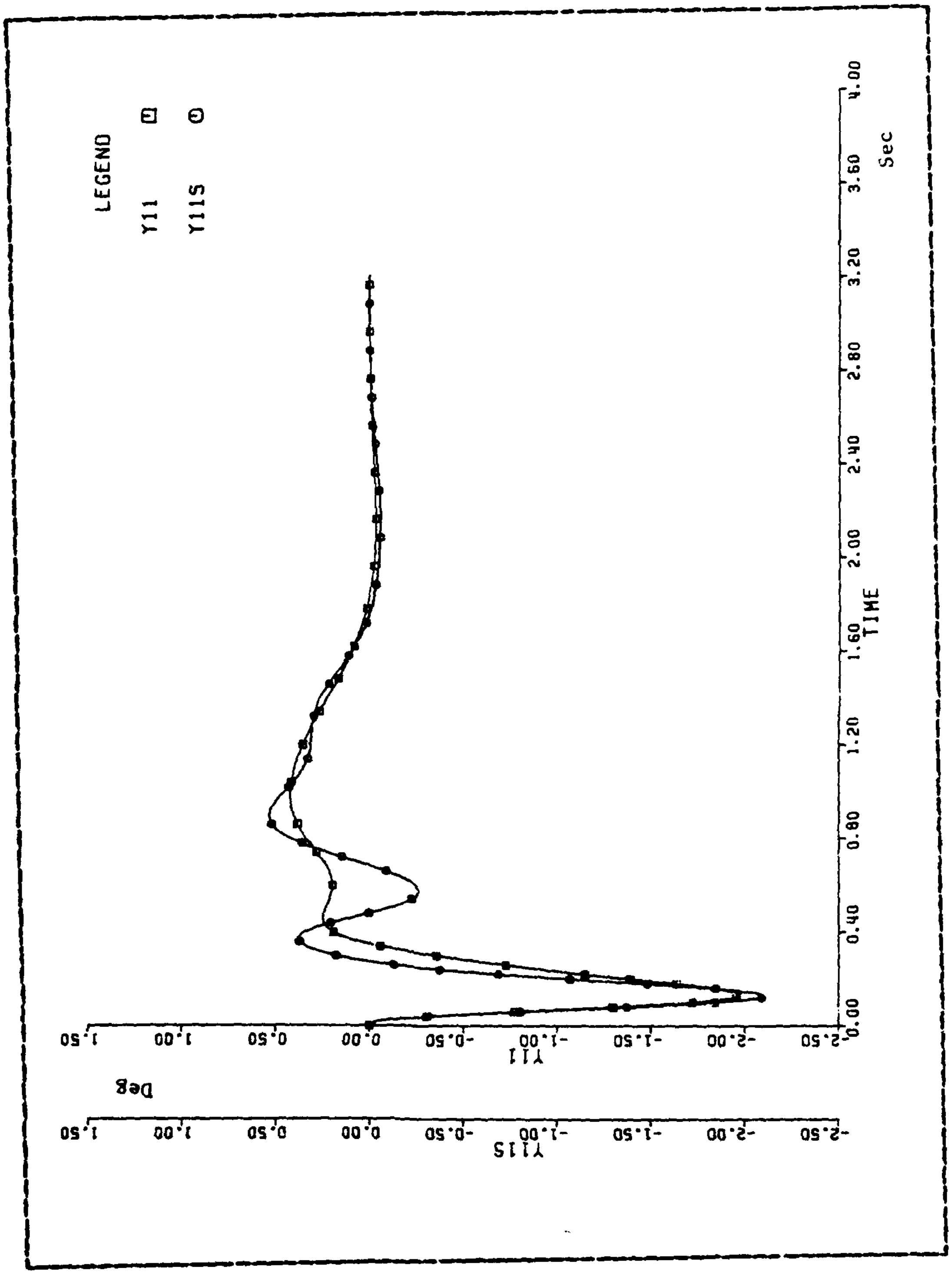

Figure 3.66 Actual and Nominal Output of X11(40% variation).

IV. APPLICATION OF SENSITIVITY ANALYSIS TO NONLINEAR SYSTEMS

A. INTRODUCTION

The three dimensional nonlinear system used here for sensitivity analysis purposes is presented in appendix C. The missile configuration, size and mass properties are the same as those used in the linear system and were presented in appendix A.

In section B the sensitivity equations with respect to parameters of nominal system is shown and in section C the sensitivity equations with respect to parameters that showed to be of more effect in the previous analysis is presented.

Section D shows the results of the trajectory sensitivity equations when step inputs are applied at specific time as mentioned in appendix C.

Section E gives the parameter-induced output analysis when each parameter is varied 10% from the nominal value.

B. NONLINEAR EQUATIONS OF THE NOMINAL SYSTEM

From the block diagram of Fig.C.1 and Eqn.C.3 through C.21 as given in appendix C one have the following nominal equations.

$$\dot{X1} = -\ C4\ X1 - C02\ C4\ A2 \tag{4.1}$$

$$\dot{X2} = -\ C5\ X2 - C6\ NZC + C6\ C19\ \cos(X7) + C7\ X1 \tag{4.2}$$

$$\dot{X3} = (C5\ C8 - C9)\ X2 + C6\ C8\ NZC \tag{4.3}$$

$$- C6\ C8\ C19\ \mathrm{Cos}(X7) - C7\ C8\ X1 + (C8/Conv^2)\ X17\ X18$$
$$+ C1\ C8\ A1 + (C9/Conv)\ X5$$

$$\dot{X4} = - C3\ X4 + C3\ Conv\ X3 \tag{4.4}$$

$$\dot{X5} = (X17\ X18)/Conv + C1\ Conv\ A1 \tag{4.5}$$

$$\dot{X6} = X5 - KB\ CO2\ A2 - (X16\ X18)/Conv \tag{4.6}$$

$$\dot{X7} = X5\ \mathrm{Cos}(X19)/Conv - X17\ \mathrm{Sin}(X19)/Conv \tag{4.7}$$

$$\dot{X8} = - C10\ X8 + (C12 - C11\ C13)\ X10 \tag{4.8}$$

$$+ C11\ C14\ PHC - C11\ C14\ X19 - C11\ C0(A8\ X16 + A7\ X15$$
$$+ A3\ X13) - (C12/Conv)\ X18$$

$$\dot{X9} = - X9/T1 + K1/T1\ CO2(AA\ X16 + A6\ X15) \tag{4.9}$$

$$\dot{X10} = - C13\ X10 + C14\ PHC - C14\ X19 \tag{4.10}$$

$$\dot{X11} = - C15\ X11 + C16\ C0(A8\ X16 + A7\ X15 + A3\ X13) \tag{4.11}$$

$$\dot{X}12 = -C17\ X12 + K(C17 - (C10/C18)\ X8 \qquad (4.12)$$

$$(K/C18)(C12 - C11\ C13)\ X10 + K\ C11\ (C14/C18)\ PHC$$
$$- K\ C11\ (C14/C18)\ X19 - (K/C18)\ C0\ A8(C11 - (K/C18)$$
$$C0\ A3(C11 + C16)\ X13 - (K/C12)(C18\ Conv)\ X18$$
$$+ K(C15/C18) - C17)\ X11$$

$$\dot{X}13 = -C3\ X13 + C3\ Conv\ X12 \qquad (4.13)$$

$$\dot{X}14 = (K2/10)(-(X9/T1)\ C02(AA\ X16 + A6\ X15) \qquad (4.14)$$

$$- X15\ (X18/Conv^2) + C01(A9\ X16 + A4\ X15 + A5\ X13)$$
$$- (KYP/Conv^2)(X5 - KB\ C02\ A2 - X16(X18/Conv)\ X18$$
$$- (KYP/Conv)\ XX6\ C0(a8\ X16 + A7\ X15 + A3\ X13)$$
$$+ (K2/Conv)\ X17 - K2(KYP/Conv^2)\ XX6\ X18$$

$$\dot{X}15 = -C3\ X15 + C3\ Conv\ X14 \qquad (4.15)$$

$$\dot{X}16 = KB\ C02(AA\ X16 + A6\ X15) + X5(X18/Conv) - X17 \qquad (4.16)$$

$$\dot{X}17 = -(X5\ X18)/Conv + C01\ Conv(A9\ X16 \qquad (4.17)$$

$$+ A4\ X15 + A5\ X13)$$

$$\dot{X}18 = C0\ Conv(A8\ X16 + A7\ X15\ A3\ X13) \qquad (4.18)$$

$$\dot{X}19 = X18/Conv \qquad (4.19)$$

The correspondence of nonlinear state vectors[1] as given in the above equations with relation to the nonlinear system presented in Fig.C.1 in appendix C are :

$X1 = X$, $X2 = Y$, $X3 = \delta_{Pc}$, $X4 = \delta_P$, $X5 = q$,
$X6 = \alpha$, $X7 = \theta$, $X8 = Y1$, $X9 = Y2$, $X10 = X1$,
$X11 = X2$, $X12 = \delta_{Rc}$, $X13 = \delta_R$, $X14 = \delta_{Yc}$, $X15 = \delta_Y$,
$X16 = \beta$, $X17 = r$, $X18 = p$, and $X19 = \phi$.

The parameters of interest for this nonlinear system are given by

$A1 = C_m(\alpha,\delta_p)$, $A2 = C_N(\alpha,\delta_p)$, $A3 = C_{q_{\delta_R}}(\alpha)$, $A4 = C_{n_{\delta_Y}}(\alpha)$, and $A5 = C_{n_{\delta_R}}(\alpha)$.

Definition of the constants C0 through C19 are given in appendix C.

As previously, this nomenclature is used here to easily apply the sensitivity theory.

C. NONLINEAR SENSITIVITY EQUATIONS

From the sensitivity theory given in chapter 2 one knows from Eqn.2.12 that for nonlinear systems one has

$$\frac{\partial \dot{\bar{x}}}{\partial \alpha_j} = \frac{\partial \bar{f}}{\partial \bar{x}} \frac{\partial \bar{x}}{\partial \alpha_j} + \frac{\partial \bar{f}}{\partial \alpha_j} , \frac{\partial \bar{x}^0}{\partial \alpha_j} = 0 \qquad (4.20)$$

Here

$$\bar{X} = \bar{f}\{X1,X2,\ldots,X19,t,NZC,A1,A2,\ldots,A5\} \qquad (4.21)$$

---

[1]The state vectors and parameters in the nonlinear case have no correspondence to the state vectors and parameters used in the linear case.The correspondence can be found in the appendices B and C.

From Eqn.2.14 one knows that the trajectory sensitivity vector is given as

$$\dot{\bar{\lambda}}_j = \frac{\partial \bar{f}}{\partial \bar{x}}\Big|_{\bar{\alpha}_0} \bar{\lambda}_j + \frac{\partial \bar{f}}{\partial \alpha_j}\Big|_{\bar{\alpha}_0}, \quad \bar{\lambda}(0) = 0 \tag{4.22}$$

Applying the above theory one obtains the nonlinear sensitivity equations as given in Eqn.4.24 through &4e42, where A_1 was considered the parameter to be varied.

$$\dot{\lambda}11 = - C4\,\lambda 11 - C02\ C4(DA26\,\lambda 61 + DA24\,\lambda 41) \tag{4.23}$$

$$\dot{\lambda}21 = - C5\,\lambda 21 - C6\ C19\ Sin(X7)\,\lambda 71 + C7\,\lambda 11 \tag{4.24}$$

$$\dot{\lambda}31 = (C5\ C8 - C9)\,\lambda 21 + C6\ C8\ C19\ Sin(X7)\,\lambda 71 \tag{4.25}$$

$$- C7\ C8\,\lambda 11 + C8(x18\,\lambda 171 + (X17\,\lambda 181)/(Conv^2)$$

$$+ C1\ C8(DA16\,\lambda 61 + DA14\,\lambda 41 + 1.) + (C9\,\lambda 51)/Conv$$

$$\dot{\lambda}41 = - C3\,\lambda 41 + C3\ Conv\,\lambda 31 \tag{4.26}$$

$$\dot{\lambda}51 = (X17\,\lambda 181 + X18\,\lambda 171)/Conv \tag{4.27}$$

$$+ C1\ Conv(DA16\,\lambda 61 + DA14\,\lambda 41 + 1.)$$

$$\dot{\lambda}61 = \lambda 51 - KB\ C02(DA26\,\lambda 61 + DA24\,\lambda 41) \tag{4.28}$$

$$- ((X16\,\lambda 181 + X18\,\lambda 161)/Conv$$

$$\dot{\lambda}71 = (\mathrm{Cos}(X19)\,\lambda51 - X5\ \mathrm{Sin}(X19)\,\lambda191)/\mathrm{Conv} \qquad (4.29)$$

$$- (\mathrm{Sin}(X19)\,\lambda171 + X17\ \mathrm{Cos}(X19)\,\lambda171)/\mathrm{Conv}$$

$$\dot{\lambda}81 = - C10\,\lambda81 + (C12 - C11\ C13)\,\lambda101 \qquad (4.30)$$

$$- C11\ C14\,\lambda191 - C11\ C0\ (A8\,\lambda161 + A7\,\lambda151 + A3\,\lambda131$$
$$+ \lambda61(X16\ Da8 + X15\ Da7 + X13\ DA13)) - (C12\,\lambda181)/\mathrm{Conv}$$

$$\dot{\lambda}91 = -\lambda91/T1 + K1/(T1\ C02)\,(AA\,\lambda161 \qquad (4.31)$$

$$+ A6\,\lambda151 + \lambda61(X16\ DAA + X15\ Da6))$$

$$\dot{\lambda}101 = - C13\,\lambda101 - C14\,\lambda191 \qquad (4.32)$$

$$\dot{\lambda}111 = - C15\,\lambda111 + C16\ C0(A8\,\lambda161 + A7\,\lambda151 \qquad (4.33)$$

$$+ A3\,\lambda131 + \lambda61(X16\ DA8 + X15\ DA7 + X13\ DA3))$$

$$\dot{\lambda}121 = - C17\,\lambda121 + K(C17 - C10/C18)\,\lambda81 \qquad (4.34)$$

$$+ (K/C18)\,(C12 - C11\ C13)\,\lambda101 - K\ C11\ (C14/C18)\,\lambda191$$
$$- (K/C18)\ C0(C11 + C16)\,(A8\,\lambda161 + A7\,\lambda151 + A3\,\lambda131$$
$$+ A3\,\lambda131 + \lambda61(X16\ DA8 + X15\ DA7 + X13\ Da3))$$
$$- K\ C12/(C18\ \mathrm{Conv})\,\lambda181 + K\ (C15/C18 - C17)\,\lambda111$$

$$\dot{\lambda}131 = - C3\,\lambda131 + C3\ \mathrm{Conv}\,\lambda121 \qquad (4.35)$$

$$\dot{\lambda}141 = (K2/10)(-\ X9/T1 + (K1/T1)\ C02(AA \tag{4.36}$$

$$\begin{aligned}
&\lambda 161 + A6\lambda 151 + \lambda 61(X16\ DAA + X15\ DA6)) - (X15\lambda 181 \\
&+ X18\lambda 151)/(Conv^2) + C01(A9\lambda 161 + A4\lambda 151 + A5\lambda 131 \\
&+ \lambda 61(X16\ DA9 + X15\ DA4 + X13\ DA5)) - KYP/(Conv^2) \\
&(X5\lambda 181 + X18\lambda 51 - KB\ C02(A2\lambda 181 + X18(DA25\lambda 61 \\
&+ DA24\lambda 41) - (X18^2\lambda 161 + 2\ X18\ X16\lambda 181)\ /Conv)) \\
&- (KYP/Conv)\ C0(\ xx6\lambda 161 + X16\lambda 161) + A7\lambda 161) \\
&+ A7(XX6\lambda 151 + X15\lambda 161) + A3(XX6\lambda 131 + X13\lambda 161) \\
&+ X16(XX6\ DA8\lambda 61 + A8\lambda 161) + X15(XX6\ DA7\lambda 61 \\
&+ A7\lambda 161) + X13(XX6\ DA3\lambda 61 + A3\lambda 161))) \\
&+ (K2/Conv)\lambda 171 + K2\lambda 91 - K2\ KYP/(Conv^2) \\
&(XX6\lambda 181 + X18\lambda 161))
\end{aligned}$$

$$\dot{\lambda}151 = -\ C3\lambda 151 + C3\ Conv\lambda 141 \tag{4.37}$$

$$\dot{\lambda}161 = KB\ C02(AA\lambda 61 + A6\lambda 151 + \lambda 61(\tag{4.38}$$

$$X16\ DAA + X15\ DA6)) + (X6\lambda 181 + X18\lambda 61)/Conv - \lambda 171$$

$$\dot{\lambda}171 = -\ (X5\lambda 181 + X18\lambda 51)/Conv \tag{4.39}$$

$$\begin{aligned}
&+ C01\ Conv(A9\lambda 161 + A4\lambda 151 + A5\lambda 131 + \lambda 61(X16\ DA9 \\
&+ X15\ DA4 + X13\ DA5))
\end{aligned}$$

$$\dot{\lambda}181 = C0\ Conv(A8\lambda 161 + A7\lambda 151 \tag{4.40}$$

$$+A3\lambda 131 + \lambda 61(X16\ DA8 + X15\ DA7 + X13\ DA3))$$

$$\dot{\lambda}191 = \lambda 181/Conv \tag{4.41}$$

The same procedure above must be made for the other parameters that most affected the previous analysis and will be given in the nonlinear sensitivity analysis presented in the next section.

D. NONLINEAR SENSITIVITY ANALYSIS

A computer program in appendix 3 shows the simulation of the sensitivity equations with respect to the selected parameters.

The number of equations solved are 19 for the nominal equations and 95 for the sensitivity equations. Here, the parameters were selected from the previous two cases given in Chapter III that showed to have most effect in the time response of the systems.

From the analysis performed, two parameters were selected from the uncoupled pitch autopilot and three from the coupled roll-yaw autopilot, which has been shown to be more sensible to parameter variations. These parameters are respectively $C_{N_{\delta_P}}$ (A2) and $C_{m_{\delta_P}}$ (A4) in the uncoupled pitch autopilot and $C_{\ell_{\delta_R}}$ (A3), $C_{n_{\delta_P}}$ (A5), and $C_{n_{\delta_R}}$ (A8) in the coupled roll-yaw autopilot.

In order to compare the influence of the parameter of interest in the nonlinear case some state variables were chosen to be analysed. The state variables selected were:

From the uncoupled pitch autopilot :

$X3 = \delta_{P_c}$, $X4 = \delta_P$, $X5 = q$, $X6 = \alpha$

From the coupled roll-yaw autopilot :

$X12 = \delta_{R_c}$, $X13 = \delta_R$, $X14 = \delta_{Y_c}$, $X15 = \delta_Y$,

$X16 = \beta$, $X17 = r$, $X18 = p$, and $X19 = \phi$.

The results are plotted in Fig.4.1 throught 4.12 and Table IV and V. Each state variable output is plotted separately with the correspondent five output sensitivity functions.

From the plots, the following observations can be made:

Fig.4.1 indicates that the state variable X3(δ_{P_c}) is little affected in the rise time by the parameters A1 and A2. The overshoot is strongly affected by A5, A1 and A4 and little affected by A2 and A3. The steady state is strongly affected by A1 and little affected by A2.

Fig.4.2 indicates that the state variable X4(δ_P) is little affected in the rise time by A1 and A2. The overshoot is strongly affected by A1, A3, and A4 with little effect by A2 and A5. The steady state is strongly affected by A1 and little affected by A2.

Fig.4.3 indicates that the state variable X5(q) is strongly affected in the rise time by A1 and A2. The overshoot is strongly affected by A3, A4, A5, and little affected by A1 and A2. The steady state is not affected by parameter variations.

Fig.4.4 indicates that the state variable X6(α) is strongly affected in the rise time by A1 and A2. The overshoot is strongly affected by A3, A4, and A5. The steady state is little affected by A1 and A2.

Fig.4.5 indicates that the rise time and steady state of the state variable X12(δ_{R_c}) are not affected by parameter variations. The overshoot is strongly affected by A3 and A5 and little affected by A1, A2, and A4.

Fig.4.6 indicates that the rise time and steady state of the state variable X13(δ_R) are not affected by parameter variations. The overshoot is strongly affected by A3, A4, and A5 and little affected by A1 and A2.

Fig.4.7 indicates that the rise time and the steady state of the state variable X14(δ_{Y_c}) are not affected by the

parameter variations. The overshoot is strongly affected by A4 and A5 and little affected by A1, A2, and A3.

Fig.4.8 indicates that the rise time and the steady state of the state variable X15(δ_{γ}) are not affected by the parameter variations. The overshoot is strongly affected by A3, A4, and A5 and little affected by A1, and A2.

Fig.4.9 indicates that the rise time and the steady state of the state variable X16(β) are not affected by the parameter variations. The overshoot is strongly affected by A3, A4, and A5 and little affected by A1, and A2.

Fig.4.10 indicates that the rise time and the steady state of the state variable X17(r) are not affected by the parameter variations. The overshoot is strongly affected by A3, A4, and A5 and little affected by A1, and A2.

Fig.4.11 indicates that the rise time and the steady state of the state variable X18(p) are not affected by the parameter variations. The overshoot is strongly affected by A3, A4, and A5 and little affected by A1, and A2.

Fig.4.12 indicates that the rise time and the steady state of the state variable X19(ϕ) are not affected by the parameter variations. The overshoot is strongly affected by A3, A4, and A5 and little affected by A1, and A2.

E. PARAMETER-INDUCED OUTPUT ANALYSIS

As shown previousy, if $\Delta\alpha \ll \alpha_0$, the actual output can be written as

$$y(t,\alpha) \triangleq y(t,\alpha_0) + \sigma(t,\alpha_0)\,\Delta\alpha \qquad (4.42)$$

The computer program given in appendix G was written for simulating the system when variations from the nominal value

of each parameter is assumed. Figs.4.13 throught 4.24 give the plots of the actual and nominal output of the state variables that are present in the unclouped pitch autopilot and coupled roll-yaw autopilot when 10% of parameter variation is assumed. The others state variables are not given here because they are just intermediate state variables.

these results confirm the theory presented in chapter 2 when small parameter variations are assumed. Figs.4.25 and 4.26 present the actual and nominal output of the state variables X3 and X11 that showed pronounced variations when 30% of parameter variation is assumed. Figs.4.27 and 4.28 present the actual and nominal output of X3 and X11 when 40% of parameter variation is assumed. From these polts one note that the modeling is starting to break down.

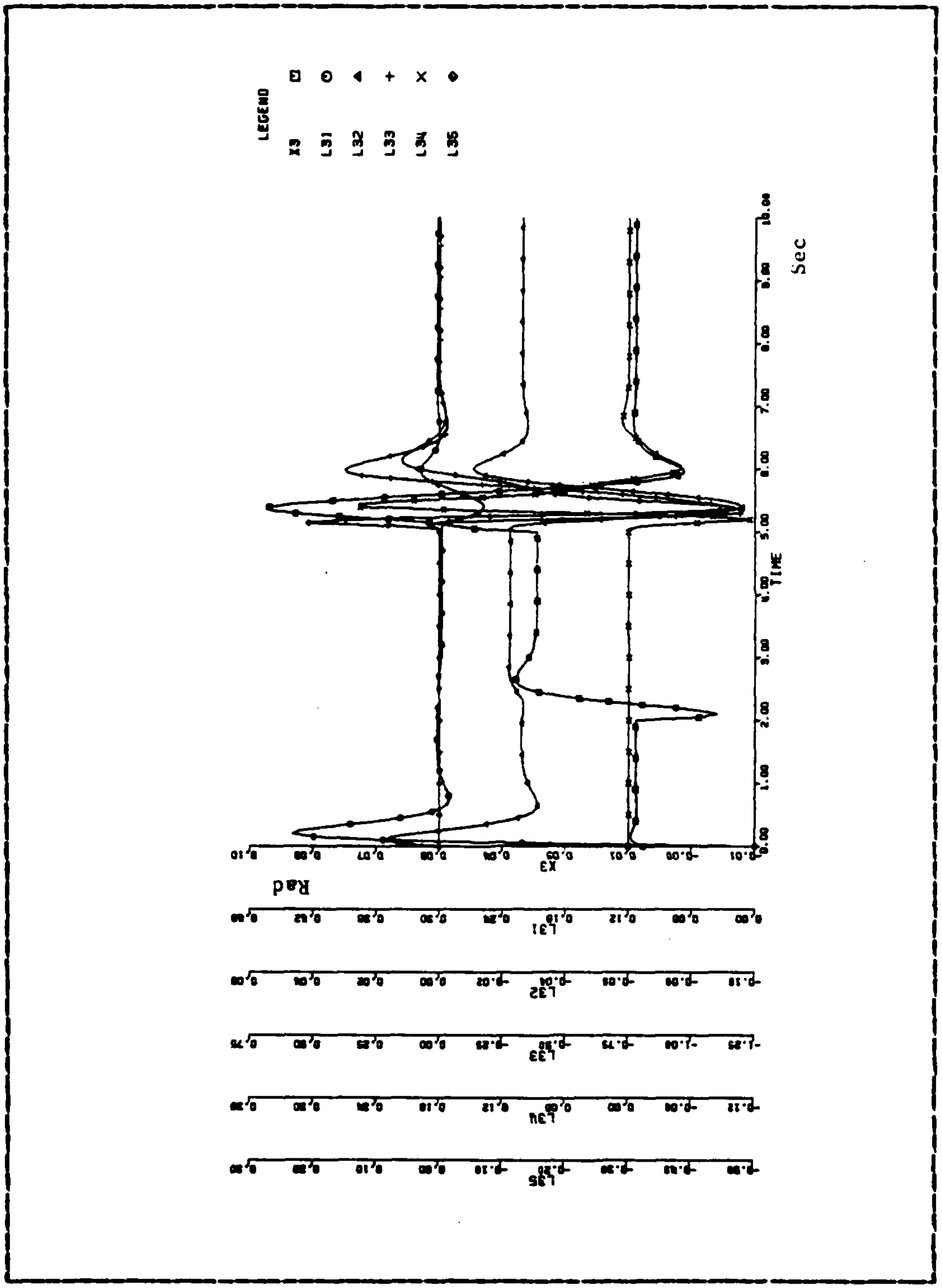

Figure 4.1 Sensitivity of X3 with respect to λ1,λ2,λ3,λ4,λ5.

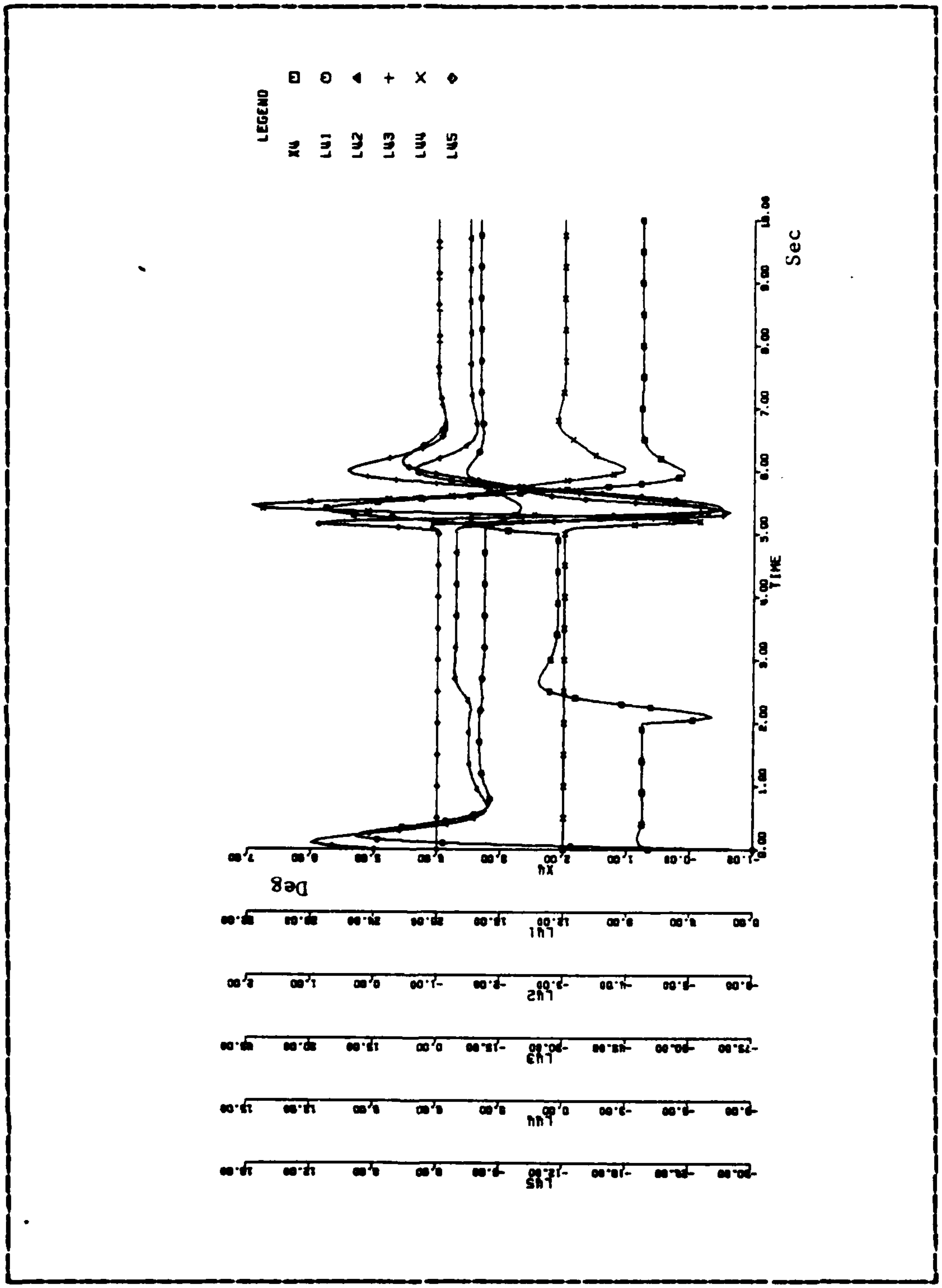

Figure 4.2 Sensitivity of X4 with respect to A1,A2,A3,A4,A5.

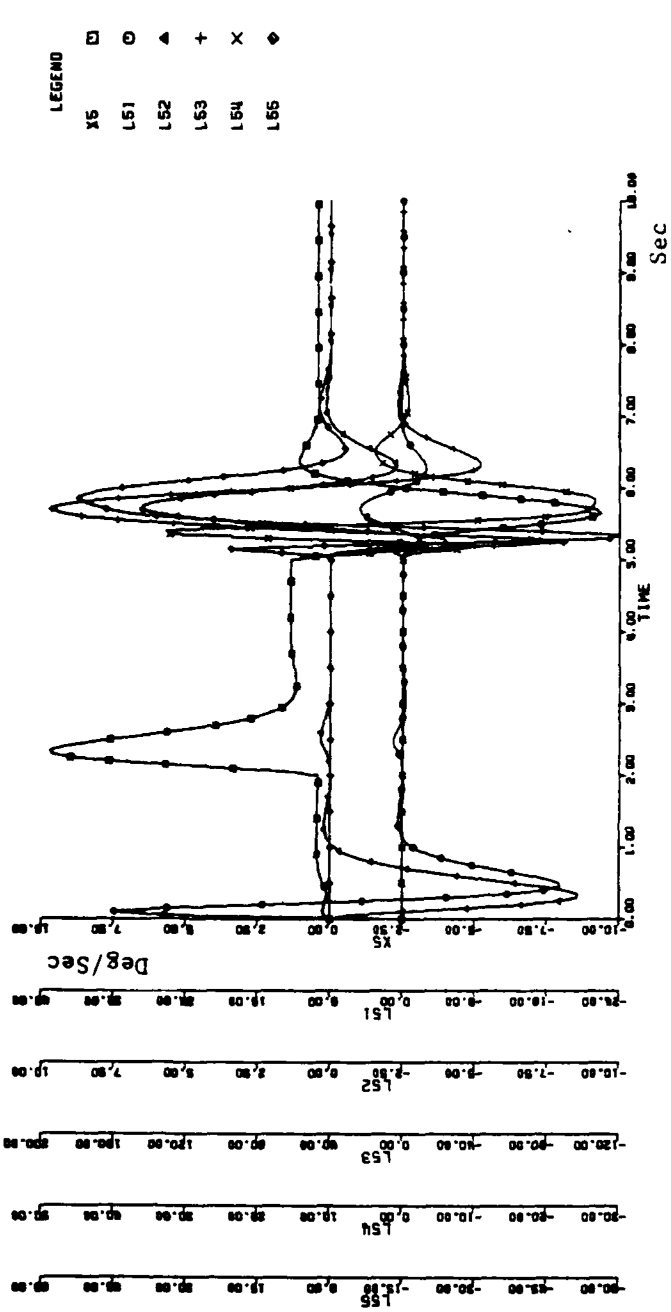

Figure 4.3 Sensitivity of X5 with respect to A1,A2,A3,A4,A5.

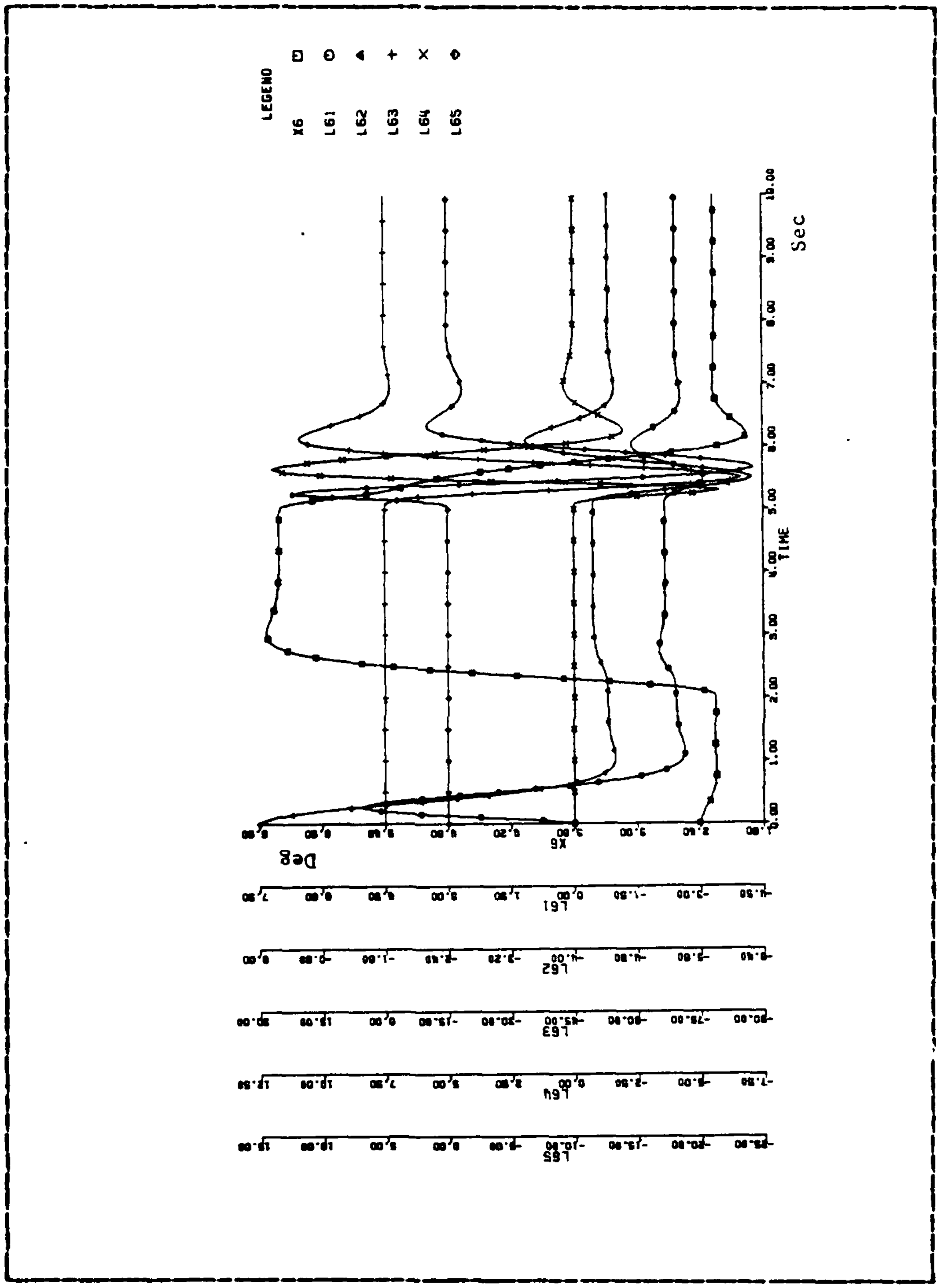

Figure 4.4 Sensitivity of X6 with respect to A1,A2,A3,A4,A5.

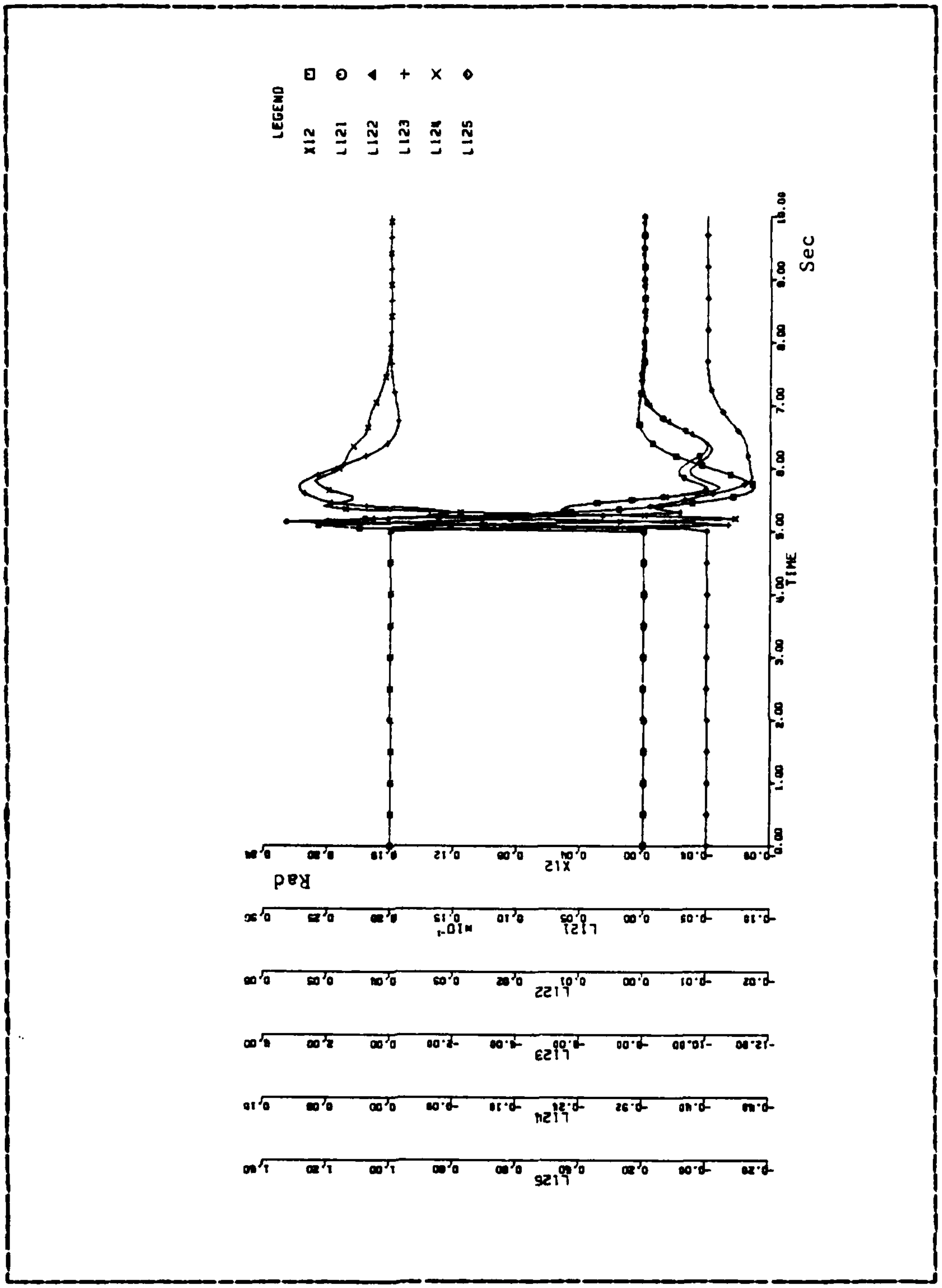

Figure 4.5 Sensitivity of X12 with respect to A1,A2,A3,A4,A5.

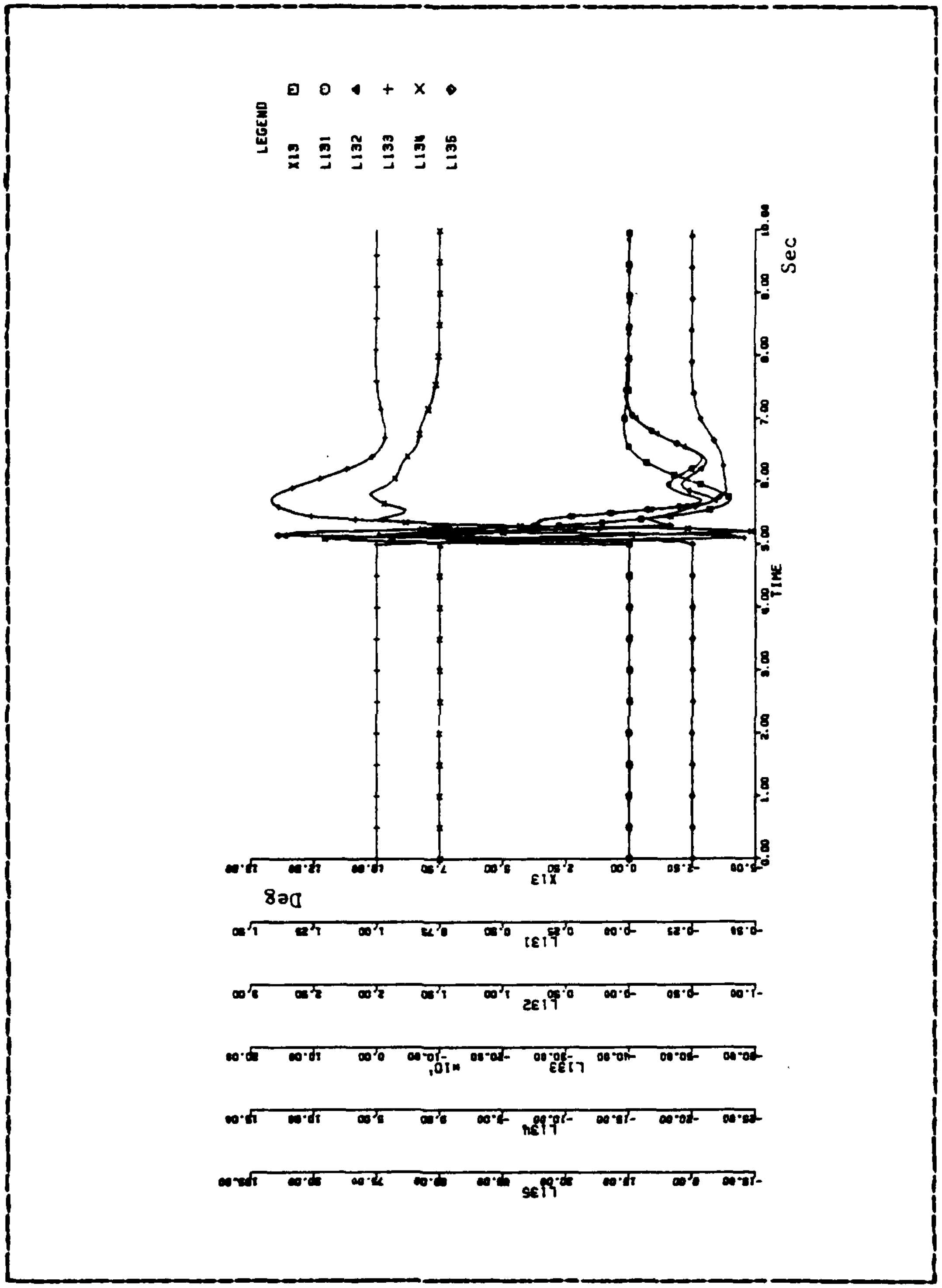

Figure 4.6   Sensitivity of X13 with respect to A1,A2,A3,A4,A5.

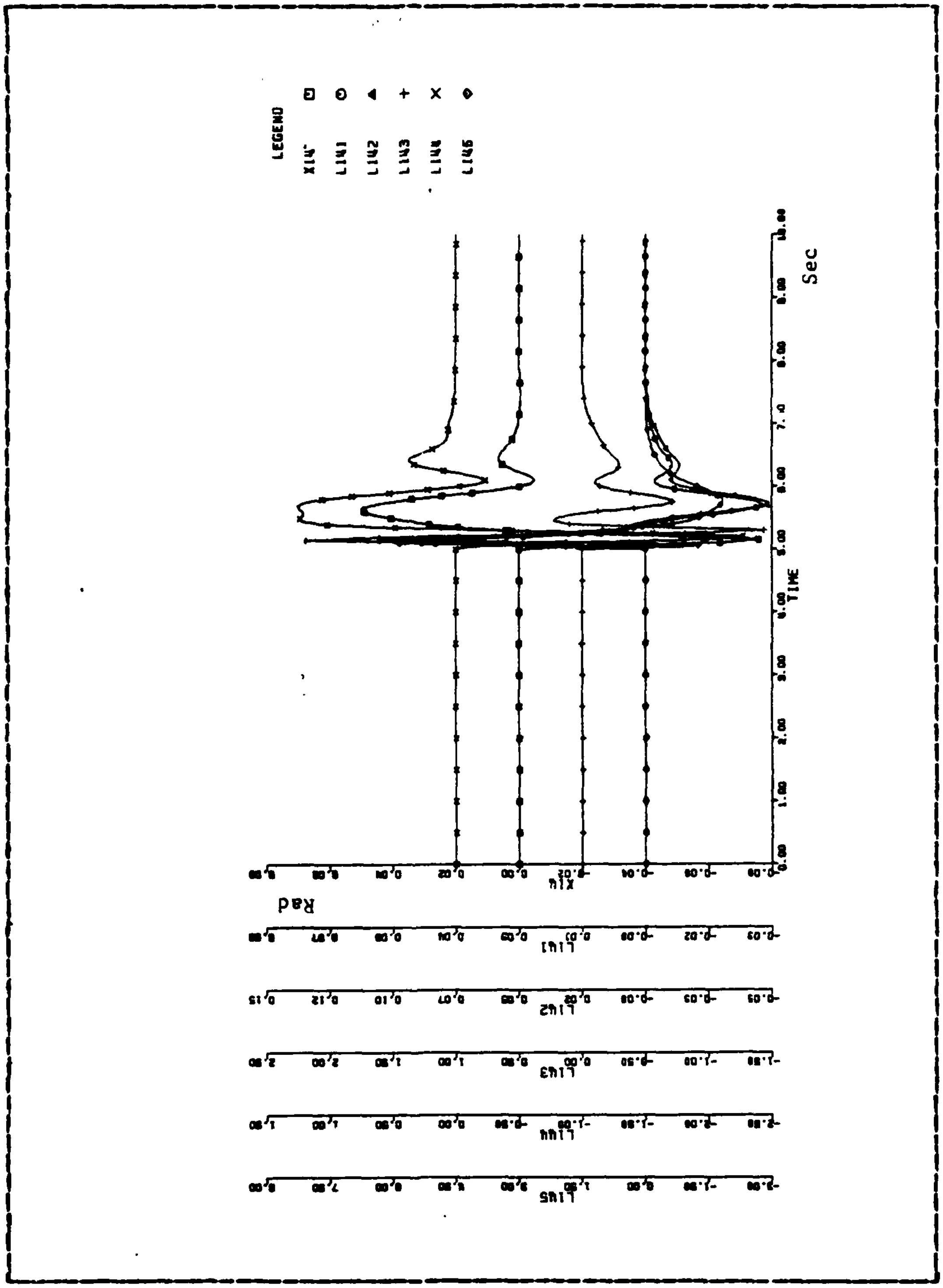

Figure 4.7 Sensitivity of X14 with respecc to A1,A2,A3,A4,A5.

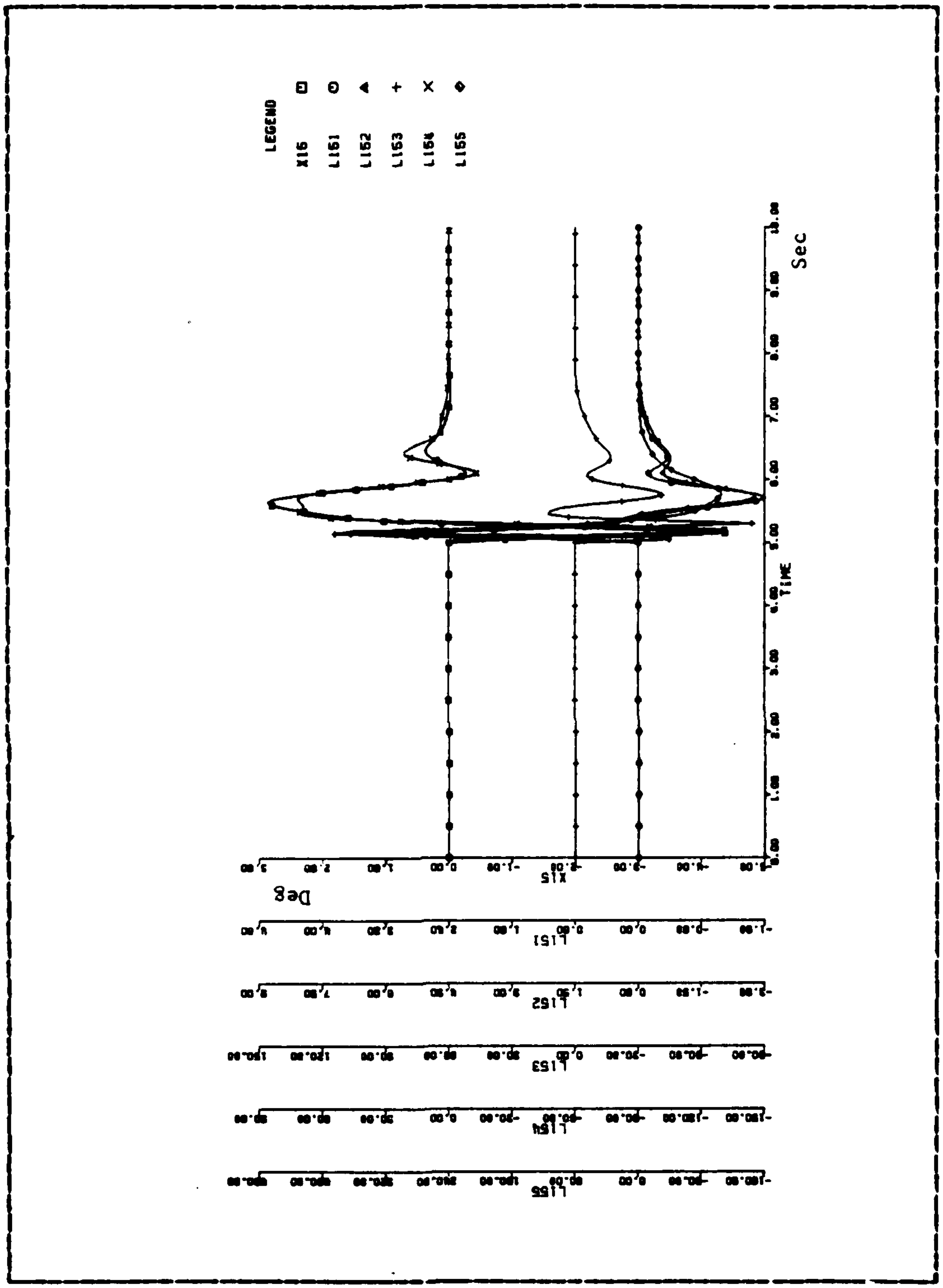

Figure 4.8 Sensitivity of X15 with respect to A1,A2,A3,A4,A5.

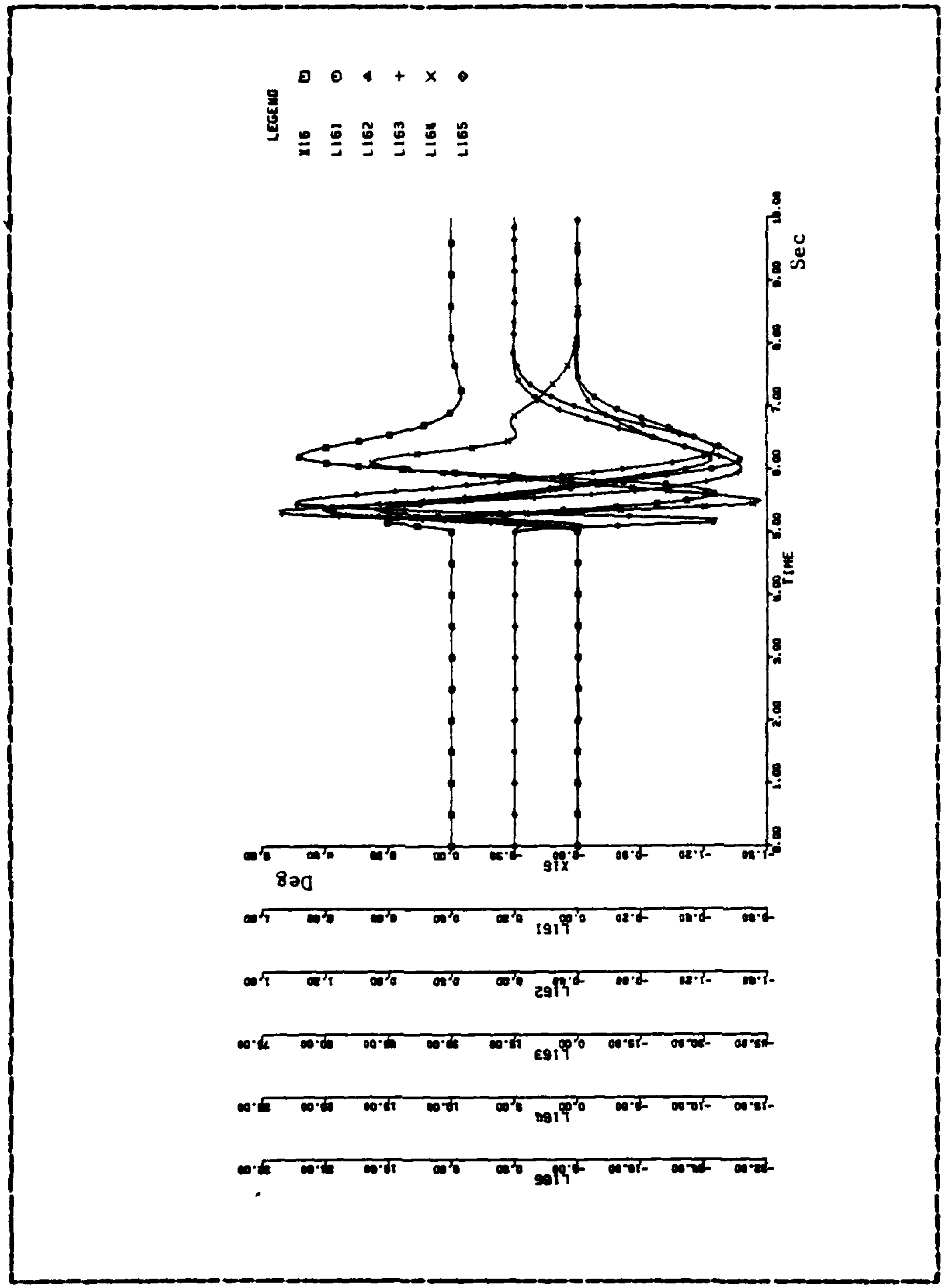

Figure 4.9   Sensitivity of X16 with respect to A1,A2,A3,A4,A5.

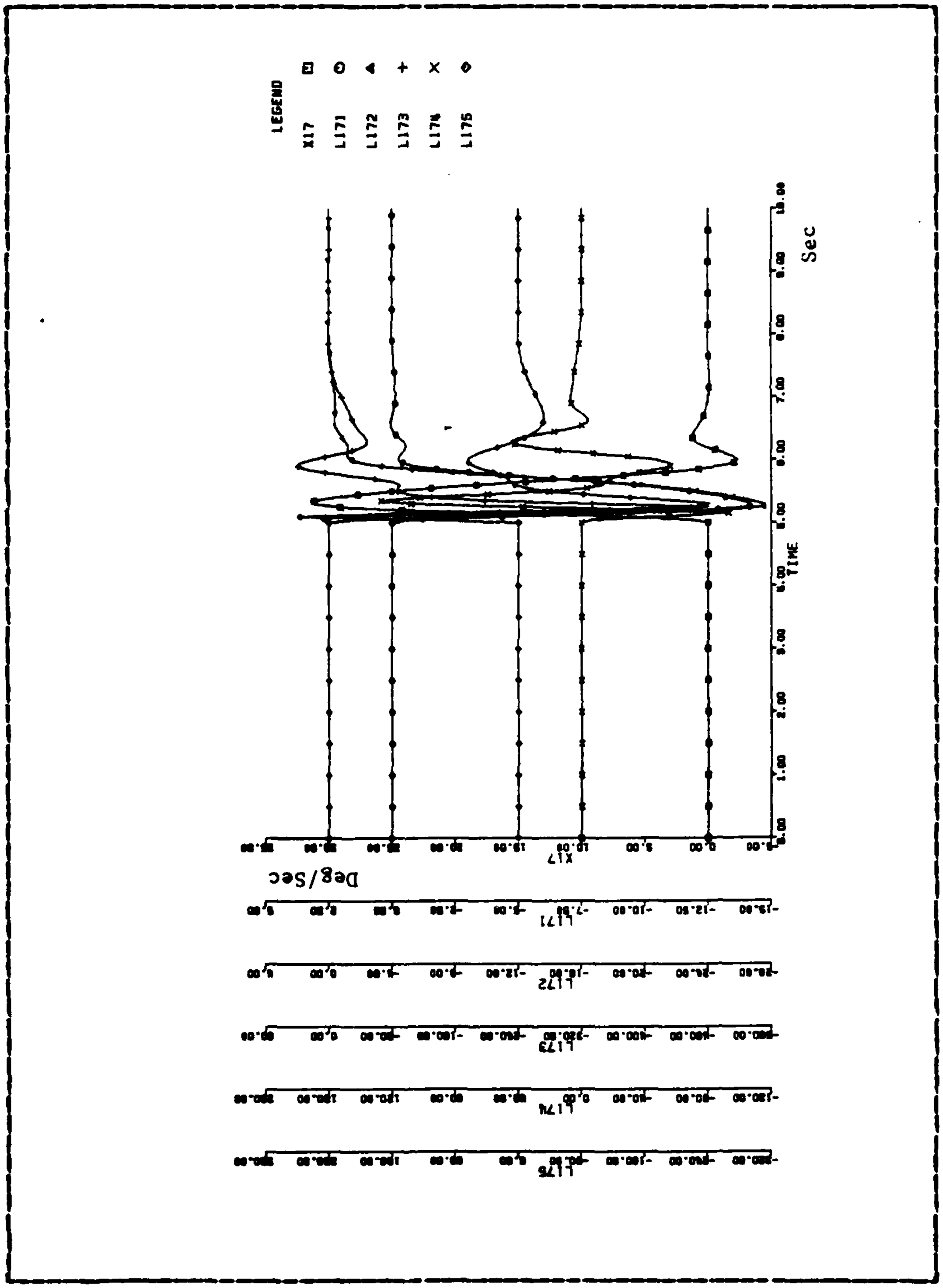

Figure 4.10 Sensitivity of X17 with respect to A1,A2,A3,A4,A5.

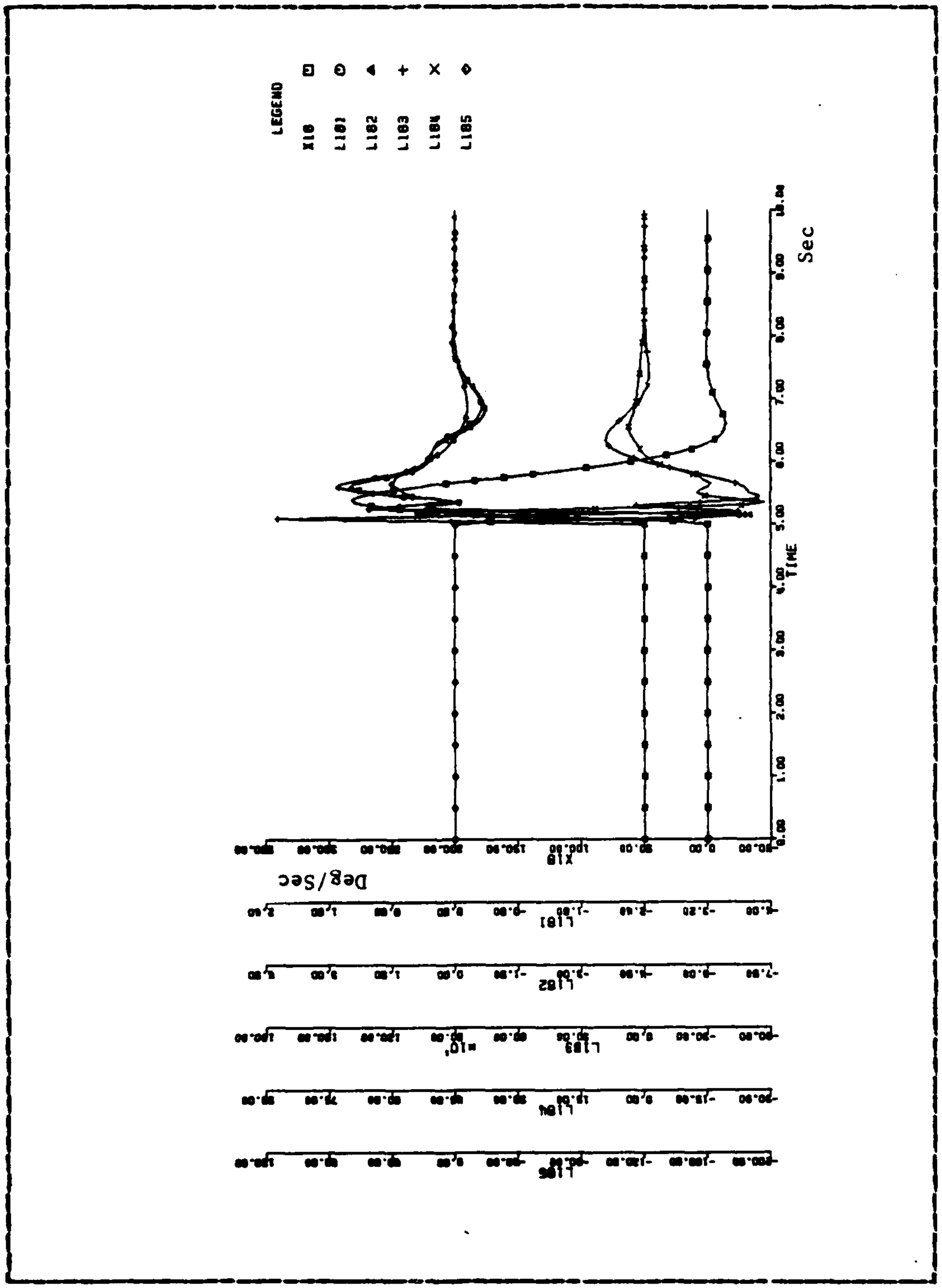

Figure 4.11 Sensitivity of X18 with respect to A1,A2,A3,A4,A5.

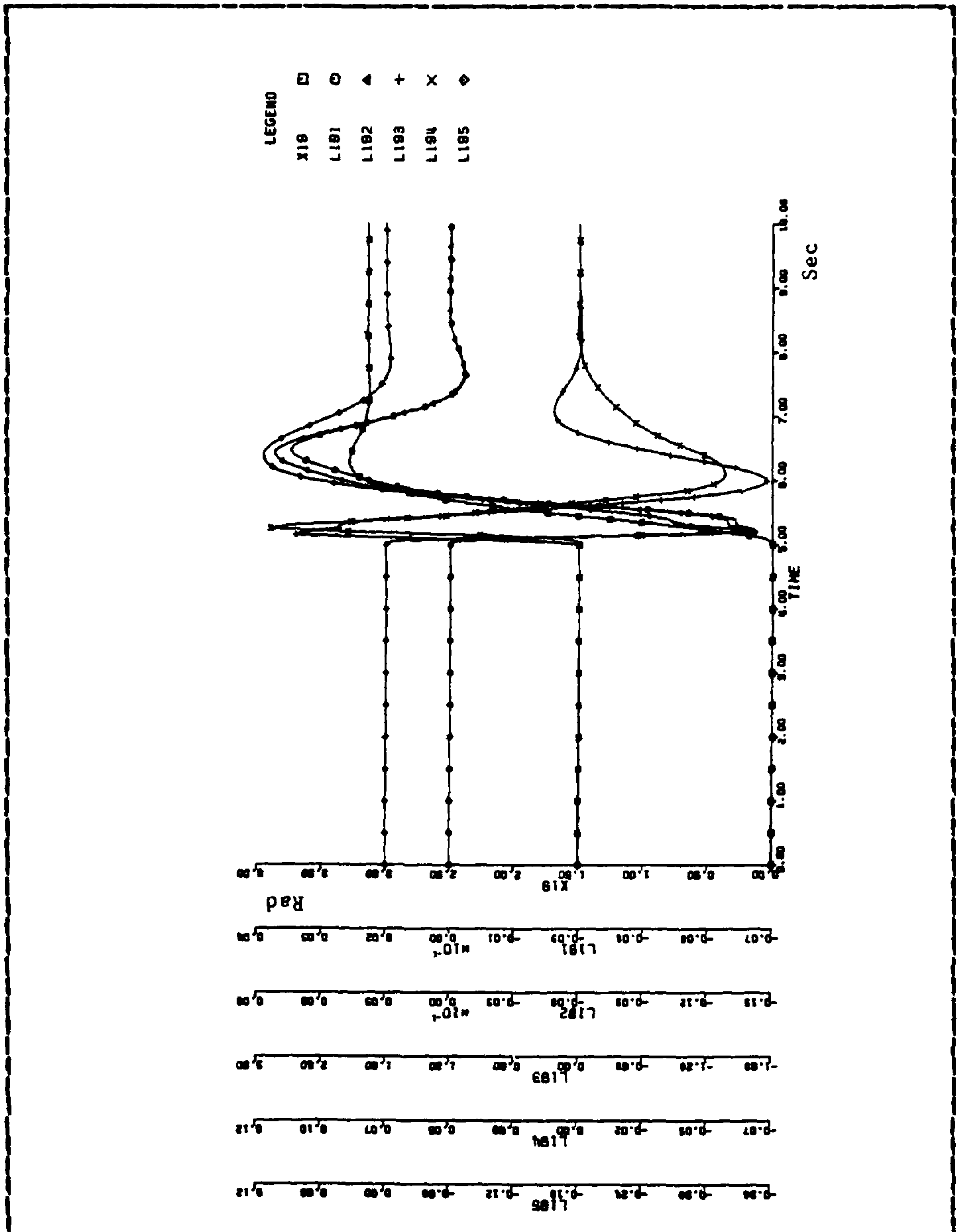

Figure 4.12 Sensitivity of X19 with respect to A1,A2,A3,A4,A5.

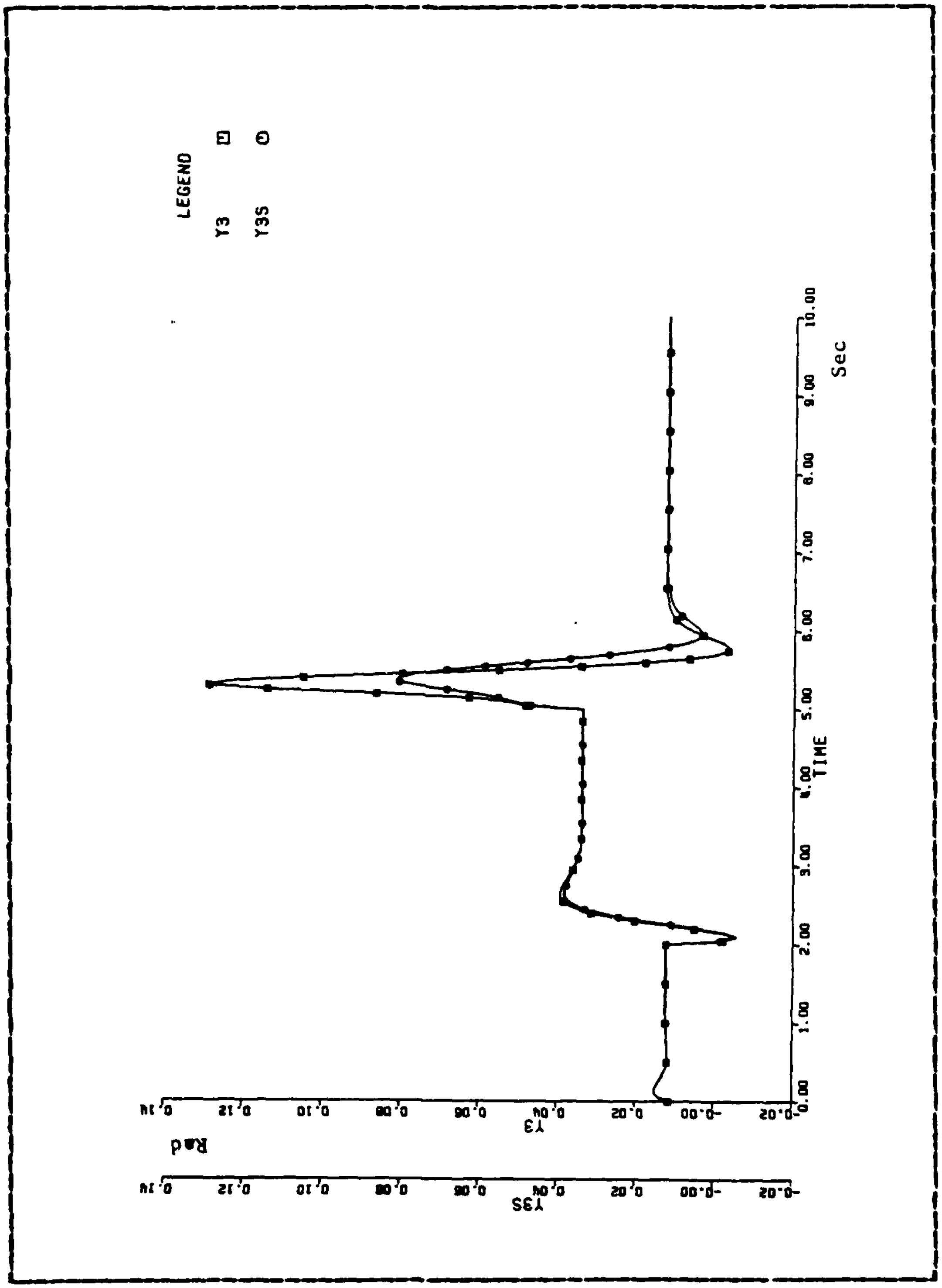

Figure 4.13 Actual and Nominal Output of X3 (10% variation).

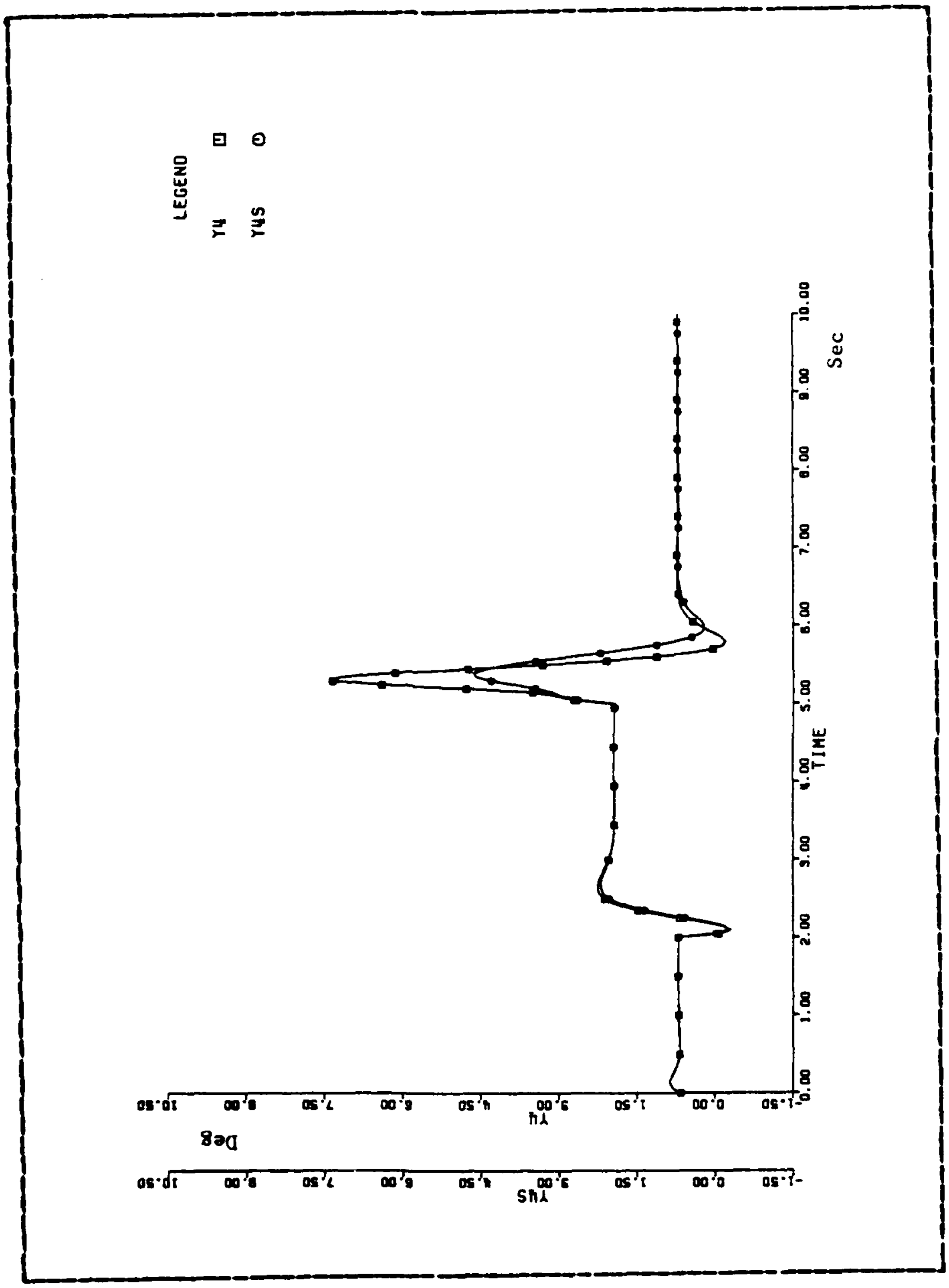

Figure 4.14 Actual and Nominal Output of X4 (10% variation).

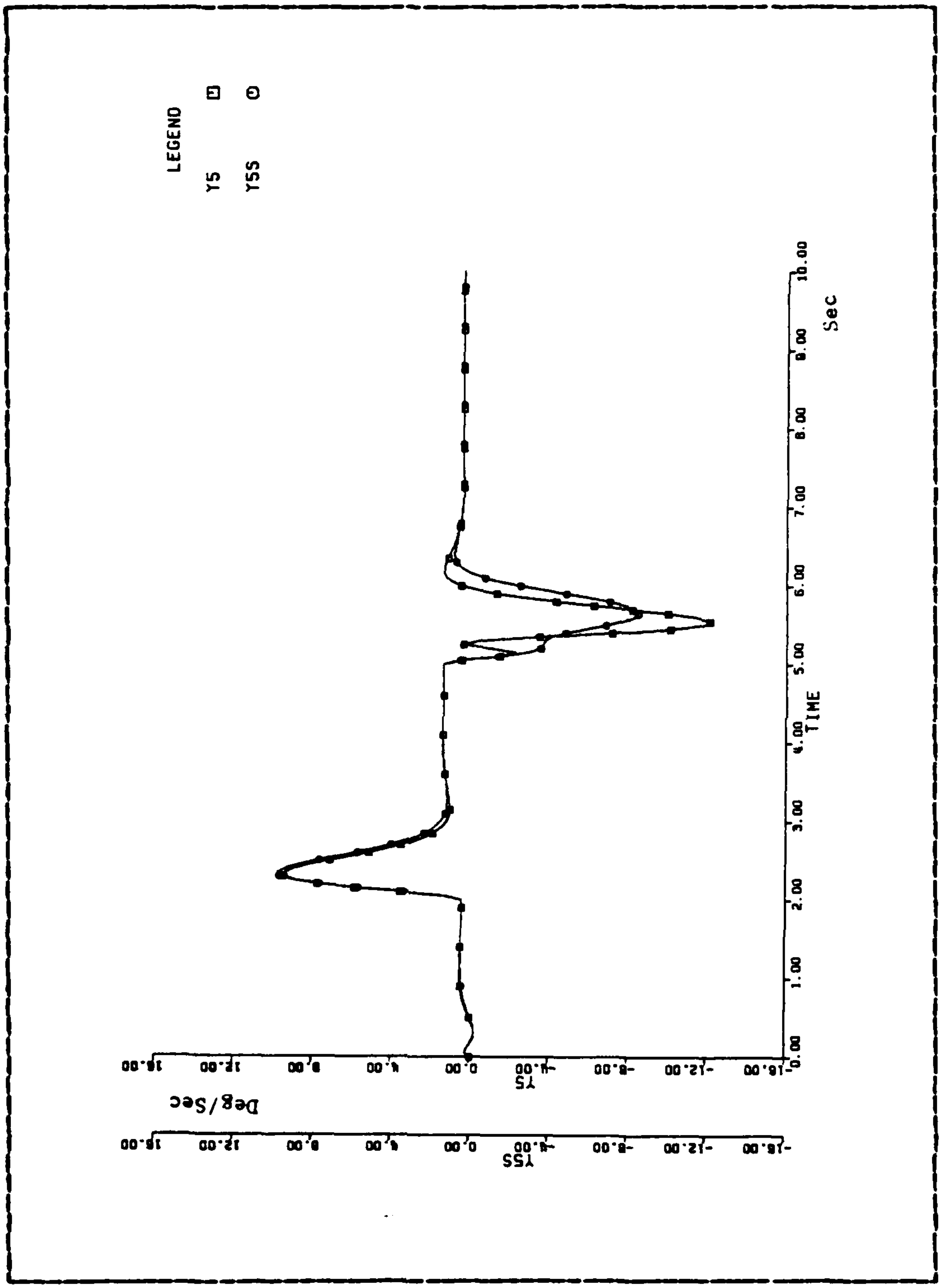

Figure 4.15 Actual and Nominal Output of X5 (10% variation).

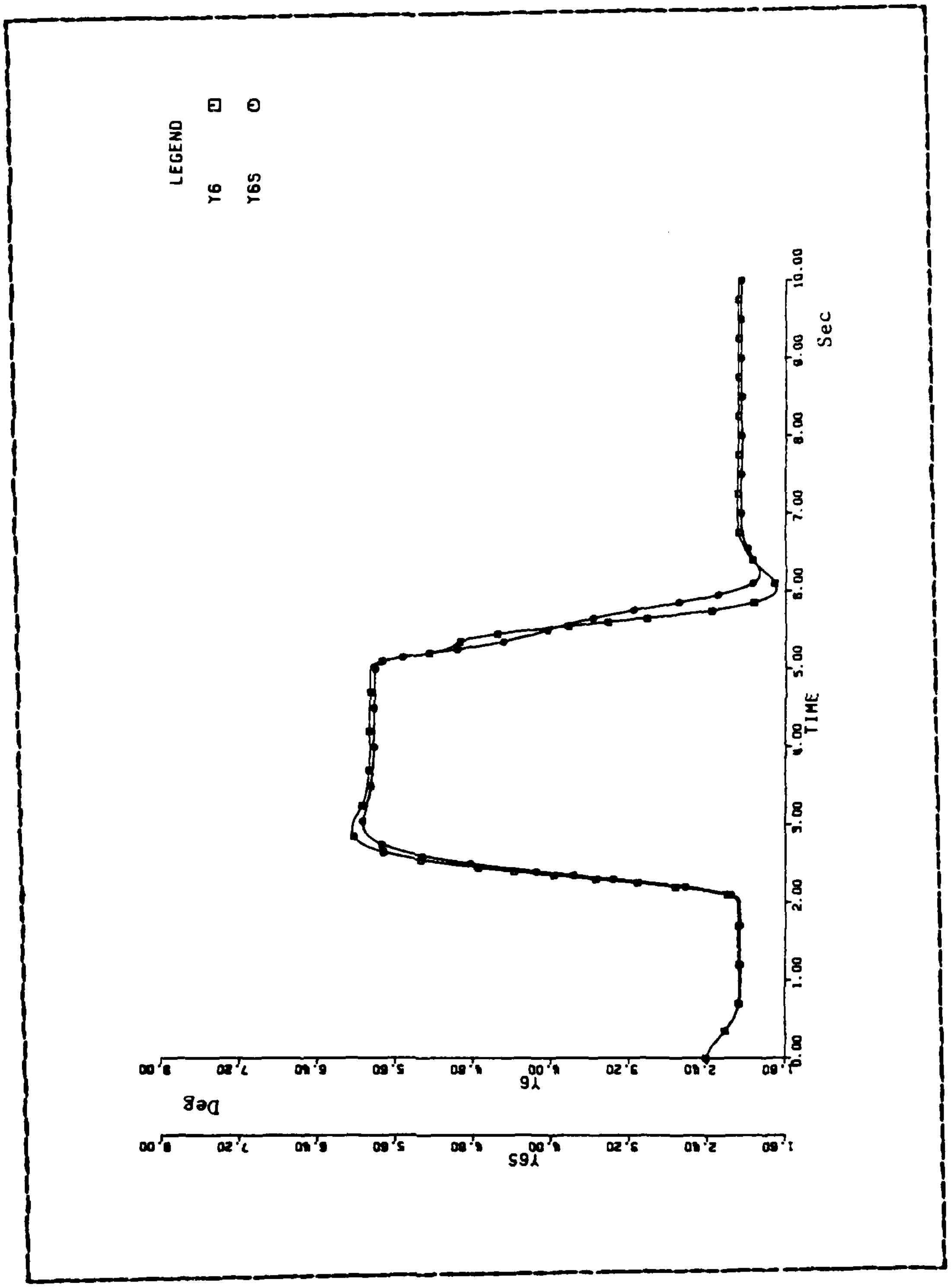

Figure 4.16   Actual and Nominal Output of X6 (10% variation).

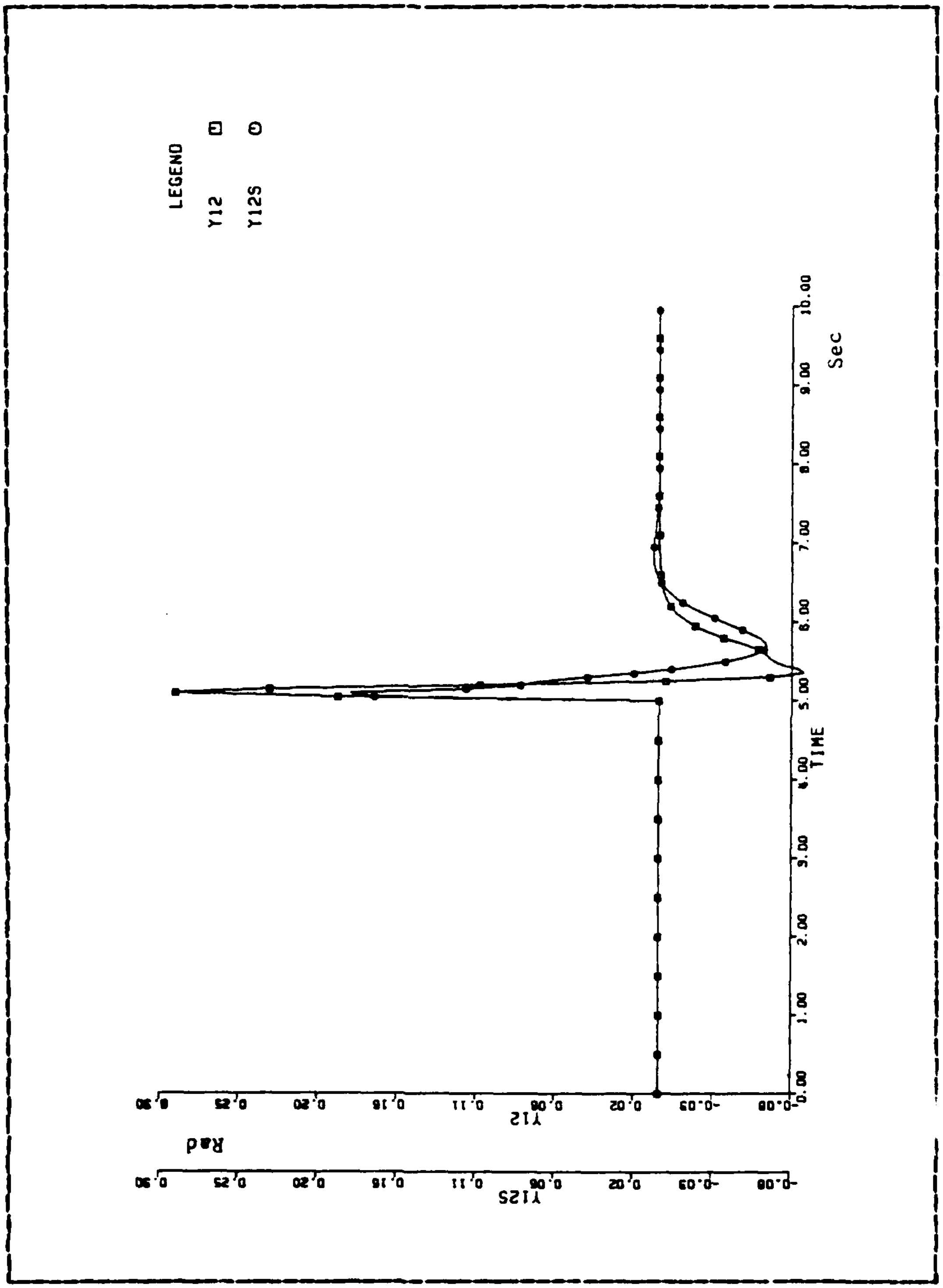

Figure 4.17   Actual and Nominal Output of X12 (10% variation).

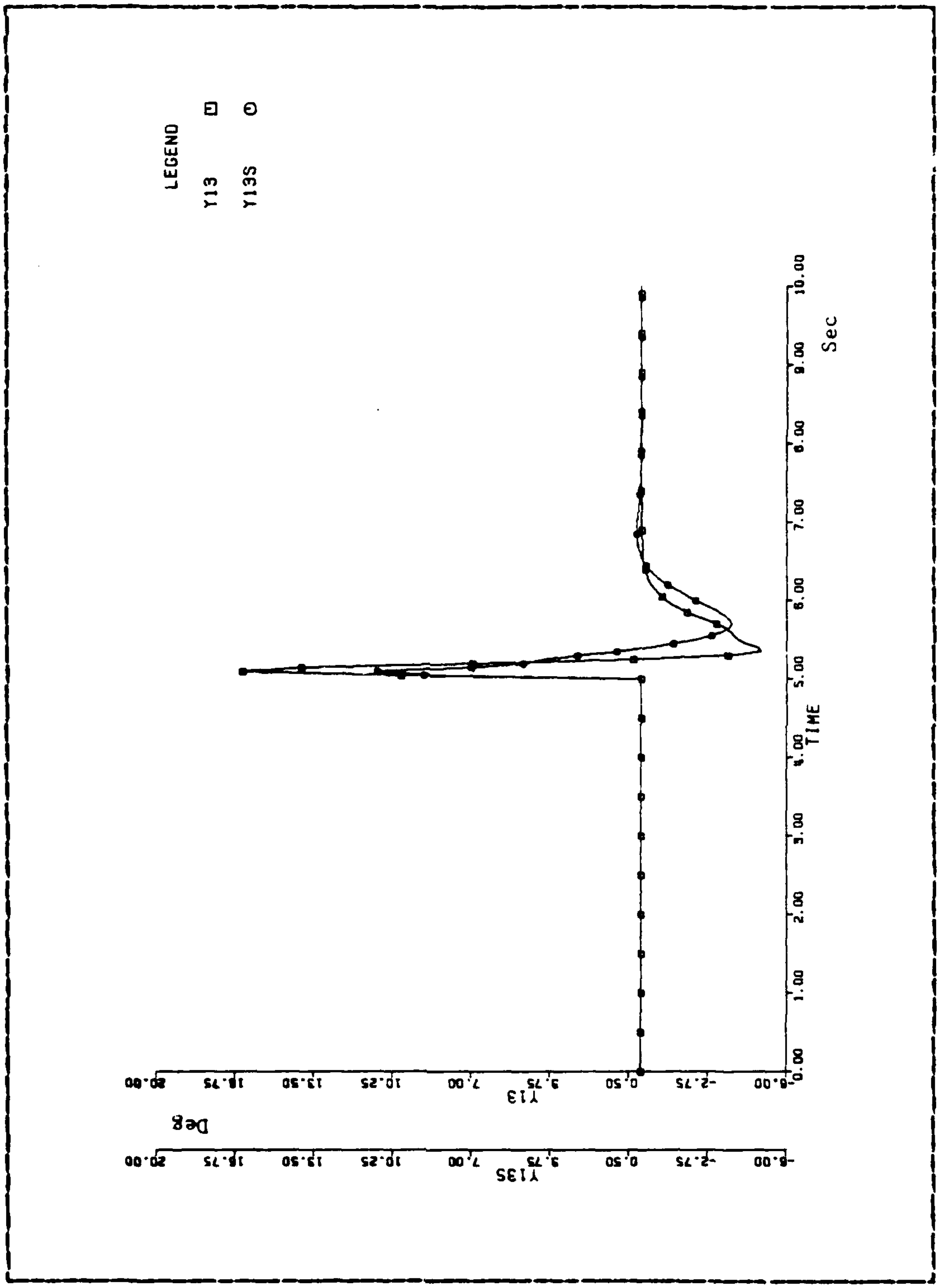

Figure 4.18 Actual and Nominal Output of X13 (10% variation).

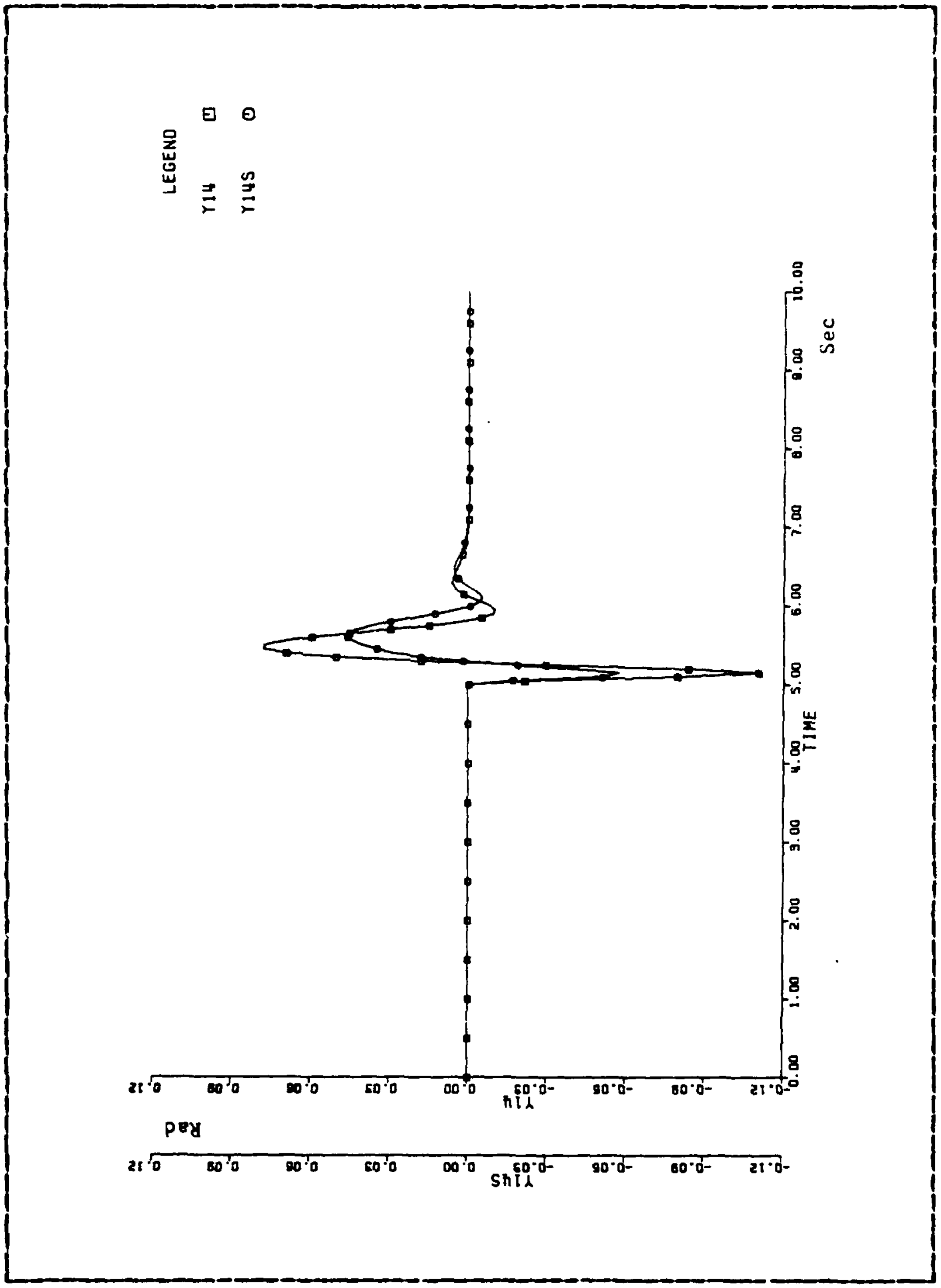

Figure 4.19 Actual and Nominal Output of X14 (10% variation).

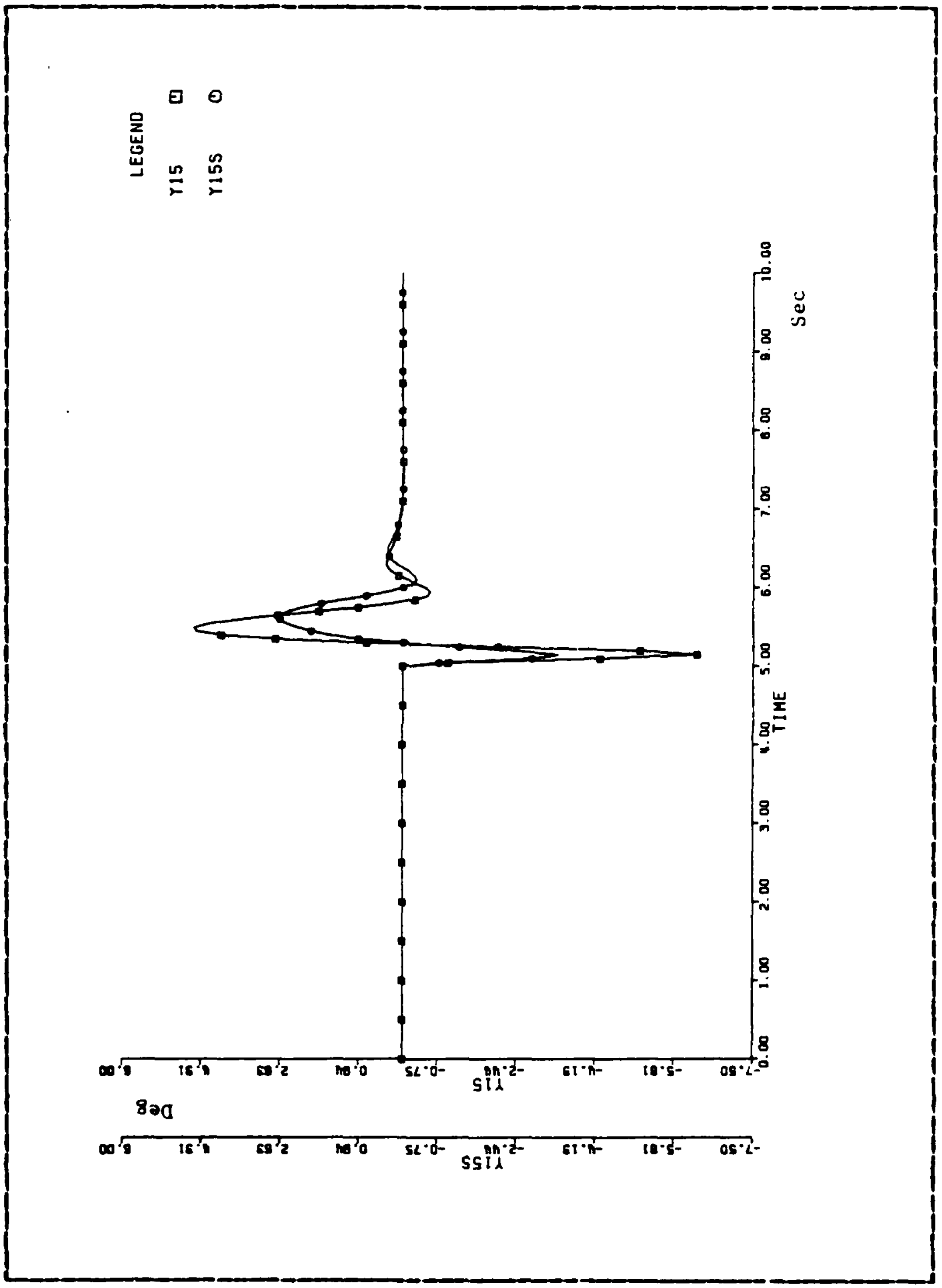

Figure 4.20 Actual and Nominal Output of X15 (10% variation).

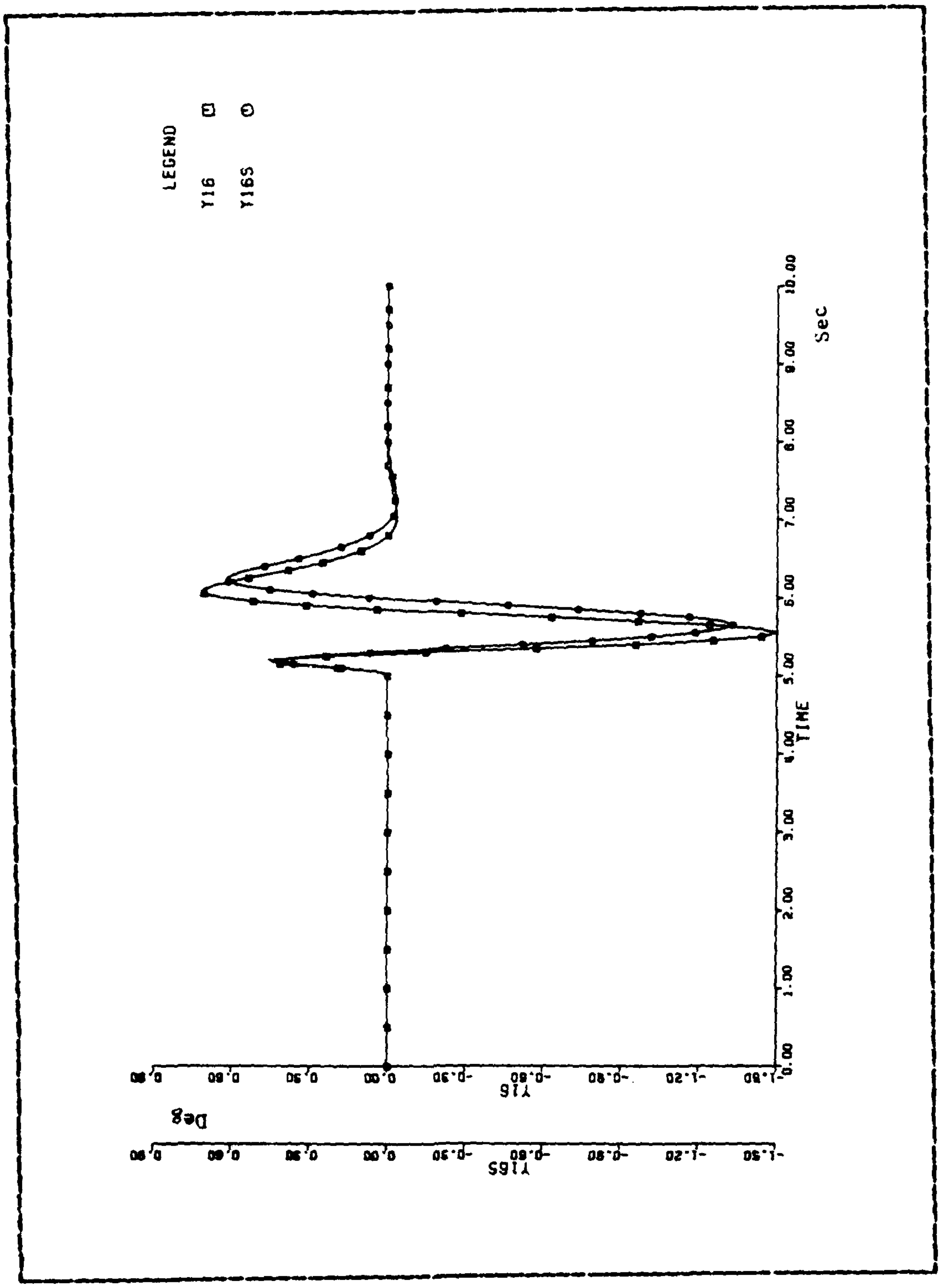

Figure 4.21   Actual and Nominal Output of X16 (10% variation).

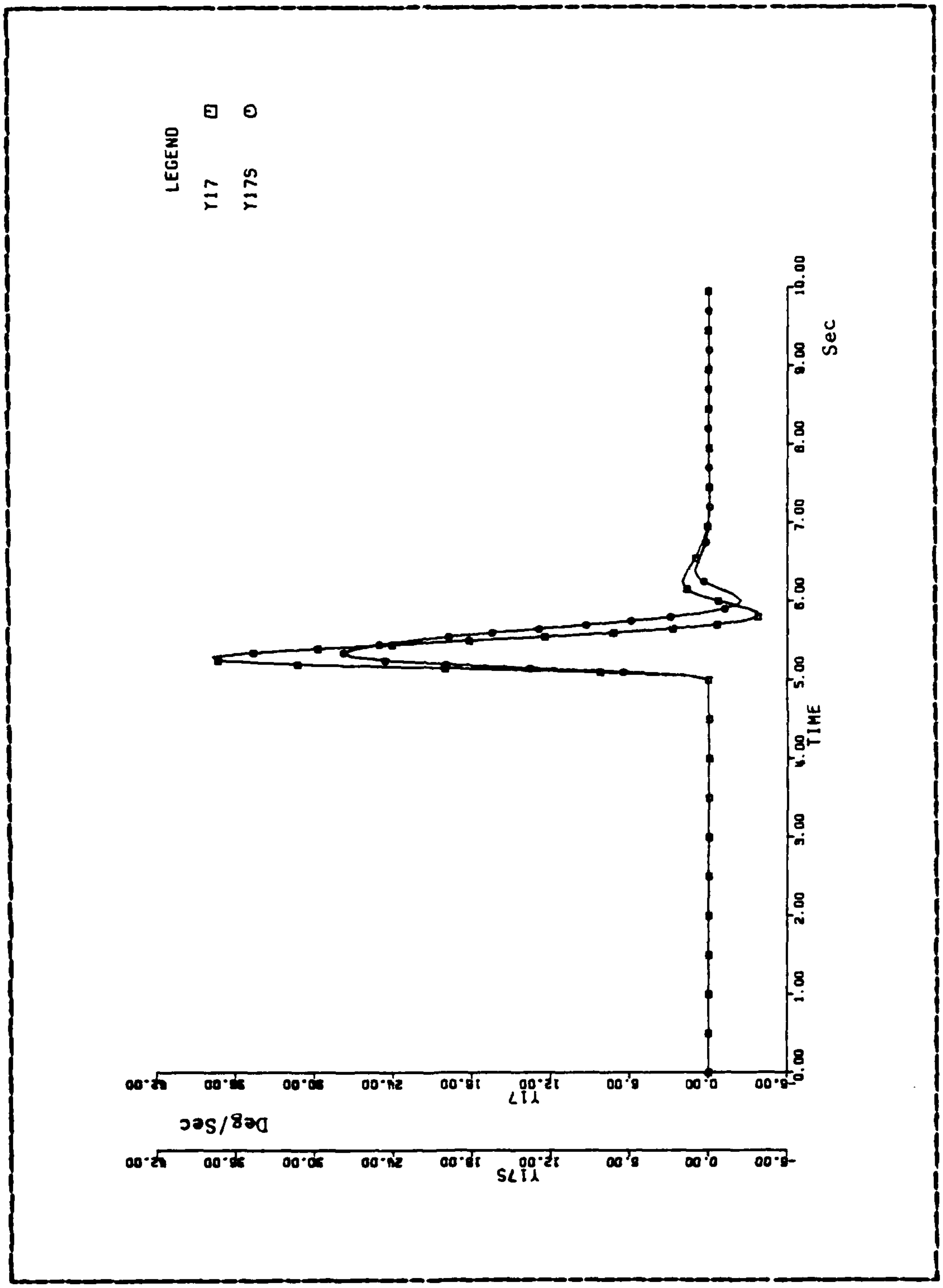

Figure 4.22 Actual and Nominal Output of X17 (10% variation).

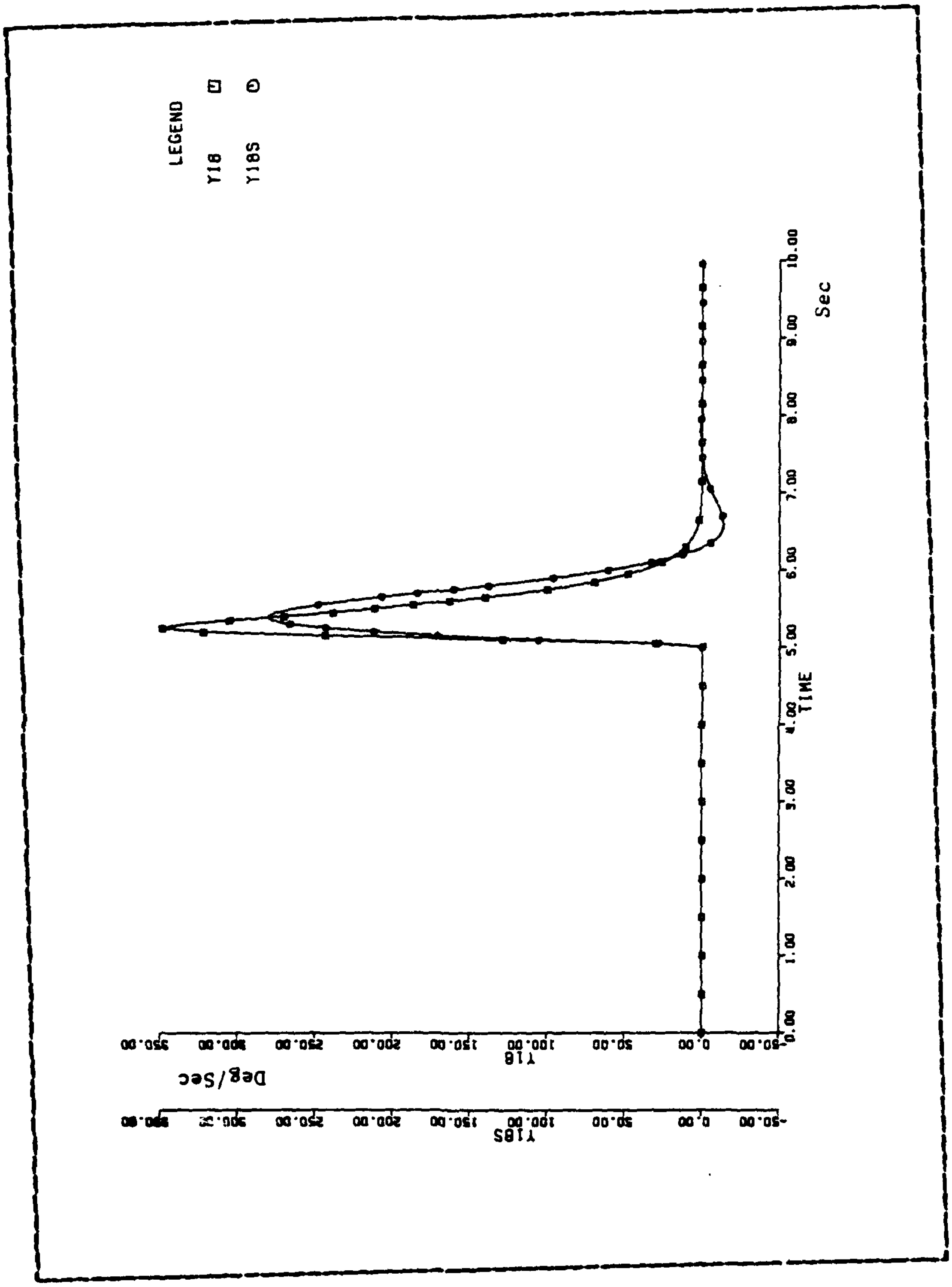

Figure 4.23   Actual and Nominal Output of X18 (10% variation).

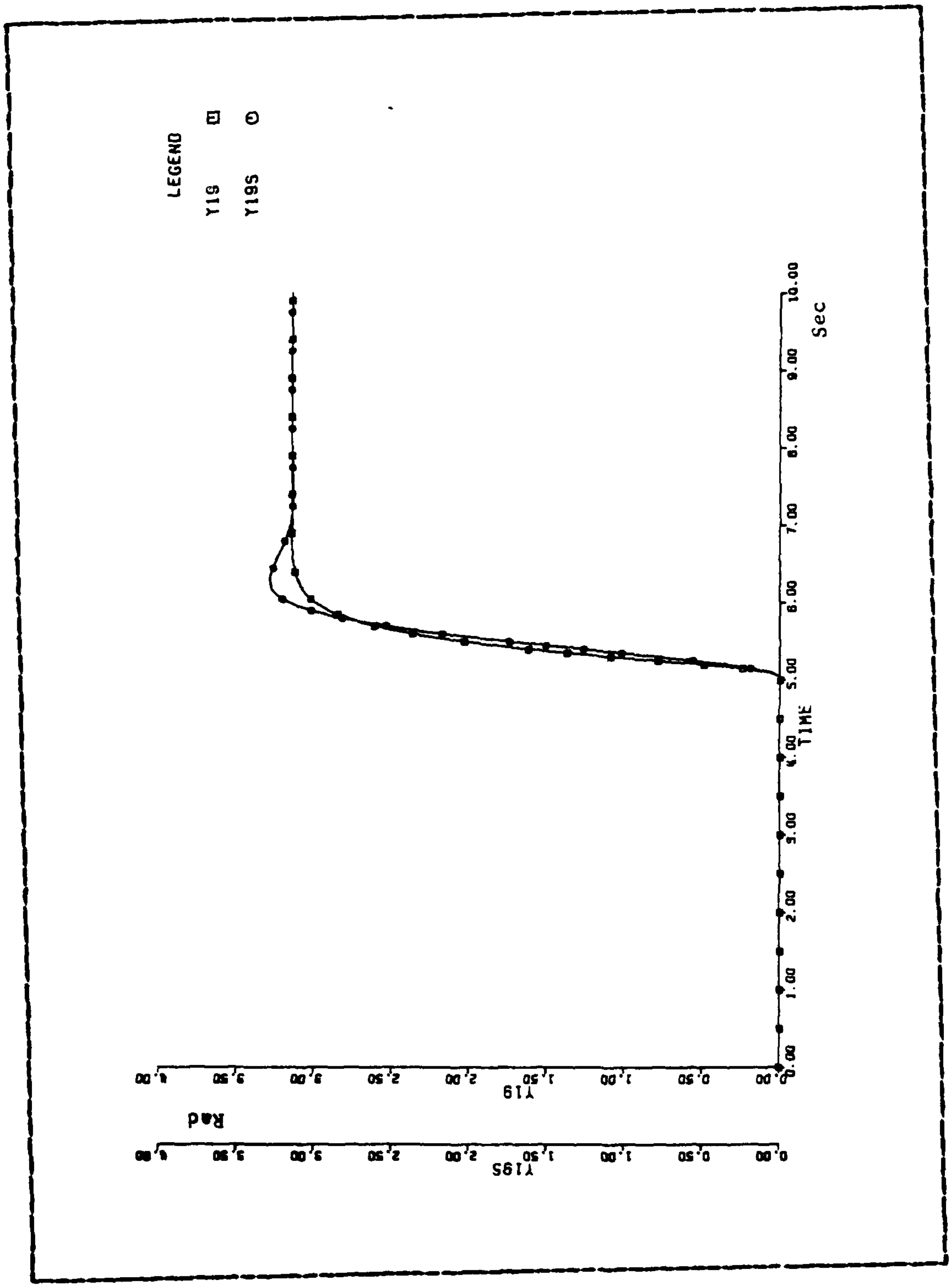

Figure 4.24   Actual and Nominal Output of X19 (10% variation).

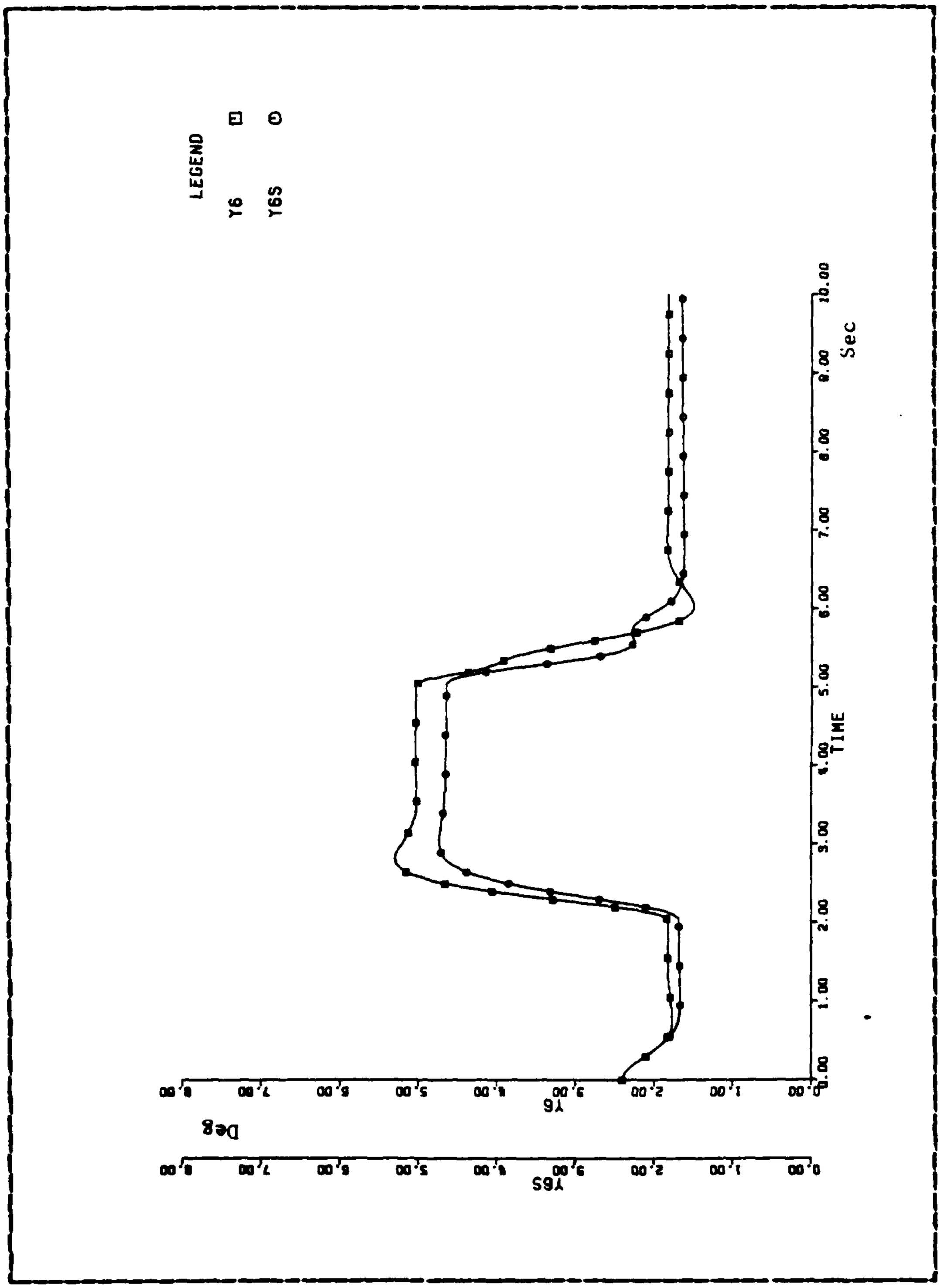

Figure 4.25 Actual and Nominal Output of X6 (30% variation).

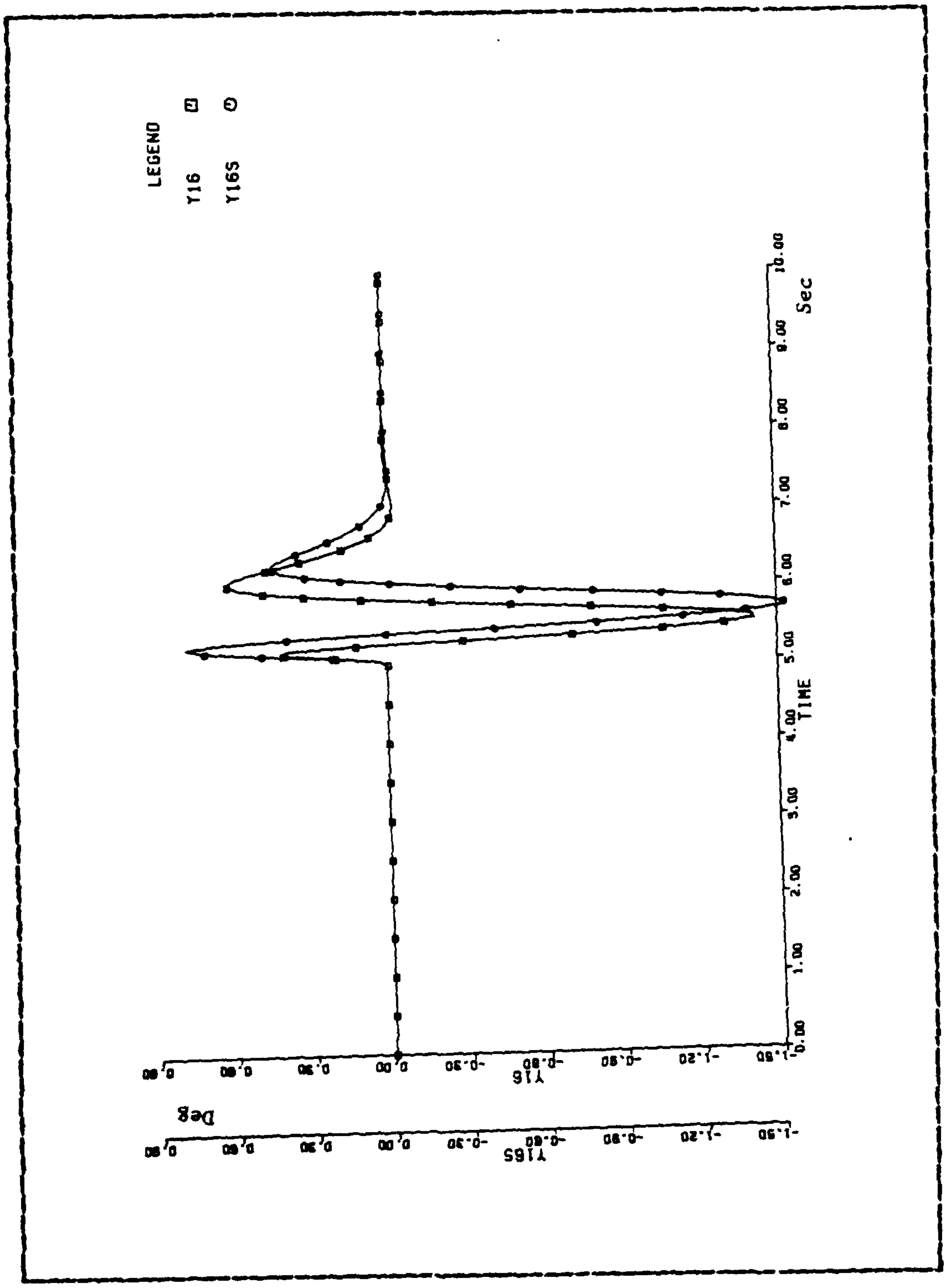

Figure 4.26   Actual and Nominal Output of X16 (30% variation).

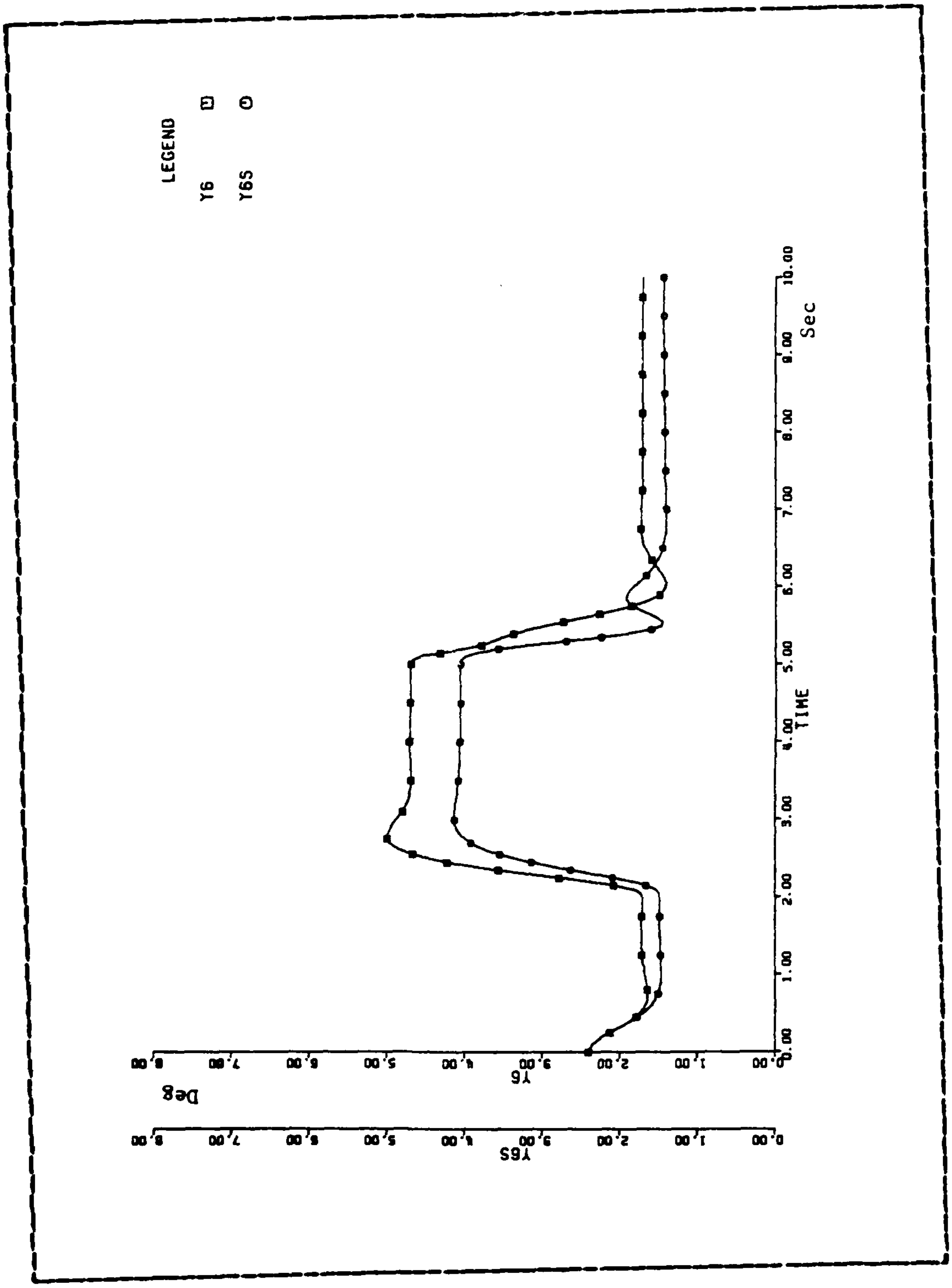

Figure 4.27 Actual and Nominal Output of X6 (40% variation).

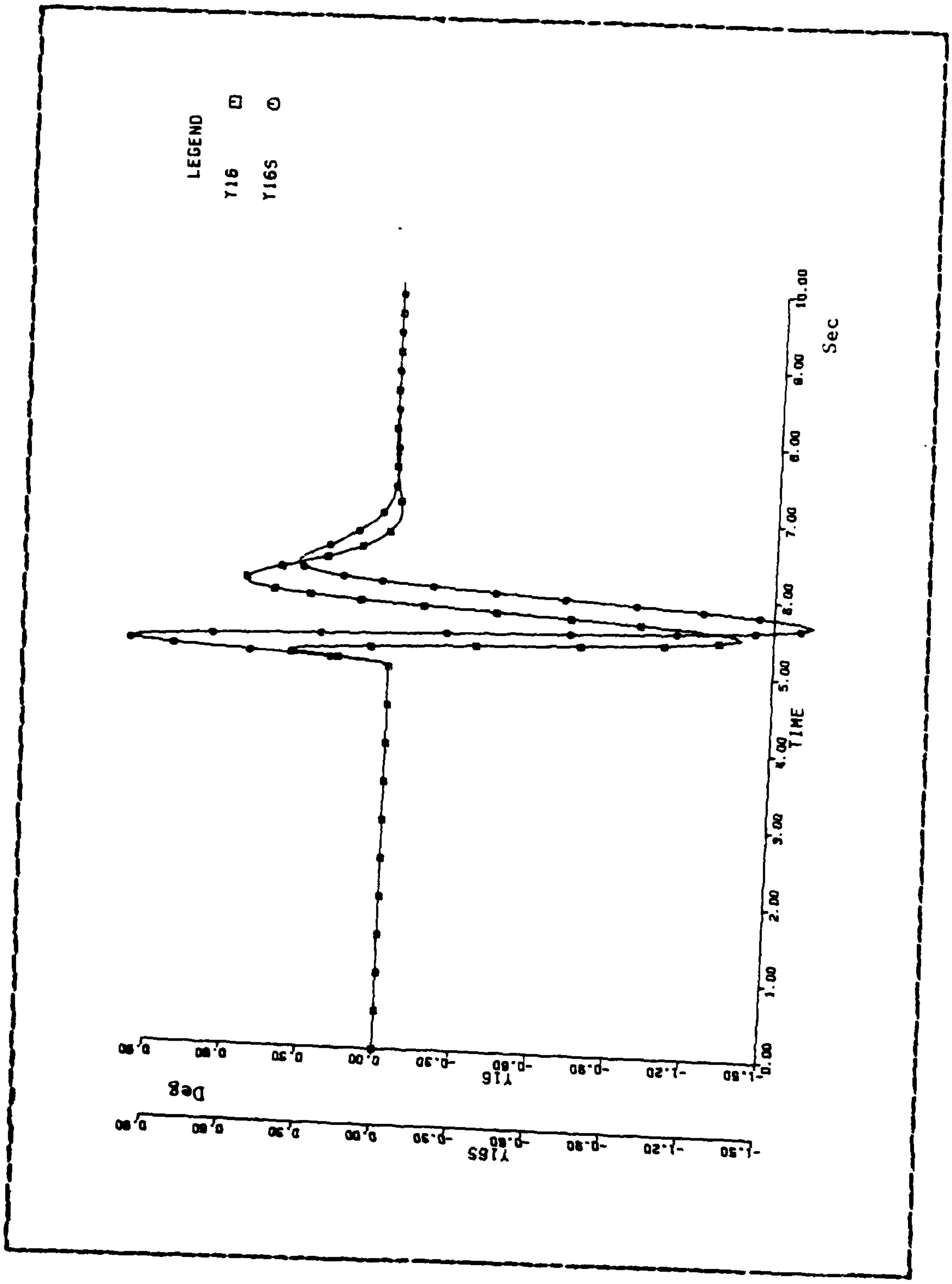

Figure 4.28 Actual and Nominal Output of X16 (40% variation).

TABLE IV

Influence of Parameters in the Time Response

X_3	λ_{31}	λ_{32}	λ_{33}	λ_{34}	λ_{35}
RISE TIME	LE	LE	NE	NE	NE
OVERSHOOT	SE	LE	LE	SE	SE
STEADY STATE	SE	LE	NE	NE	NE

X_4	λ_{41}	λ_{42}	λ_{43}	λ_{44}	λ_{45}
RISE TIME	LE	LE	NE	NE	NE
OVERSHOOT	SE	LE	SE	SE	LE
STEADY STATE	SE	LE	NE	NE	NE

X_5	λ_{51}	λ_{52}	λ_{53}	λ_{54}	λ_{55}
RISE TIME	SE	SE	NE	NE	NE
OVERSHOOT	LE	LE	SE	SE	SE
STEADY STATE	NE	NE	NE	NE	NE

X_6	λ_{61}	λ_{62}	λ_{63}	λ_{64}	λ_{65}
RISE TIME	SE	SE	NE	NE	NE
OVERSHOOT	LE	LE	SE	SE	SE
STEADY STATE	LE	LE	NE	NE	NE

X_{12}	λ_{121}	λ_{122}	λ_{123}	λ_{124}	λ_{125}
RISE TIME	NE	NE	NE	NE	NE
OVERSHOOT	LE	LE	SE	LE	SE
STEADY STATE	NE	NE	NE	NE	NE

X_{13}	λ_{131}	λ_{132}	λ_{133}	λ_{134}	λ_{135}
RISE TIME	NE	NE	NE	NE	NE
OVERSHOOT	LE	LE	SE	SE	SE
STEADY STATE	NE	NE	NE	NE	NE

TABLE V

Influence of Parameters in the Time Response (cont.)

X_{14}	λ_{141}	λ_{142}	λ_{143}	λ_{144}	λ_{145}
RISE TIME	NE	NE	NE	NE	NE
OVERSHOOT	LE	LE	LE	SE	SE
STEADY STATE	NE	NE	NE	NE	NE

X_{15}	λ_{151}	λ_{152}	λ_{153}	λ_{154}	λ_{155}
RISE TIME	NE	NE	NE	NE	NE
OVERSHOOT	LE	LE	SE	SE	SE
STEADY STATE	NE	NE	NE	NE	NE

X_{16}	λ_{161}	λ_{162}	λ_{163}	λ_{164}	λ_{165}
RISE TIME	NE	NE	NE	NE	NE
OVERSHOOT	LE	LE	SE	SE	SE
STEADY STATE	NE	NE	NE	NE	NE

X_{17}	λ_{171}	λ_{172}	λ_{173}	λ_{174}	λ_{175}
RISE TIME	NE	NE	NE	NE	NE
OVERSHOOT	LE	LE	SE	SE	SE
STEADY STATE	NE	NE	NE	NE	NE

X_{18}	λ_{181}	λ_{182}	λ_{183}	λ_{184}	λ_{185}
RISE TIME	NE	NE	NE	NE	NE
OVERSHOOT	LE	LE	SE	SE	SE
STEADY STATE	NE	NE	NE	NE	NE

X_{19}	λ_{191}	λ_{192}	λ_{193}	λ_{194}	λ_{195}
RISE TIME	NE	NE	NE	NE	NE
OVERSHOOT	LE	LE	SE	SE	SE
STEADY STATE	NE	NE	NE	NE	NE

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The results of the nonlinear 3-D parameter sensitivity analysis presented in Chapter IV verify the linear analysis of Chapter III. These conclusions can be quickly made, using Tables I, II, III and IV that give a brief review of the time response of the linear and nonlinear system. Another means of comparison of the linear and nonlinear analysis is to use the figures that give a precise visualization of the time response.

The comparison of the linear and nonlinear analysis can be done by means of the plots as follows:

B. EFFECT ON STATE VARIABLES *DUE TO* PARAMETER VARIATIONS

1. Effect on State Variable Due to Variation in $C_m(\alpha, \delta_e)$:

Figs.3.7 and 4.1 (λ_{64} and λ_{31}) show that δp_c is little affected in the rise time, strongly affected in the overshoot and steady state.

Figs.3.4 and 4.2 (λ_{34} and λ_{41}) show that δp is little affected in the rise time and strongly affected in the overshoot and steady state.

Figs.3.2 and 4.3 (λ_{14} and λ_{51}) show that q is strongly affected in the rise time, little affected in the overshoot and not affected in the steady state.

Figs.3.3 and 4.4 (λ_{24} and λ_{61}) show that α is strongly affected in the rise time, little affected in the overshoot and steady state.

2. Effect on State Variable Due to Variation in $C_{N}(\alpha, \delta_P)$:

Figs.3.7 and 4.1 (λ_{62} and λ_{32}) show that δ_{Pc} is little affected in the rise time, overshoot and steady state.

Figs. 3.4 and 4.2 (λ_{32} and λ_{42}) show that δ_P is little affected in the rise time, overshoot and steady state.

Figs.3.2 and 4.3 (λ_{12} and λ_{52}) show that q is strongly affected in the rise time and little affected in the overshoot and steady state.

Figs.3.3 and 4.4 (λ_{22} and λ_{62}) show that α is strongly affected in the rise time and little affected in the overshoot and steady state.

3. Effect on State Variable Due to Variation in $C_{\ell\delta R}$.

Figs.3.40 and 4.6 (λ_{78} and λ_{123}) show that δ_{Rc} is little affected in the rise time, strongly affected in the overshoot and not affected in the steady state.

Figs.3.42 and 4.7 (λ_{88} and λ_{133}) show that δ_R is not affected in the rise time and steady state, and strongly affected in the overshoot.

Figs.3.46 and 4.8 (λ_{108} and λ_{143}) show that δ_{Yc} is little affected in the rise time, little affected in the overshoot and not affected in the steady state.

Figs.3.48 and 4.9 (λ_{118} and λ_{153}) show that δ_Y is little effected in the rise time, strongly affected in the overshoot and not affected in the steady state.

Figs.3.32 and 4.10 (λ_{38} and λ_{163}) show that β is little affected in the rise time, stronglu affected in the overshoot and not affected in the steady state.

Figs.3.28 and 4.11 (λ_{18} and λ_{173}) show that r is little affected in the rise time, strongly affected in the overshoot and not affected in the steady state.

Figs.3.30 and 4.12 (λ_{28} and λ_{183}) show that p is little affected in the rise time, strongly affected in the overshoot and not affected in the steady state.

Figs.3.50 and 4.13 (λ_{128} and λ_{193}) show that ϕ is not affected in the rise time and steady state and strongly affected in the overshoot.

4. <u>Effect on State Variable Due to Variation in $C_{n_{\delta p}}$</u>:

Figs.3.40 and 4.6 (λ_{75} and λ_{124}) show that δ_{Rc} is not affected in the rise time and steady state and little affected in the overshoot.

Figs.3.42 and 4.7 (λ_{85} and λ_{134}) show that δ_R is not affected in the rise time and steady state and strongly affected in the overshoot.

Figs.3.46 and 4.8 (λ_{105} and λ_{144}) show that $\delta_{\gamma c}$ is little affected in the rise time, strongly affected in the overshoot and not affected in the steady state.

Figs.3.48 and 4.9 (λ_{115} and λ_{154}) show that δ_γ is not affected in the rise time and steady state and strongly affected in the overshoot.

Figs.3.32 and 4.10 (λ_{35} and λ_{164}) show that β is little affected in the rise time, strongly affected in the overshoot and not affected in the steady state.

Figs.3.28 and 4.11 (λ_{15} and λ_{174}) show that r is not affected in the rise time and steady state and strongly affected in the overshoot.

Figs.3.30 and 4.12 (λ_{25} and λ_{184}) show that p is little affected in the rise time, strongly affected in the overshoot and not affected in the steady state.

Figs.3.50 and 4.13 (λ_{125} and λ_{194}) show that ϕ is not affected in the rise time and steady state and strongly affected in the overshoot.

5. Effect on State Variable Due to Variation in $C_{n\delta R}$:

Figs.3.39 and 4.6 (λ_{73} and λ_{125}) show that δ_{Rc} is little affected in the rise time, strongly affected in the overshoot and not affected in the steady state.

Figs.3.41 and 4.7 (λ_{83} and λ_{135}) show that δ_R is little affected in the risr time, stronglu affected in the overshoot and not affected in the steady state.

Figs.3.45 and 4.8 (λ_{103} and λ_{145}) show that δ_{Yc} is little affected in the rise time, strongly affected in the overshoot and not affected in the steady state.

Figs.3.47 and 4.9 (λ_{113} and λ_{155}) show that δ_Y is little affected in the rise time, strongly affected in the overshoot and not affected in the steady state.

Figs.3.31 and 4.10 (λ_{33} and λ_{165}) show that β is little affected in the rise time, stongly affected in the overshoot and not affected in the steady state.

Figs.3.27 and 4.11 (λ_{13} and λ_{175}) show that r is little affected in the rise time, strongly affected in the overshoot and not affected in the steady state.

Figs.3.29 and 4.12 (λ_{23} and λ_{185}) show that P is little affected in the rise time, strongly affected in the overshoot and not affected in the steady state.

Figs.3.49 and 4.13 (λ_{123} and λ_{195}) show that ϕ is little affected in the rise time, strongly affected in the overshoot and not affected in the steady state.

C. EFFECT OF TRAJECTORY SENSITIVITY FUNCTIONS

1. Uncoupled Pitch Autopilot

The trajectory sensitivity functions $\lambda 11$, $\lambda 12$, $\lambda 13$, and $\lambda 14$ in the linear case and the correspondent trajectory sensitivity functions 151 and 152 in the nonlinear case show strong effect on the rise time and overshoot of the state variable q.

The trajectory sensitivity functions $\lambda 23$ and $\lambda 24$ in the linear case and the correspondent trajectory sensitivity function 161 in the nonlinear case show strong effect on the rise time of the state variable α .

The trajectory sensitivity functions $\lambda 33$ and $\lambda 34$ in the linear case and the correspondent trajectory sensitivity function $\lambda 41$ in the nonlinear case show strong effect on the overshoot and steady state of the state variable δp .

The trajectory sensitivity functions $\lambda 53$ and $\lambda 64$ in the linear case and the correspondent sensitivity trajectory function $\lambda 31$ in the nonlinear case show strong effect on the overshoot and steady state of the state variable δ_{p_c}.

2. <u>Coupled Roll-Yaw Autopilot</u>

The trajectory sensitivity functions $\lambda 13$, $\lambda 15$, and $\lambda 18$ in the linear case and the correspondent trajectory sensitivity functions $\lambda 175$, $\lambda 174$, and $\lambda 173$ in the nonlinear case show strong effect on the overshoot of the state variable r.

The trajectory sensitivity functions $\lambda 23$, $\lambda 25$, and $\lambda 28$ in the linear case and the correspondent trajectory sensitivity functions $\lambda 185$, $\lambda 184$, and $\lambda 183$ in the nonlinear case show strong effect on the overshoot of the state variable p.

The trajectory sensitivity functions $\lambda 33$, $\lambda 35$, and $\lambda 38$ in the linear case and the correspondent trajectory sensitivity functions $\lambda 165$, $\lambda 164$, and $\lambda 163$ in the nonlinear case show strong effect on the overshoot of the state variable β .

The trajectory sensitivity functions $\lambda 73$, $\lambda 75$, and $\lambda 78$ in the linear case and the correspondant trajectory sensitivity functions $\lambda 125$, $\lambda 124$, and $\lambda 123$ in the nonlinear case show little effect on the overshoot of the state variable δ_{R_c}.

The trajectory sensitivity functions $\lambda 83$, $\lambda 85$, and $\lambda 88$ in the linear case and the correspondent trajectory sensitivity functions $\lambda 135$, $\lambda 134$, and $\lambda 133$ in the nonlinear case show strong effect on the overshoot of the state variable δ_R .

The trajectory sensitivity functions $\lambda 103$ and $\lambda 105$ in the linear case and the correspondent trajectory sensitivity functions $\lambda 145$ and $\lambda 144$ in the nonlinear case show strong effect on the overshoot of the state variable $\delta\gamma_c$. The trajectory sensitivity function $\lambda 108$ in the linear case and the correspondent trajectory sensitivity function $\lambda 143$ in the nonlinear case show little effect on the overshoot of the state variable $\delta\gamma_c$.

The trajectory sensitivity functions $\lambda 113$, $\lambda 115$, and $\lambda 118$ in the linear case and the correspondent trajectory sensitivity functions $\lambda 155$, $\lambda 154$, and $\lambda 153$ in the nonlinear case show strong effect on the overshoot of the state variable $\delta\gamma$.

The trajectory sensitivity functions $\lambda 123$, $\lambda 125$, and $\lambda 128$ in the linear case and the correspondent trajectory sensitivity functions $\lambda 195$, $\lambda 194$, and $\lambda 193$ in the nonlinear case show strong effect on the overshoot of the state variable ϕ .

The trajectory sensitivity functions have the following range of values :

	Minimum	Maximum
Linear	- 385.25	632.35
Nonlinear	- 642.9	1772.9

D. GENERAL CONCLUSIONS

The parameter C_{m_α} strongly affect the overshoot in almost all case.

The parameter C_{N_α} little affect the overshoot in all case.

The parameter $C_{\ell_{\delta R}}$ strongly affect the overshoot in almost all case.

The parameter $C_{n_{\delta P}}$ strongly affect the overshoot in almost all case.

The parameter $C_{n_{\delta R}}$ strongly affect the overshoot in almost all case.

E. RECOMMENDATIONS

It was not possible to run a CSMP computer program using all state variable (19) and parameters (10) in the nonlinear case due to the work area available. The above restriction occurred just when the actual and nominal output were required to be printed. The computational time in this case may be reduced using Fortran subroutines imbedded in the CSMP program in order to calculate the necessary parameter derivatives.

In the linear cases the computational time can be reduced by means of using the method of Sensitivity Points that uses one model for all parameters instead of using one model for each parameter.

The present analysis can be repeated for the case of the Circular[2] Airframe given in the [Ref. 2], and comparisons between both airframes can be performed.

Future study can be made modeling the system in the frequency domain where the "root sensitivity" is analysed.

Influence of parameter variations on miss distance can be analysed using an augumented system and a scenario of reference.

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[2]The current CSMP programs in the appendices are prepared to run the Circular case just by inserting the data of the correspondant airframe.

APPENDIX A

MISSILE SIZING, MASS PROPERTIES AND AERODYNAMIC DATA

A. INTRODUCTION

In this appendix, one considers a model that was assumed to be 1/6 scale of the actual elliptical missile configuration as given in Fig.A.1.

Missile configuration was sized to provide realistc mass properties needed for this study.

Since the main purpose of this work was to perform the parameters sensitivity analysis applied to a Bank-to-Turn missile, no effort was expended on a detailed design and analysis of the various parts envolved, which is given in [Ref. 2]. Some of figures given in this appendix were taken from the [Ref. 2] for easy visualization of the system in study.

B. GEOMETRIC AND MASS PROPERTIES OF MISSILE CONFIGURATION

One can find in Table VI the properties used in the development of the equations for applying the Parameters Sensitivity Analysis.

Fig.A.2 shows the aerod amic sign convention, nomenclature and body-fixed e the following three assumptions were made:

- Plane $\bar{x}_B$ A.2 is the maneuver plane.
- Missile is in pitch (i.e. , My=0 , at fixed values of , and δ_p).
- Missile roll rate is constant.

Fig. A.3 shows a block diagram of a BTT autopilot.

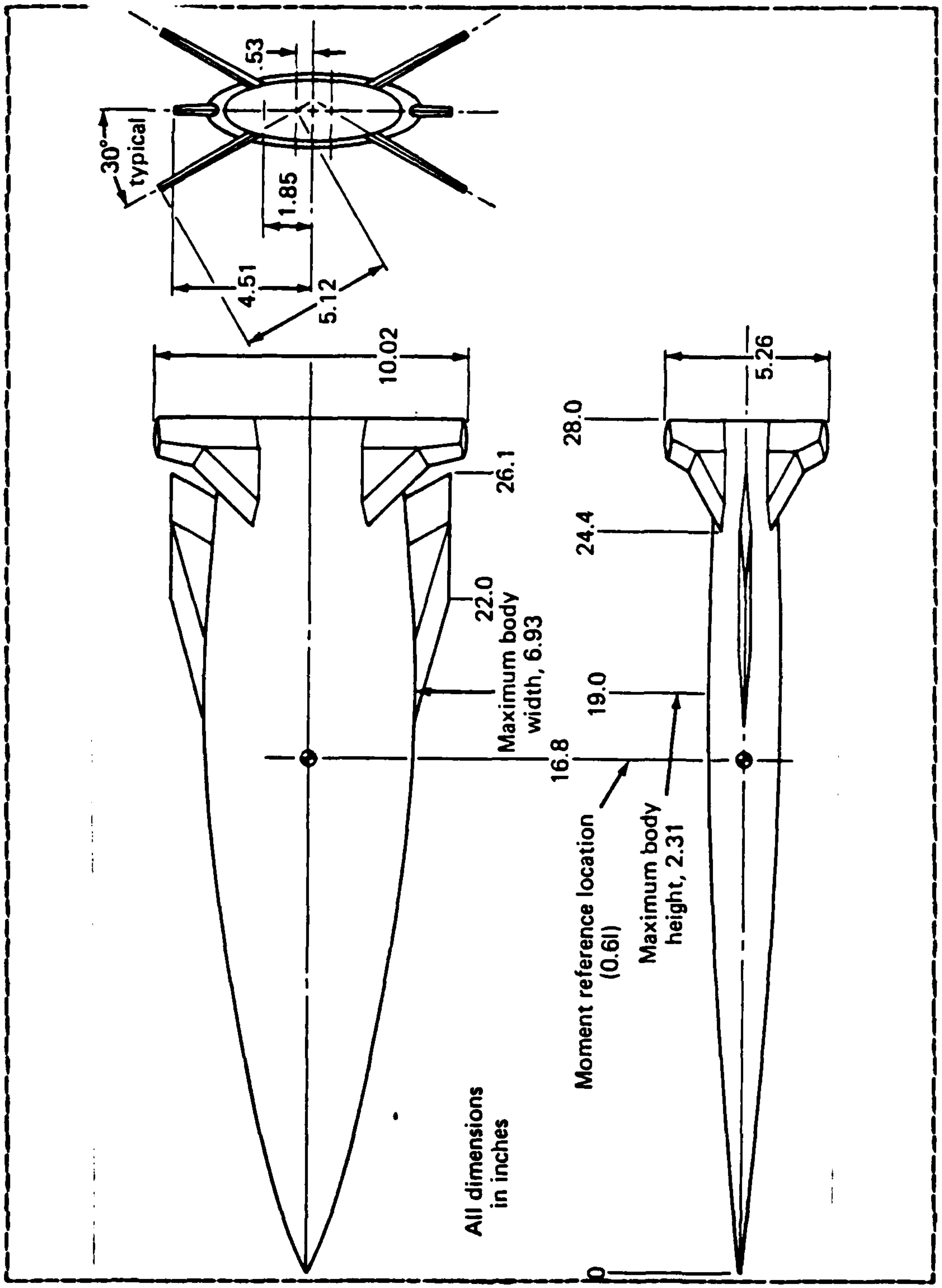


Figure A.1 Model of elliptical configuration (1/6 scale).

TABLE VI

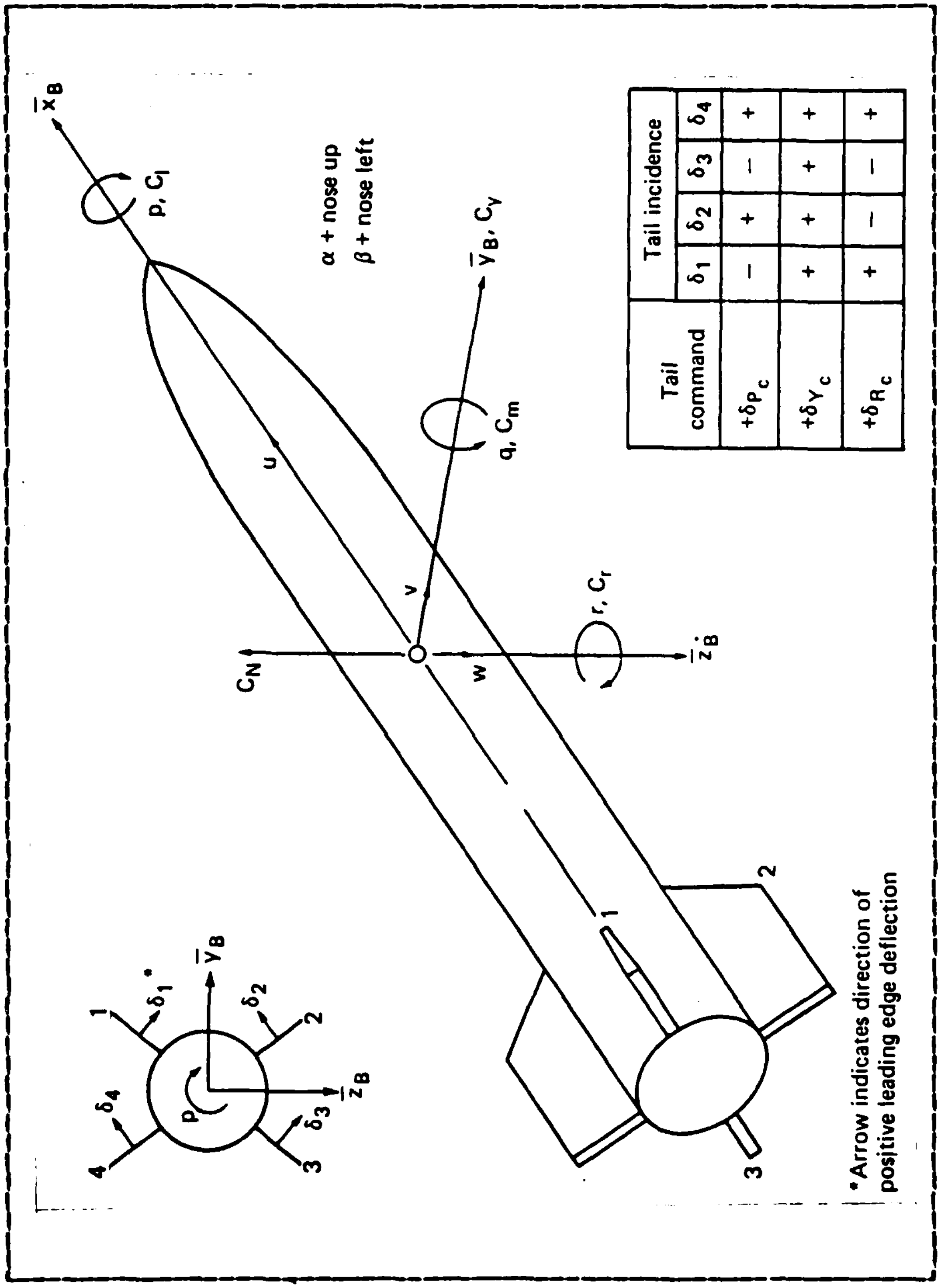
Geometric and Mass Properties of Missile Configuration.

WEIGHT (lbs)	2475
I_{xx} (slug sq ft)	110
I_{yy} (slug sq ft)	790
I_{zz} (slug sq ft)	853
LENGTH (in), l	168
CENTER OF GRAVITY distance from nose (in)	100.8(0.6 l)
MAX. MAJOR AXIS (in)	41.7
MAX. MINOR AXIS (in)	13.86

Inertial acceleration commands are applied in polar coordinates (i.e., magnitude of command (η_C) applied to the pitch autopilot and the direction (ϕ_C) is applied to the roll autopilot).

The yaw autopilot is slaved to the roll autopilot to minimize sideslip angle by coordinating the missile yaw and roll motion.

Achieved maneuver plane or inertial acceleration in rectangular coordinates (i.e., η_{Z_V} and η_{Y_V}) is determined by resolving achieved body-fixed accelerations (i.e., η_{Z_B} and η_{Y_B}) through missile roll angle(ϕ) (i.e., Euler angles θ and ψ are assumed to be sufficiently small).



Tail command	Tail incidence δ_1	δ_2	δ_3	δ_4
$+\delta_{P_c}$	−	+	−	+
$+\delta_{Y_c}$	+	+	+	+
$+\delta_{R_c}$	+	−	−	+

Figure A.2 Aerodynamic Sign Convention.

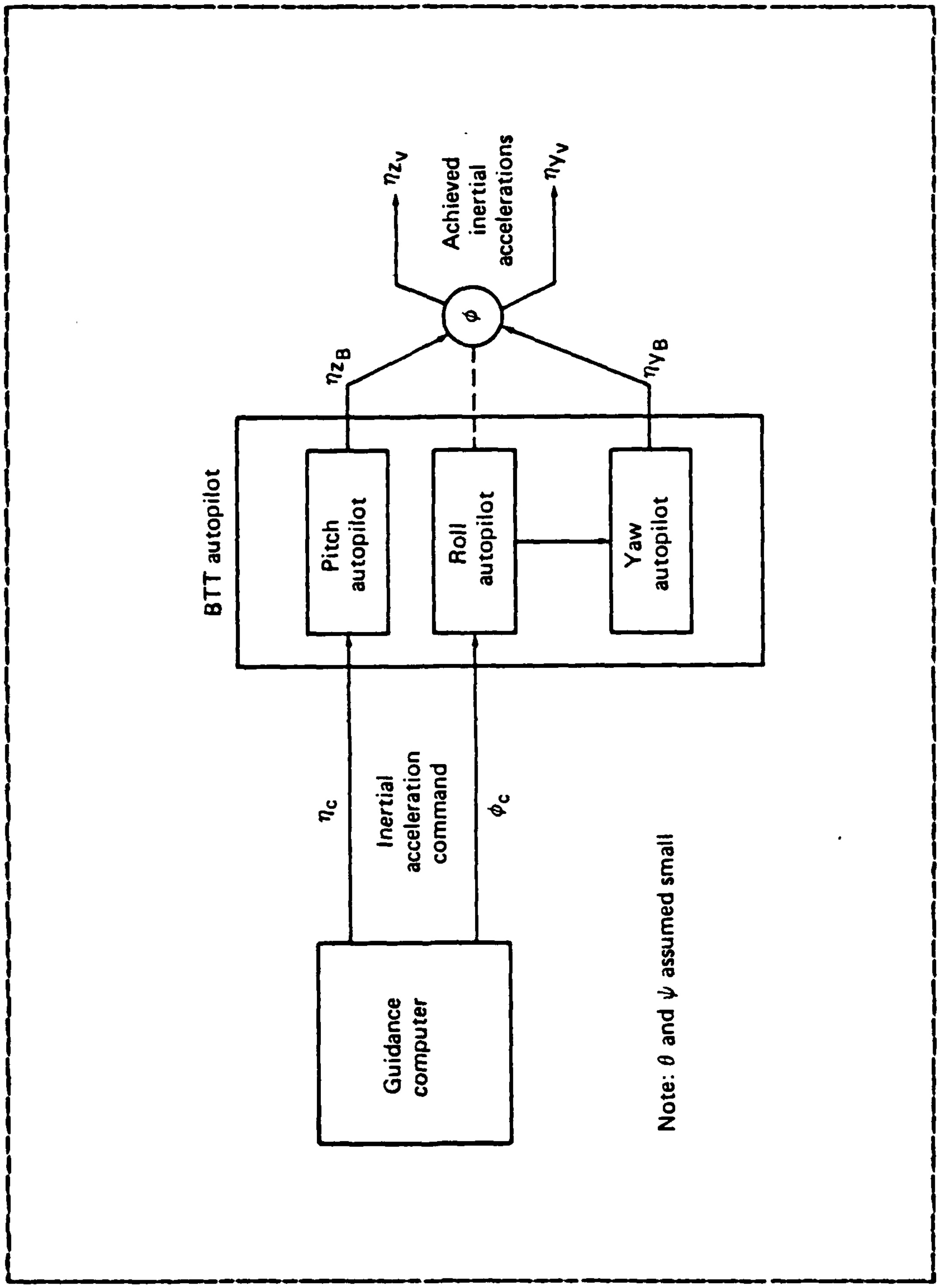


Figure A.3 BTT AUTOPILOT.

APPENDIX B

LINEAR SYSTEM MODELS

A. INTRODUCTION

The following appendix addresses the informations of the linear uncoupled pitch channel autopilot and the roll-yaw coupled channel autopilot of a elliptical airframe taken from [Ref. 2].

The linear time domain analysis of the CBTT autopilot was assumed that the missile is initially in the desired maneuver plane and trimmed at ten degrees angles-of-attack (i.e., the equilibrium or trim angle-of-attack in the model of Fig.A.1 equals 10 degrees and the equilibrium roll rate Pe in the models of Fig.A.3 equal zero). When Pe = 0, Qe (i.e., equilibrium pitch rate) has been formed to have negligible influence in the lateral model compared to α_e and was therefore set equal to zero.

B. UNCOUPLED PITCH CHANNEL AUTOPILOT

A general block diagram of an uncoupled pitch channel autopilot is shown in Fig. B.1.

A normal acceleration command (η_z, g's) is applied to the pitch control law which uses measurements of missile body pitch angular rate (q) and pitch normal acceleration (η_z) to determine the required actuator command (δ_{P_c}). The actuator is modeled as a first-order lag at 30Hz (188.4 rad/sec).

The dynamic model is linearized about a trim angle-of-attack as described in appendix A.

From the block diagram of Fig.B.1, the parameters of interest for the present analysis are given respectively as C_{N_α}, $C_{N_{\delta_P}}$, C_{m_α}, $C_{m_{\delta_P}}$.

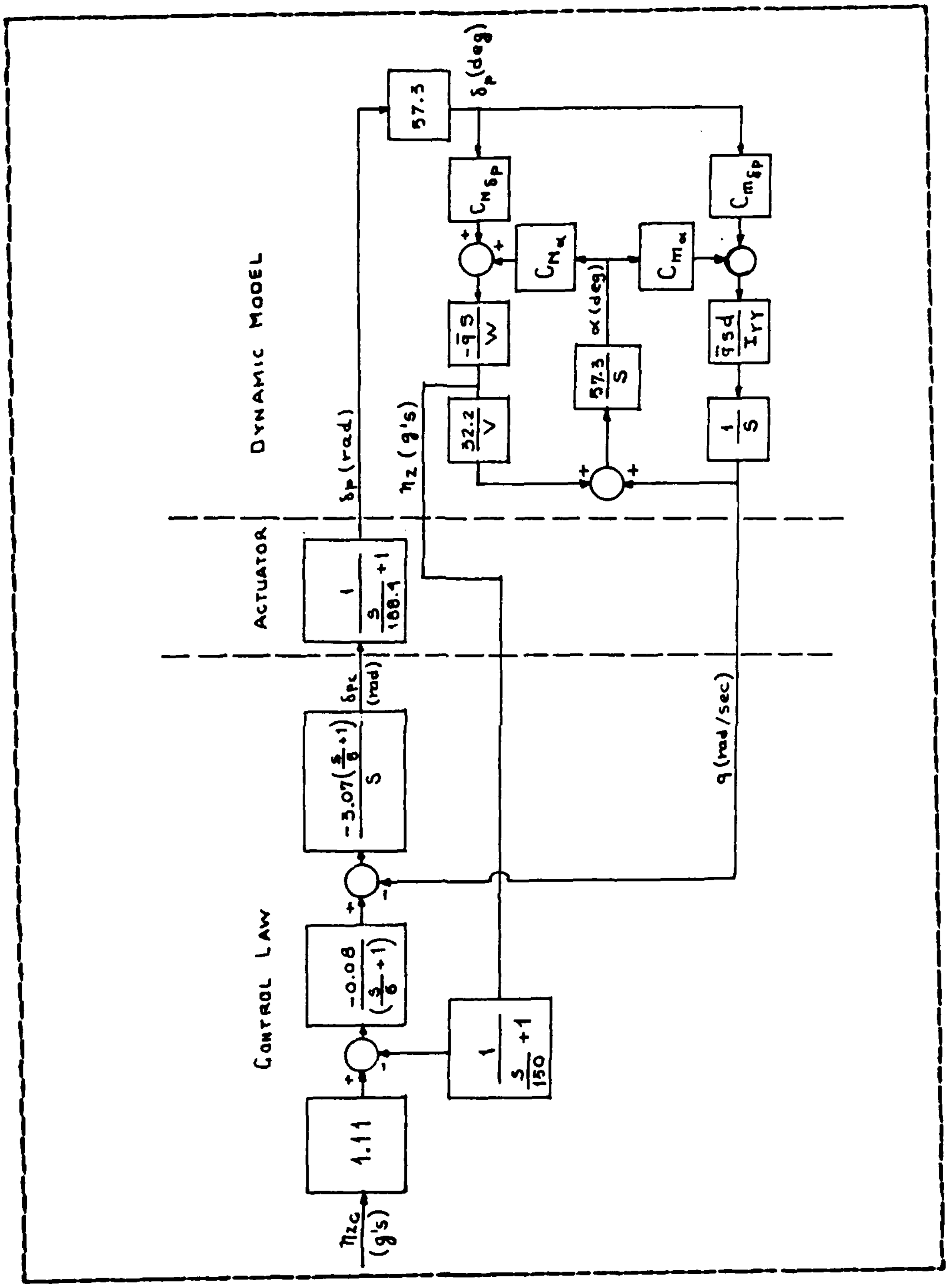


Figure B.1   Uncoupled Pitch Channel Autopilot.

For ease notation the correspondence of state variables and constants are given in Table VII

Performing block diagram manipulations see [Ref. 5], one yields to the following state variable equations:

Aerodynamic equations

$$\dot{X}1 = C2\ A3\ X2 + C2\ A4\ X3 \tag{B.1}$$

$$\dot{X}2 = X1 - K\ C1\ A1\ X2 - K\ C1\ A2\ X3 \tag{B.2}$$

Control law equations

$$\dot{X}3 = -\ C3\ X3 + C3\ Conv\ X6 \tag{B.3}$$

$$\dot{X}4 = -\ C1\ C4\ A1\ X2 - C1\ C4\ X3 - C4\ X4 \tag{B.4}$$

$$\dot{X}5 = C7\ X4 - C5\ X5 - C6\ NZC \tag{B.5}$$

Actuator equation

$$\dot{X}6 = C\ E\ A7\ X3 - 6\ X6 + C\ E\ A8\ X8 + C\ E\ A6\ X11 \tag{B.6}$$

C. COUPLED ROLL-YAW CHANNEL AUTOPILOT

A general block diagram of a coupled roll-yaw channel autopilot is shown in Fig.B.2.

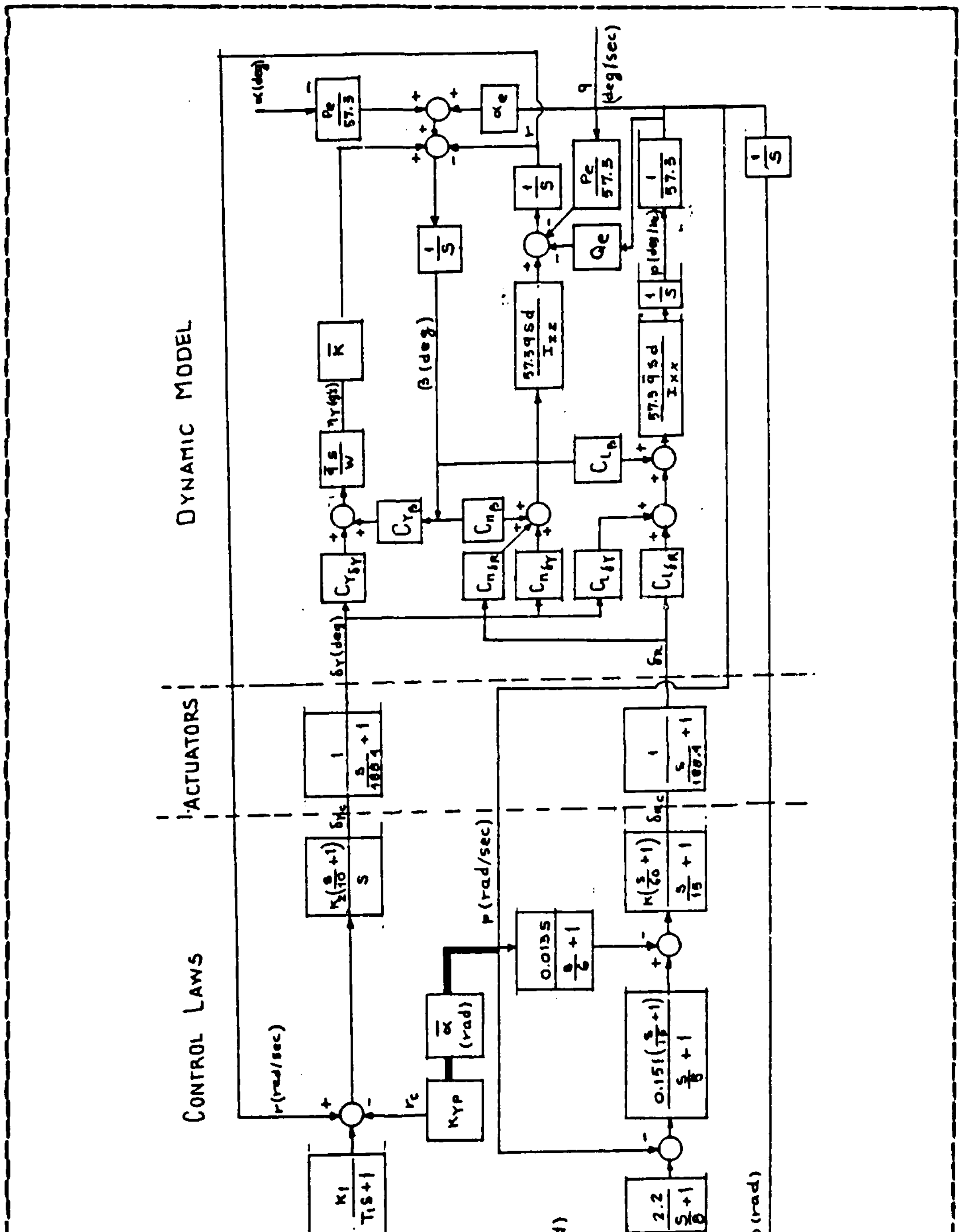


Figure B.2 Coupled Roll-Yaw Channel Autopilot.

The block diagram (Fig. B.2) shows, eight parameters of interest that are:

$A1 = C_{Y_{\delta_Y}}$, $A2 = C_{Y_\beta}$, $A3 = C_{n_{\delta_R}}$, $A4 = C_{n_\beta}$,

$A5 = C_{n_{\delta_Y}}$, $A6 = C_{\ell_{\delta_Y}}$, $A7 = C_{\ell_\beta}$, and $A8 = C_{\ell_{\delta_R}}$.

For ease of notation the correspondence of state variables and constants is given in Table VIII.

Eqns. B.7 through B.17 give the state variable equations of the coupled roll-yaw channel autopilot:

Aerodynamic equations

$$\dot{X}1 = C\ \mathrm{Conv}(A4\ X3 + A3\ X8 + A5\ X11) \tag{B.7}$$

$$\dot{X}2 = C\ \mathrm{Conv}(A7\ X3 + A8\ X8 + A6\ X11) \tag{B.8}$$

$$\dot{X}3 = -\ X1 - (\mathrm{alpha}/\mathrm{Conv})\ X2 \tag{B.9}$$

$$+\ KB\ A\ A2\ X3 + KB\ A\ A1\ X11$$

$$\dot{X}4 = -8\ X4 - 17.6\ X12 + 17.6\ PHC \tag{B.10}$$

Control law equations (roll)

$$\dot{X}5 = -\ (0.755/\mathrm{Conv})\ X2 - C\ D\ A7\ X3 \tag{B.11}$$

$$+\ (0.755 - 8\ D)\ X4 - 5\ X5 - C\ D\ A3\ X8 - C\ D\ A6\ X11$$

$$-\ 17.6\ D\ X12 + 17.6\ PHC$$

$$\dot{X}6 = C\ E\ A7\ X3 - 6\ X6 + C\ E\ A8\ X8 + C\ E\ A6\ X11 \qquad (B.12)$$

$$\dot{X}7 = -F\ KC\ (0.755/Conv)\ X2 - (D + E)\ F\ KC\ A7\ X3 \qquad (B.13)$$

$$+ F\ KC(0.755 - 8\ D)\ X4 + KC(15 - 5\ F)\ X5$$
$$+ KC(6F - 15)\ X6 - 15\ X7 - (D + E)\ F\ KC\ C\ A8\ X3$$
$$- (D + E)\ F\ KC\ C\ A6\ X11 - F\ KC\ D\ 17.6\ X12$$
$$+ F\ KC\ D\ 17.6\ PHC$$

$$\dot{X}8 = 188.4\ Conv\ X7 - 188.4\ X8 \qquad (B.14)$$

Control law equations (yaw)

$$\dot{X}9 = (K1\ A\ A2/T1)\ X3 - X9/T1 + (K1\ A\ a1/T1)\ X11 \qquad (B.15)$$

$$\dot{X}10 = (K2/Conv)\ X1 - (K2\ H\ ALPHAB)/Conv\ X2 \qquad (B.16)$$

$$+ (K2/10)((K1\ A\ A2)/T1 + B\ A4 - H\ ALPHAB\ C\ A7\ X3$$
$$+ (K2/10)(B\ A3 - H\ ALPHAB\ C\ A8\ X3 + K2(1 - (1/10\ T1)\ X9$$
$$+ (K2/10)((K1\ A\ A1)/T1) + B\ A5 - H\ ALPHAB\ C\ A6)\ X11$$

Actuator equations

$$\dot{X}11 = 188.4\ Conv\ X10 - 188.4\ X11 \qquad (B.17)$$

$$\dot{X}12 = X2/Conv \qquad (B.18)$$

D. AERODYNAMIC DATA-LINEAR APPROXIMATION

A linear approach was used in the design and stability analysis of the aultopilots of the pitch, yaw, and roll channels, both uncoupled and coupled. According, a linear approximation of the aerodynamic derivatives at m=3.95 was provided, about which the system could be perturbed. These linearized aerodynamic derivatives are given in [Ref. 2] and are presented here in Table IX.

TABLE VII

Correspondence of Symbols (Uncoupled Pitch Autopilot)

q	X1
α	X2
δ_P	X3
X	X4
Y	X5
δ_{P_c}	X6
C_{N_α}	A1
$C_{N_{\delta_P}}$	A2
C_{m_α}	A3
$C_{m_{\delta_P}}$	A4
$\bar{q}s/d$	C1
$57.3\bar{q}sd/I$	C2
188.4	C3
150.0	C4
6.0	C5
0.53544	C6
0.48	C7
0.38375	C8
3.07	C9
0.48	K
57.3	Conv

TABLE VIII

Correspondence of Symbols (Coupled Roll-Yaw Autopilot)

r	X1	3.1416	S
p	X2	57.3	Conv
β	X3	$\bar{q}s/d$	A
X	X4	$\bar{q}sd\,1/I_{zz}$	B
Y1	X5	$\bar{q}sd\,1/I_{xx}$	C
X1	X6	0.05033	D
δ_{Rc}	X7	0.078	E
δ_R	X8	0.25	F
γ	X9	α_e	ALPHA
$\delta_{\gamma c}$	X10	$\alpha_e/57.3$	ALPHAB
δ_γ	X11	0.48	KB
ϕ	X12	0.25	T1
$C_{Y\delta_\gamma}$	A1	0.84	K1
$C_{Y\beta}$	A2	6.08	K2
$C_{n\delta_R}$	A3	1.0	KYP
$C_{n\beta}$	A4	4.17	KC
$C_{n\delta_\gamma}$	A5	0,48	K
$C_{\ell\delta_\gamma}$	A6	2.0	D1
$C_{\ell\beta}$	A7	$\bar{q}$	QB
$C_{\ell\delta_R}$	A8		

TABLE IX

Linearized Aerodynamic Derivatives

	$\alpha = 10°$
C_{Y_β}	- 0.054
C_{n_β}	+ 0.024
C_{ℓ_β}	- 0.027
$C_{Y_{\delta_Y}}$	+ 0.015
$C_{n_{\delta_Y}}$	- 0.039
$C_{\ell_{\delta_Y}}$	- 0.010
$C_{Y_{\delta_R}}$	- 0.006
$C_{n_{\delta_R}}$	+ 0.014
$C_{\ell_{\delta_R}}$	+ 0.023
C_{N_α}	+ 0.220
$C_{N_{\delta_P}}$	+ 0.020
C_{m_α}	+ 0.0137
$C_{m_{\delta_P}}$	- 0.055

APPENDIX C
NONLINEAR SYSTEM MODEL

A. INTRODUCTION

A general block diagram of the nonlinear model is shown in Fig.C.1 The three dimensional nonlinear aerodynamic model presented here is for the same conditions used in the linear model shown in appendix B. The same flight condition used for the linear case is the same for the nonlinear model (i. e. , 60 kft altitude, Mach 3.95). The control laws are the same used for the linear models except for a minor modification to the coordinating branch dependence on angle-of-attack and also the inclusion of anti-gravity bias as stated in [Ref. 2].

B. CONTROL LAWS

The control laws used for the following nonlinear 3-D studies were the same as those used for the linear models, except for the gain $\bar{\alpha}$ shown in the bold line of coordination branch in Fig. C.1. The new gain $\bar{\alpha}$ is held constant at one degree magnitude for angle-of-attack less than one degree positive. For angle-of-attack greater than one degree positive, the gain $\bar{\alpha}$ is equal to the angle-of-attack. This keeps coordination for very small angle-of-attack.

Gravity effects were not included in the linear models in appendix B, because it was assumed to have a negligible influence on autopilot stability and response for pertubations about a missile trim condition. However, gravity

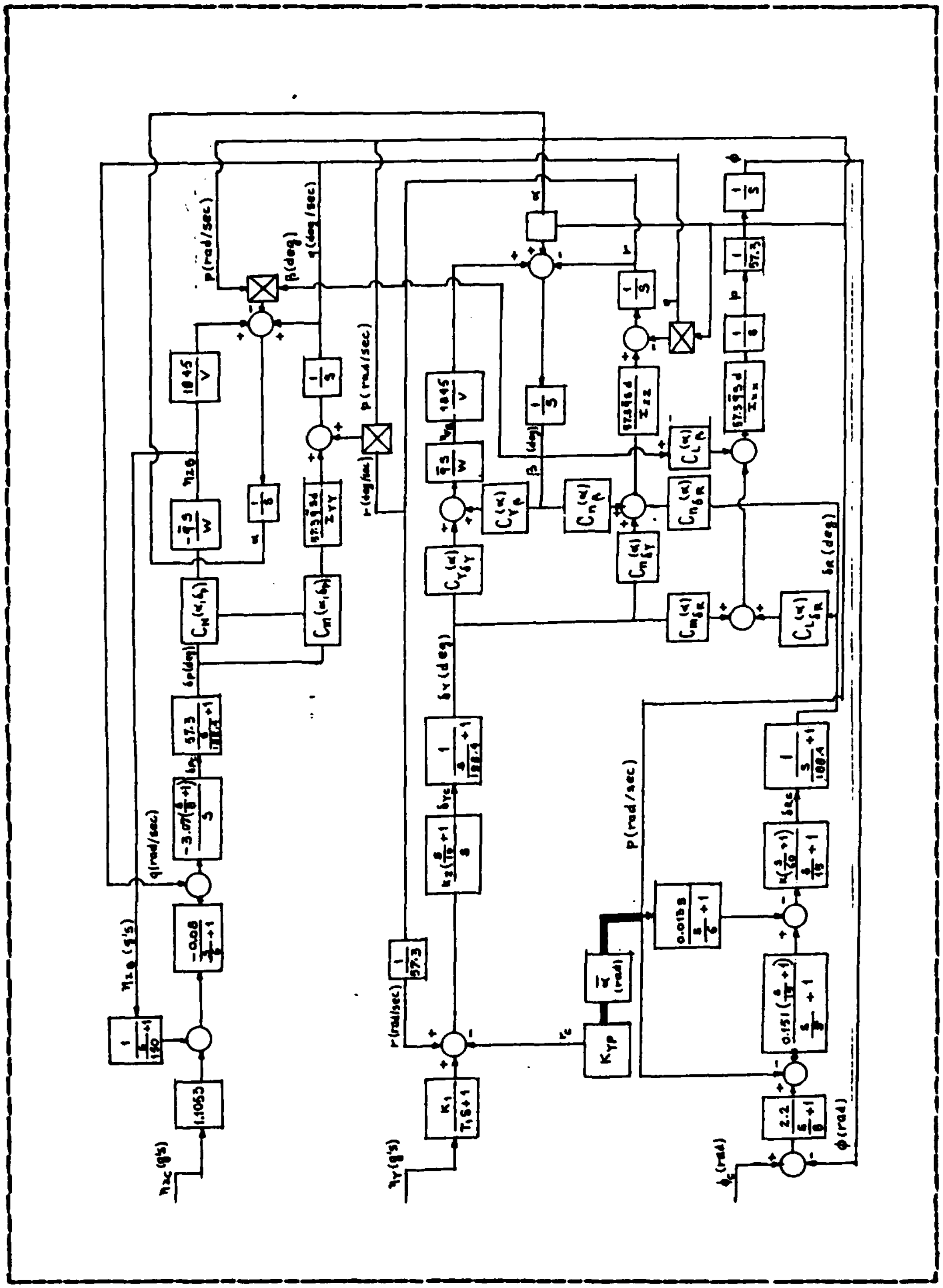


Figure C.1   Nonlinear Model.

effects were included in the following nonlinear model where the missile body-fixed yaw axis will be subjected to the full force of gravity and may, therefore, have a significant influence on sideslip.

The gravity effects are compensated for by pitch and roll motion of the missile which have less influence on sideslip than yaw motion. In inertial rectangular coordinates one has:

η_C =acceleration command in inertial $\bar{Z}_V$ direction

$$\eta_C = \eta_{Z_C} - \cos(\Theta) \tag{C.1}$$

where

η_{Z_C} = guidance command(g's)

$-\cos\Theta$ = anti-gravity bias command (g's)

η_{Y_C} = acceleration command in the inertial $\bar{Y}_V$ direction guidance command(g's)

There is no gravity effect in Y direction. In order to assure an anti-gravity bias of just one g, one needs to modify the anti-gravity command as follows:

elliptical airframe inertial acceleration command

η_C = Nzc-0.913cos(ϕ)

C. PERFOMANCE COMMANDS

The commands 2 gees is first applied in the 0° or upward direction at 2 seconds. Since both the missile roll angle and roll angle command are at zero degrees, there is no roll motion and the missile turns upward as a skid-to-turn controlled missile. At 5 seconds, a second 2 g's inertial guidance command is applied in the downward

or 180° direction. The missile is commanded to roll through 180° while moving in a coordinated manner in yaw and roll to minimize sideslip angle and prevent or minimize negative angle-of-attack.

The response of achieved maneuver and cross-plane accelerations during the first 2 seconds are due to initial conditions, gravity and anti-gravity bias effects. The initial conditions were added to minimize the transients which result when gravity bias commands the autopilot for constant altitude missile flight. The initial conditions were:

α = angle-of-attack = 2.41 deg

δ_p = pitch tail angle = 0.658 deg

Θ = pitch Euler angle = 3.65 deg

output of pitch acceleration feedback lag = -1.0 g's

pitch control law acceleration error lag prior to dc gain = - 0.0105

δ_{p_c} = pitch actuator command = 0.658 deg

The achieved maneuver plane acceleration (Eqn.C.2) is calculated from the body-fixed acceleration η_{z_B}, η_{Y_B}, and the roll angle ϕ as follows:

$$\eta_z = \eta_{z_B} \cos\phi + \eta_{Y_B} \sin\phi \tag{C.2}$$

During the first command, achieved body-fixed yaw acceleration (η_{Y_B}) and missile roll angle are equal to zero, because the roll channel is not commanded. Therefore, achieved maneuver plane acceleration is equal to the body-fixed acceleration η_{z_B}. During the second command the missile roll angle has the same roll angle response of the roll autopilot.

D. AERODYNAMIC DATA - NONLINEAR REPRESENTATION

The aerodynamic data are given in [Ref. 2]. The entire study was conducted using M=3.95 and the aerodynamic coefficients are based on a body-fixed axis system of Fig.B.2.

For reference, the normal force and pitching moment plots given in appendix B of [Ref. 2] at M=3. 95 were reproduced in Table X and XI, which give just the values for points of interest. These tables present, too, the correspondent derivatives that were found by simulation using a fortran program shown in appendix F. The aerodynamic derivative of C_ℓ, C_n, and C_Y with respect to sideslip angle β, yaw control δ_Y, and roll control δ_R are presented in Table XII, XIII and XIV which give just values for points of interest and includes the correspondent derivatives that were found by simulation using a fortran program given in appendix F.

The block diagram of Fig.C.1 shows 10 parameter of interest that are :

$A1 = C_m(\alpha, \delta_P)$, $A2 = C_N(\alpha, \delta_P)$, $A3 = C_{\ell_{\delta_R}}(\alpha)$,

$A4 = C_{n_{\delta_P}}(\alpha)$, $A5 = C_{n_{\delta_R}}(\alpha)$, $A6 = C_{Y_{\delta_Y}}(\alpha)$,

$A7 = C_{\ell_{\delta_Y}}(\alpha)$, $A8 = C_{\ell_\beta}(\alpha)$, $A9 = C_{n_\beta}(\alpha)$,

and $AA = C_{Y_\beta}(\alpha)$.

Using the same procedure as given in appendix B one can have the Tables XV and XVI that give the correspondence of symbols for the nonlinear model. Eqns.C.3 through C.21 give the state variable equations of the nonlinear model.

$$\dot{X}1 = - C4\ X1 - C02\ C4\ CN(X6, X4) \tag{C.3}$$

$$\dot{X}2 = C7\ X1 - C5\ X2 + C6\ C19\ \mathrm{Cos}(X17) - C6\ NZC \tag{C.4}$$

$$\dot{X}3 = - C7\ C8\ X1 + (C5\ C8 - C9)\ X2 + (C9/\mathrm{Conv})\ X5 \tag{C.5}$$

$$+ C6\ C8\ NZC$$

$$\dot{X}4 = - C3\ X4 + C3\ \mathrm{Conv}\ X3 \tag{C.6}$$

$$\dot{X}5 = (X17\ X18)/\mathrm{Conv} + C1\ \mathrm{Conv}\ CM(X6, X4) \tag{C.7}$$

$$\dot{X}6 = X5 - KB\ CM(X6, X4) - (X16\ X18)/\mathrm{Conv} \tag{C.8}$$

$$\dot{X}7 = X5\ \mathrm{Cos}(X19)/\mathrm{Conv} - X17\ \mathrm{Sin}(X19)/\mathrm{Conv} \tag{C.9}$$

$$\dot{X}8 = - C10\ X8 + (C12 - C11\ C13)\ X10 \tag{C.10}$$

$$+ C11\ C14\ PHC - C11\ C14\ X19 - C0\ C11\Big(C_{Y\delta_Y}(X6)\ X15$$
$$+ C_{\ell\delta_R}(X6)\ X13\Big) - C12\ X18/\mathrm{Conv}$$

$$\dot{X}9 = - X9/T1 + (K1/T1)\ C02\big(C_{Y\beta}(X6)\ X16 \tag{C.11}$$

$$+ C_{Y\delta_Y}(X6)\ X15\big)$$

$$\dot{X}10 = - C13\ X10 + C14\ PHC - C14\ X19 \tag{C.12}$$

$$\dot{X}11 = - C15\ X11 + C16\ C0\,(C_{\ell\beta}(x6)\ X16 \tag{C.13}$$

$$+ C\quad (X6)\ X15 + C\quad (X6)\ X13)$$

$$\dot{X}12 = - C7\ X12 + K\,(C17 - C10/C18)\ X18 \tag{C.14}$$

$$+ (K/C18)\,(C12 - C11\ C13)\ X10 + K\ C11\ (C14/C18)\ PHC$$
$$- K\ C11\ C14/C18\ X19 - (K/C18)\ C0\ C\quad (X6)\,(C11$$
$$+ C16)\ X16 - (K/C18)\ C0\ C_{\ell\beta}(X6)\,(C11 + C16)\ X15$$
$$- (K/C18)\ C0\ C_{\ell\beta}(X6)\ (C11 + C16)\ X13$$
$$- (K\ C12/Conv)\ X18 + (C15/C18 - C17)\ X11$$

$$\dot{X}13 = - C3\ X13 + C3\ Conv\ X12 \tag{C.15}$$

$$\dot{X}14 = (K2/10)\,(- X9/T1 + K1\ T1\ C02\ (C_{\gamma\beta}(X6)\ X16 \tag{C.16}$$

$$+ C_{\gamma\delta\gamma}(X6)\ X15) - (X15\ X18)/Conv^2(X5 - KB\ C02\ C_N(X6, X4)$$
$$- (X16\ X18)/Co\text{[illegible]}\ X18 - (KYP/Conv)\,(X5 - KB\ C02\ C_N(X6, X4))$$
$$- (X16\ X18)/Conv\ X18 - (KYP/Conv)\ C0\ XX6\ (C_{\ell\beta}(X6)\ X16$$
$$- C_{\ell\delta\gamma}(X6)\ X15 + C_{\ell\delta R}(X6)\ X13) + (K2/Conv)\ X17$$
$$+ K2\ X9 - (K2\ KYP/Conv^2)\ XX6\ X18$$

$$\dot{X}15 = - C3\ X15 + C3\ Conv\ X14 \tag{C.17}$$

$$\dot{X}16 = KB\ C02\,(C_{\gamma\beta}(X6)\ X16 + C_{\gamma\delta\gamma}(X6)\ X15 \tag{C.18}$$

$$+ (X16\ X18)/Conv - X17$$

$$\dot{X}17 = -(X5\ X18)/Conv + C01\ Conv(C_{n_\beta}(X6)\ X16 \qquad (C.19)$$

$$+ C_{n_{\delta_R}}(X6)\ X13)$$

$$\dot{X}18 = C0\ Conv(C_{\ell_\beta}(X6)\ X16 + C_{\ell_{\delta_Y}}(X6)\ X15 \qquad (C.20)$$

$$+ C_{n_{\delta_R}}(X6)\ X13)$$

$$\dot{X}19 = X18/Conv \qquad (C.21)$$

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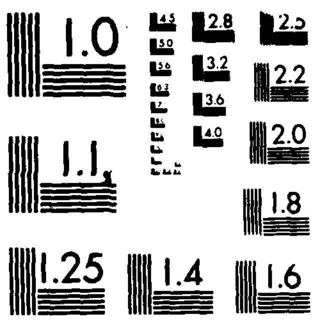

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TABLE X

Normal Force Coefficient and Derivatives M=3.95

α	δ_p	$C_N(\alpha,\delta_p)$	$\frac{\partial C_N(\alpha,\delta_p)}{\partial \alpha}$	$\frac{\partial C_N(\alpha,\delta_p)}{\partial \delta_p}$
-4.0000	-10.0000	-1.0500	0.1842	0.0308
-4.0000	0.0000	-0.8000	0.1453	0.0208
-4.0000	5.0000	-0.7000	0.2496	0.0196
-4.0000	10.0000	-0.6000	0.2141	0.0208
0.0000	-10.0000	-0.2500	0.2110	-0.0332
0.0000	0.0000	-0.0800	0.2074	0.0448
0.0000	5.0000	0.1300	0.1827	0.0336
0.0000	10.0000	0.2000	0.1929	-0.0112
4.0000	-10.0000	0.6000	0.2092	0.0200
4.0000	0.0000	0.8000	0.2253	0.0200
4.0000	5.0000	0.9000	0.2196	0.0200
4.0000	10.0000	1.0000	0.2141	0.0200
8.0000	-10.0000	1.4500	0.2272	0.0042
8.0000	0.0000	1.7000	0.2265	0.0342
8.0000	5.0000	1.8500	0.2288	0.0229
8.0000	10.0000	1.9000	0.2256	-0.0058
12.0000	-10.0000	2.4000	0.2321	0.0200
12.0000	0.0000	2.6000	0.2188	0.0200
12.0000	5.0000	2.7000	0.2153	0.0200
12.0000	10.0000	2.8000	0.2335	0.0200
16.0000	-10.0000	3.3000	0.2319	0.0083
16.0000	0.0000	3.5000	0.2482	0.0283
16.0000	5.0000	3.6500	0.2599	0.0308
16.0000	10.0000	3.8000	0.2654	0.0283
20.0000	-10.0000	4.3000	0.2653	0.0300
20.0000	0.0000	4.6000	0.2884	0.0300
20.0000	5.0000	4.7500	0.2825	0.0300
20.0000	10.0000	4.9000	0.2798	0.0300
24.0000	-10.0000	5.4000	0.2819	0.0300
24.0000	0.0000	5.7000	0.2482	0.0300
24.0000	5.0000	5.8500	0.2599	0.0300
24.0000	10.0000	6.0000	0.2654	0.0300

TABLE XI

Pitching Moment Coefficient and Derivatives M=3.95

α	δ_p	$C_m(\alpha,\delta_p)$	$\frac{\partial C_m(\alpha,\delta_p)}{\partial \alpha}$	$\frac{\partial C_m(\alpha,\delta_p)}{\partial \delta_p}$
-4.0000	-10.0000	0.4850	0.0130	-0.0159
-4.0000	-5.0000	0.2800	-0.0200	-0.0610
-4.0000	0.0000	-0.0750	0.0190	-0.0759
-4.0000	10.0000	-0.6300	0.0197	-0.0149
0.0000	-10.0000	0.5500	0.0185	-0.0550
0.0000	-5.0000	0.2750	0.0131	-0.0550
0.0000	0.0000	-0.0000	0.0186	-0.0550
0.0000	10.0000	-0.5500	0.0195	-0.0550
4.0000	-10.0000	0.6250	0.0180	-0.0551
4.0000	-5.0000	0.3500	0.0200	-0.0550
4.0000	0.0000	0.0750	0.0190	-0.0551
4.0000	10.0000	-0.4800	0.0147	-0.0561
8.0000	-10.0000	0.7100	0.0294	-0.0563
8.0000	-5.0000	0.4300	0.0232	-0.0558
8.0000	0.0000	0.1500	0.0178	-0.0563
8.0000	10.0000	-0.4300	0.0118	-0.0603
12.0000	-10.0000	0.8500	0.0331	-0.0640
12.0000	-5.0000	0.5250	0.0184	-0.0655
12.0000	0.0000	0.2000	0.0034	-0.0640
12.0000	10.0000	-0.3500	0.0057	-0.0520
16.0000	-10.0000	0.9500	0.0182	-0.0725
16.0000	-5.0000	0.5750	0.0120	-0.0763
16.0000	0.0000	0.2000	0.0059	-0.0725
16.0000	10.0000	-0.4000	-0.0122	-0.0425
20.0000	-10.0000	1.0000	0.0065	-0.0756
20.0000	-5.0000	0.6220	0.0065	-0.0755
20.0000	0.0000	0.2450	0.0067	-0.0752
20.0000	10.0000	-0.5000	-0.0395	-0.0736
24.0000	-10.0000	1.0000	-0.0068	-0.0841
24.0000	-5.0000	0.5870	-0.0290	-0.0818
24.0000	0.0000	0.1750	-0.0516	-0.0837
24.0000	10.0000	-0.7300	-0.0772	-0.1001

TABLE XII

Sideslip Derivatives and Derivatives

α	$C_{l_\beta}(\alpha)$	$\frac{\partial C_{l_\beta}(\alpha)}{\partial \alpha}$
-4.0000	0.0110	-0.0027
0.0000	0.0000	-0.0027
4.0000	-0.0110	-0.0022
5.2000	-0.0135	-0.0024
8.0000	-0.0220	-0.0027
10.0000	-0.0270	-0.0021
12.6000	-0.0310	-0.0015
16.0000	-0.0360	-0.0014
20.0000	-0.0410	-0.0011

α	$C_{n_\beta}(\alpha)$	$\frac{\partial C_{n_\beta}(\alpha)}{\partial \alpha}$
-4.0000	0.0240	0.0000
0.0000	0.0240	0.0000
4.0000	0.0240	0.0000
5.2000	0.0240	0.0000
8.0000	0.0240	0.0000
10.0000	0.0240	0.0005
12.6000	0.0270	0.0009
16.0000	0.0290	0.0007
20.0000	0.0320	0.0008

α	$C_{Y_\beta}(\alpha)$	$\frac{\partial C_{Y_\beta}(\alpha)}{\partial \alpha}$
-4.0000	-0.0490	0.0024
0.0000	-0.0440	0.0001
4.0000	-0.0480	-0.0009
5.2000	-0.0490	-0.0009
8.0000	-0.0520	-0.0013
10.0000	-0.0550	-0.0013
12.6000	-0.0580	-0.0010
16.0000	-0.0610	-0.0009
20.0000	-0.0650	-0.0011

TABLE XIII

Yaw Control Derivatives and Derivatives

α	$C_{l_{\delta_Y}}(\alpha)$	$\frac{\partial C_{l_{\delta_Y}}(\alpha)}{\partial \alpha}$
-4.0000	0.0025	-0.0007
0.0000	0.0000	-0.0006
4.0000	-0.0020	-0.0014
5.2000	-0.0040	-0.0015
8.0000	-0.0075	-0.0013
10.0000	-0.0100	-0.0014
12.6000	-0.0140	-0.0015
16.0000	-0.0190	-0.0014
20.0000	-0.0240	-0.0011
α	$C_{n_{\delta_Y}}(\alpha)$	$\frac{\partial C_{n_{\delta_Y}}(\alpha)}{\partial \alpha}$
-4.0000	-0.0400	0.0000
0.0000	-0.0400	0.0000
4.0000	-0.0400	0.0000
5.2000	-0.0400	0.0000
8.0000	-0.0400	0.0000
10.0000	-0.0400	0.0000
12.6000	-0.0400	-0.0003
16.0000	-0.0420	-0.0007
20.0000	-0.0453	-0.0010
α	$C_{Y_{\delta_Y}}(\alpha)$	$\frac{\partial C_{Y_{\delta_Y}}(\alpha)}{\partial \alpha}$
-4.0000	0.0150	0.0000
0.0000	0.0150	0.0000
4.0000	0.0150	0.0000
5.2000	0.0150	0.0000
8.0000	0.0150	0.0000
10.0000	0.0150	0.0000
12.6000	0.0150	0.0003
16.0000	0.0170	0.0006
20.0000	0.0195	0.0006

TABLE XIV

Roll Control Derivatives and Derivatives

α	$C_{l_{\delta_R}}(\alpha)$	$\frac{\partial C_{l_{\delta_R}}(\alpha)}{\partial \alpha}$
-4.0000	0.0232	C.0000
0.C0C0	0.0232	0.0000
4.0000	0.0232	C.00C0
5.2000	0.0232	C.0000
8.0000	0.0232	0.0000
10.0000	0.0232	0.0003
12.6000	0.C250	0.00C6
16.000C	0.0270	0.0006
20.0000	0.C298	C.00C8

α	$C_{n_{\delta_R}}(\alpha)$	$\frac{\partial C_{n_{\delta_R}}(\alpha)}{\partial \alpha}$
-4.00C0	-0.0045	0.0011
0.000C	0.C000	0.0011
4.0000	0.0045	0.00C9
5.20CC	0.C055	0.0012
8.00C0	0.C110	C.0018
10.0000	0.0145	0.0018
12.60CC	0.C195	0.0018
16.0000	0.0250	0.0017
20.0000	0.C320	0.0018

TABLE XV

Correspondence of Symbols (Nonlinear Model)

X	.... X1	p	.... X18
Y	.... X2	ϕ	.... X19
δ_{P_c}	.... X3	$C_M(\alpha,\delta_p)$	.... A1
δ_P	.... X4	$C_N(\alpha,\delta_p)$	.... A2
q	.... X5	$C_{\ell\delta_R}$	.... A3
α	.... X6	$C_{n\delta_Y}$	.... A4
θ	.... X7	$C_{n\delta_R}$	.... A5
Y1	.... X8	$C_{Y\delta_Y}$	.... A6
Y2	.... X9	$C_{\ell\delta_Y}$	.... A7
X1	.... X10	C_{ℓ_β}	.... A8
X2	.... X11	C_{m_β}	.... AA
δ_{R_c}	.... X12	C_{Y_β}	.... A10
δ_R	.... X13	$\bar{q}Sd/I_{XX}$	.... C0
δ_{Y_c}	.... X14	$\bar{q}Sd/I_{ZZ}$	.... C01
δ_Y	.... X15	$\bar{q}Sd/W$	.... C02
β	.... X16	$\bar{q}Sd/I_{YY}$	.... C1
r	.... X17	3.1416	.... C2

TABLE XVI

Correspondence of Symbols (Cont. Nonlinear Model)

188.4	.... C3	$\frac{\partial C_m(\alpha, \delta_p)}{\partial \alpha}$	.... DA16
150.0	.... C4		
6.0	.... C5	$\frac{\partial C_m(\alpha, \delta_p)}{\partial \delta_p}$	.... DA14
0.530544	.... C6		
0.48	.... C7	$\frac{\partial C_N(\alpha, \delta_p)}{\partial \alpha}$	.... DA26
0.38375	.... C8		
3.07	.... C9	$\frac{\partial C_N(\alpha, \delta_p)}{\partial \delta_p}$	.... DA24
5.0	.... C10		
0.05033	.... C11	$\frac{\partial C_{l\delta_R}(\alpha)}{\partial \alpha}$	.... DA3
0.755	.... C12		
8.0	.... C13	$\frac{\partial C_{n\delta_R}(\alpha)}{\partial \alpha}$	.... DA4
17.6	.... C14		
6.0	.... C15	$\frac{\partial C_{n\delta_a}(\alpha)}{\partial \alpha}$	.... DA5
0.078	.... C16		
15.0	.... C17	$\frac{\partial C_{n\delta_p}(\alpha)}{\partial \alpha}$	.... DA6
4.0	.... C18		
0.913	.... C19	$\frac{\partial C_{l\delta_p}(\alpha)}{\partial \alpha}$	.... DA7
0.25	.... T1		
0.839	.... K1	$\frac{\partial C_{l\beta}(\alpha)}{\partial \alpha}$	.... DA8
6.08	.... K2		
1.0	.... KYP	$\frac{\partial C_{n\beta}(\alpha)}{\partial \alpha}$	.... DA9
		$\frac{\partial C_{Y\beta}(\alpha)}{\partial \alpha}$	.... DAA

APPENDIX D

UNCOUPLED PITCH AUTOPILOT - CSMP PROGRAM

```
//ZEB JOB (0314,1797),'THESIS',CLASS=G
// EXEC CSMPXV
//X.COMPRINT DD DUMMY
//X.SYSPRINT DD DUMMY
//X.PLOTPARM DD *
 &PLOT SCALE=.5 &END
//X.SYSIN DD *
*/*MAIN ORG=NPGVM1.1797P
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
*   NAME : TIAGO DA SILVA RIBEIRO                         WT 21       *
*                                                                     *
*   SENSITIVITY ANALYSIS APPLIED TO A BANK-TO-TURN MISSILE            *
*                                                                     *
*   TITLE: UNCOUPLED PITCH AUTOPILOT                                  *
*          ELLIPTICAL CASE                                            *
*          ALPHA = 10                                                 *
*                                                                     *
*                         10  AUG  1983                               *
*                                                                     *
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
*
*
PARAM A1=.22,A2=.02,A3=.0137,A4=-.055
PARAM W=2475.,IYY=790.,CONV=57.3
PARAM S=3.1416,D=2.,QB=1650.,K=.48
PARAM C3=188.4,C4=150.,C5=6.,C6=.530544,C7=.48,C8=.38375,C9=3.07

C1=(QB*S)/W
C2=CONV*QB*S*D/IYY
NZC=STEP(0.0)

DEL1=.4*A1
DEL2=.4*A2
DEL3=.4*A3
DEL4=.4*A4

A11=1.4*A1
A21=1.4*A2
A31=1.4*A3
```

```
A41=1.4*A4

*
* NOMINAL  &  SENSITIVITY EQUATIONS
*
* AERODYNAMIC EQUATIONS

NZ=-C1*(A1*X2+A2*X3)
X1DT=C2*(A3*X2+A4*X3)
Y1DT=C2*(A31*Y2+A41*Y3)
L11DT=C2*(A3*L21+A4*L31)
L12DT=C2*(A3*L22+A4*L32)
L13DT=C2*(A3*L23+A4*L33)      +C2*X2
L14DT=C2*(A3*L24+A4*L34)      +C2*X3

X2DT=X1-K*C1*A1*X2-K*C1*A2*X3
Y2DT=Y1-K*C1*A11*Y2-K*C1*A21*Y3
L21DT=L11-K*C1*A1*L21-K*C1*A2*L31      -K*C1*X2
L22DT=L12-K*C1*A1*L22-K*C1*A2*L32      -K*C1*X3
L23DT=L13-K*C1*A1*L23-K*C1*A2*L33
L24DT=L14-K*C1*A1*L24-K*C1*A2*L34

X3DT=-C3*X3+C3*X6*CONV
Y3DT=-C3*Y3+C3*Y6*CONV
L31DT=-C3*L31+C3*L61*CONV
L32DT=-C3*L32+C3*L62*CONV
L33DT=-C3*L33+C3*L63*CONV
L34DT=-C3*L34+C3*L64*CONV

X4DT=-C4*C1*A1*X2-C4*C1*A2*X3-C4*X4
Y4DT=-C4*C1*A11*Y2-C4*C1*A21*Y3-C4*Y4

L41DT=-C4*C1*A1*L21-C4*C1*A2*L31-C4*L41      -C4*C1*X2
L42DT=-C4*C1*A1*L22-C4*C1*A2*L32-C4*L42      -C4*C1*X3
L43DT=-C4*C1*A1*L23-C4*C1*A2*L33-C4*L43
L44DT=-C4*C1*A1*L24-C4*C1*A2*L34-C4*L44

* ACTUATOR EQUATION

X5DT=C7*X4-C5*X5-C6*NZC
Y5DT=C7*Y4-C5*Y5-C6*NZC
L51DT=C7*L41-C5*L51
L52DT=C7*L42-C5*L52
L53DT=C7*L43-C5*L53
L54DT=C7*L44-C5*L54

* CONTROL LAW EQUATIONS
```

```
X6DT=C9*X1/CONV+C2*C8*A3*X2/CONV+C2*C8*A4*X3/CONV-C7*C8*X4+...
     (C5*C8-C9)*X5+C6*C8*NZC
Y6DT=C9*Y1/CONV+C2*C8*A31*Y2/CONV+C2*C8*A41*Y3/CONV-C7*C8*Y4+...
     (C5*C8-C9)*Y5+C6*C8*NZC
L61DT=C9/CONV*L11+C2*C8*A3/CONV*L21+C2*C8*A4/CONV*L31-C7*C8*L41+...
(C5*C8-C9)*L51
L62DT=C9/CONV*L12+C2*C8*A3/CONV*L22+C2*C8*A4/CONV*L32-C7*C8*L42+...
(C5*C8-C9)*L52
L63DT=C9/CONV*L13+C2*C8*A3/CONV*L23+C2*C8*A4/CONV*L33-C7*C8*L43+...
(C5*C8-C9)*L53+C2*C8/CONV*X2
L64DT=C9/CONV*L14+C2*C8*A3/CONV*L24+C2*C8*A4/CONV*L34-C7*C8*L44+...
(C5*C8-C9)*L54+C2*C8/CONV*X3

X1=INTGRL(0.0,X1DT)
X2=INTGRL(0.0,X2DT)
X3=INTGRL(0.0,X3DT)
X4=INTGRL(0.0,X4DT)
X5=INTGRL(0.0,X5DT)
X6=INTGRL(0.0,X6DT)

Y1=INTGRL(0.0,Y1DT)
Y2=INTGRL(0.0,Y2DT)
Y3=INTGRL(0.0,Y3DT)
Y4=INTGRL(0.0,Y4DT)
Y5=INTGRL(0.0,Y5DT)
Y6=INTGRL(0.0,Y6DT)

L11=INTGRL(0.0,L11DT)
L12=INTGRL(0.0,L12DT)
L13=INTGRL(0.0,L13DT)
L14=INTGRL(0.0,L14DT)

L21=INTGRL(0.0,L21DT)
L22=INTGRL(0.0,L22DT)
L23=INTGRL(0.0,L23DT)
L24=INTGRL(0.0,L24DT)

L31=INTGRL(0.0,L31DT)
L32=INTGRL(0.0,L32DT)
L33=INTGRL(0.0,L33DT)
L34=INTGRL(0.0,L34DT)

L41=INTGRL(0.0,L41DT)
L42=INTGRL(0.0,L42DT)
L43=INTGRL(0.0,L43DT)
L44=INTGRL(0.0,L44DT)

L51=INTGRL(0.0,L51DT)
```

```
L52=INTGRL(0.0,L52DT)
L53=INTGRL(0.0,L53DT)
L54=INTGRL(0.0,L54DT)

L61=INTGRL(0.0,L61DT)
L62=INTGRL(0.0,L62DT)
L63=INTGRL(0.0,L63DT)
L64=INTGRL(0.0,L64DT)

DX1=DEL1*L11+DEL2*L12+DEL3*L13+DEL4*L14
DX2=DEL1*L21+DEL2*L22+DEL3*L23+DEL4*L24
DX3=DEL1*L31+DEL2*L32+DEL3*L33+DEL4*L34
DX4=DEL1*L41+DEL2*L42+DEL3*L43+DEL4*L44
DX5=DEL1*L51+DEL2*L52+DEL3*L53+DEL4*L54
DX6=DEL1*L61+DEL2*L62+DEL3*L63+DEL4*L64

Y1S=X1+DX1
Y2S=X2+DX2
Y3S=X3+DX3
Y4S=X4+DX4
Y5S=X5+DX5
Y6S=X6+DX6

TIMER FINTIM=3.2,OUTDEL=.02

*PRINT X1,X2,X3,X4,X5,X6,X7,X8
*PRINT Y1,Y2,Y3,Y4,Y5,Y6,Y7,Y8
*PRINT Y1S,Y2S,Y3S,Y4S,Y5S,Y6S,Y7S,Y8S
*PRINT DX1,DX2,DX3,DX4,DX5,DX6,DX7,DX8
*PRINT L11,L21,L31,L41,L51,L61,L71,L81

*OUTPUT TIME,Y1,Y1S
*LABEL
*PAGE XYPLOT
OUTPUT TIME,Y2(-2.0,0.0),Y2S(-2.0,0.0)
LABEL
PAGE XYPLOT
*OUTPUT TIME,Y3,Y3S
*LABEL
*PAGE XYPLOT
*OUTPUT TIME,Y4,Y4S
*LABEL
*PAGE XYPLOT
*OUTPUT TIME,Y5,Y5S
*LABEL
*PAGE XYPLOT
*CUTPUT TIME,Y6,Y6S
*LABEL
```

```
*PAGE XYPLOT
*OUTPUT TIME,NZ
*LABEL RIBEIRO T.
*PAGE XYPLOT
END
STOP
/*
```

APPENDIX E

COUPLED ROLL_YAW AUTOPILOT - CSMP PROGRAM

```
//ZEBRA JOB (0314,1797),'THESIS',CLASS=G
// EXEC CSMPXV
//X.COMPRINT DD DUMMY
//X.SYSPRINT DD DUMMY
//X.PLOTPARM DD *
 &PLOT SCALE=.6 &END
//X.SYSIN DD *
*/*MAIN ORG=NPGVM1.1797P
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
*   NAME : TIAGO DA SILVA RIBEIRO                           WT 21      *
*                                                                      *
*        SENSITIVITY ANALYSIS FOR BANK TO TURN MISSILE                 *
*                                                                      *
*   TITLE: YAW & ROLL  AUTOPILOT                                       *
*          ELLIPTICAL CASE                                             *
*          ALPHA = 10                                                  *
*                                                                      *
*                       10  SEP  1983                                  *
*                                                                      *
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
*                                                                      *
*
PARAM QB=1650,K=0.48,D1=2,S=3.1416,ALPHA=10,CONV=57.3,D=0.05033,...
      KB=0.48,E=0.078,F=0.25

PARAM W=2475,IXX=110,IZZ=853,A1=0.015,A2=-0.054,A3=0.014,A4=0.024,...
      A5=-0.039,A6=-0.010,A7=-0.027,A8=0.023,T1=0.25,...
      H=1.0,KC=4.17,K1=.839,K2=6.08

DEL1=.4*A1
DEL2=.4*A2
DEL3=.4*A3
DEL4=.4*A4
DEL5=.4*A5
DEL6=.4*A6
DEL7=.4*A7
DEL8=.4*A8
```

```
A11=1.4*A1
A21=1.4*A2
A31=1.4*A3
A41=1.4*A4
A51=1.4*A5
A61=1.4*A6
A71=1.4*A7
A81=1.4*A8

A=QB*S/W
B=QB*S*D1/IZZ
C=QB*S*D1/IXX
ALPHAB=ALPHA/CONV

PHC=STEP(0.0)

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
* NOMINAL & SENSITIVITY EQUATIONS                                     *
* AERODYNAMIC EQUATIONS                                               *
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

NYX=A*(A2*X3+A1*X11)

*.....................................................................*

X1D=B*CONV*(A4*X3+A3*X8+A5*X11)
L11D=B*CONV*(A4*L31+A3*L81+A5*L111)
L12D=B*CONV*(A4*L32+A3*L82+A5*L112)
L13D=B*CONV*(A4*L33+A3*L83+A5*L113)+...
     B*CONV*X8
L14D=B*CONV*(A4*L34+A3*L84+A5*L114)+...
     B*CONV*X3
L15D=B*CONV*(A4*L35+A3*L85+A5*L115)+...
     B*CONV*X11
L16D=B*CONV*(A4*L36+A3*L86+A5*L116)
L17D=B*CONV*(A4*L37+A3*L87+A5*L117)
L18D=B*CONV*(A4*L38+A3*L88+A5*L118)

Y1D=B*CONV*(A41*Y3+A31*Y8+A51*Y11)
*......................................................................*

X2D=C*CONV*(A7*X3+A6*X11+A8*X8)
L21D=C*CONV*(A7*L31+A6*L111+A8*L81)
L22D=C*CONV*(A7*L32+A6*L112+A8*L82)
L23D=C*CONV*(A7*L33+A6*L113+A8*L83)
```

```
L24D=C*CONV*(A7*L34+A6*L114+A8*L84)
L25D=C*CONV*(A7*L35+A6*L115+A8*L85)
L26D=C*CONV*(A7*L36+A6*L116+A8*L86)+...
     C*CONV*X11
L27D=C*CONV*(A7*L37+A6*L117+A8*L87)+...
     C*CONV*X3
L28D=C*CONV*(A7*L38+A6*L118+A8*L88)+...
     C*CONV*X8

Y2D=C*CONV*(A71*Y3+A61*Y11+A81*Y8)
*...............................................................*

X3D=(ALPHA/CONV)*X2-X1+KB*A*A2*X3+KB*A*A1*X11
L31D=(ALPHA/CONV)*L21-L11+KB*A*A2*L31+KB*A*A1*L111+...
       KB*A*X11
L32D=(ALPHA/CONV)*L22-L12+KB*A*A2*L32+KB*A*A1*L112+...
       KB*A*X3
L33D=(ALPHA/CONV)*L23-L13+KB*A*A2*L33+KB*A*A1*L113
L34D=(ALPHA/CONV)*L24-L14+KB*A*A2*L34+KB*A*A1*L114
L35D=(ALPHA/CONV)*L25-L15+KB*A*A2*L35+KB*A*A1*L115
L36D=(ALPHA/CONV)*L26-L16+KB*A*A2*L36+KB*A*A1*L116
L37D=(ALPHA/CONV)*L27-L17+KB*A*A2*L37+KB*A*A1*L117
L38D=(ALPHA/CONV)*L28-L18+KB*A*A2*L38+KB*A*A1*L118

Y3D=(ALPHA/CONV)*Y2-Y1+KB*A*A21*Y3+KB*A*A11*Y11
*...............................................................*

X12D=(1./CONV)*X2
L121D=(1./CONV)*L21
L122D=(1./CONV)*L22
L123D=(1./CONV)*L23
L124D=(1./CONV)*L24
L125D=(1./CONV)*L25
L126D=(1./CONV)*L26
L127D=(1./CONV)*L27
L128D=(1./CONV)*L28

Y12D=(1./CONV)*Y2
*...............................................................*

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
* CONTROL LAW EQUATIONS                                             *
*    ROLL CHANNEL                                                   *
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

*...............................................................*
```

```
X4D=-8*X4-17.6*X12+17.6*PHC
L41D=-8*L41-17.6*L121
L42D=-8*L42-17.6*L122
L43D=-8*L43-17.6*L123
L44D=-8*L44-17.6*L124
L45D=-8*L45-17.6*L125
L46D=-8*L46-17.6*L126
L47D=-8*L47-17.6*L127
L48D=-8*L48-17.6*L128

Y4D=-8*Y4-17.6*Y12+17.6*PHC

*.......................................................................*

X5D=-5*X5+(0.755-8*D)*X4+17.6*D*PHC-17.6*D*X12-...
     D*C*(A7*X3+A6*X11+A8*X8)-(.755/CONV)*X2
L51D=-5*L51+(0.755-8*D)*L41-17.6*D*L121-...
     D*C*(A7*L31+A6*L111+A8*L81)-(.755/CONV)*L21
L52D=-5*L52+(0.755-8*D)*L42-17.6*D*L122-...
     D*C*(A7*L32+A6*L112+A8*L82)-(.755/CONV)*L22
L53D=-5*L53+(0.755-8*D)*L43-17.6*D*L123-...
     D*C*(A7*L33+A6*L113+A8*L83)-(.755/CONV)*L23
L54D=-5*L54+(0.755-8*D)*L44-17.6*D*L124-...
     D*C*(A7*L34+A6*L114+A8*L84)-(.755/CONV)*L24
L55D=-5*L55+(0.755-8*D)*L45-17.6*D*L125-...
     D*C*(A7*L35+A6*L115+A8*L85)-(.755/CONV)*L25
L56D=-5*L56+(0.755-8*D)*L46-17.6*D*L126-...
     D*C*(A7*L36+A6*L116+A8*L86)-(.755/CONV)*L26-...
     D*C*X11
L57D=-5*L57+(0.755-8*D)*L47-17.6*D*L127-...
     D*C*(A7*L37+A6*L117+A8*L87)-(.755/CONV)*L27-...
     D*C*X3
L58D=-5*L58+(0.755-8*D)*L48-17.6*D*L128-...
     D*C*(A7*L38+A6*L118+A8*L88)-(.755/CONV)*L28-...
     D*C*X8
Y5D=-5*Y5+(0.755-8*D)*Y4+17.6*D*PHC-17.6*D*Y12-...
     D*C*(A71*Y3+A61*Y11+A81*Y8)-(.755/CONV)*Y2
*.......................................................................*

X6D=-6*X6+E*C*(A7*X3+A6*X11+A8*X8)
L61D=-6*L61+E*C*(A7*L31+A6*L111+A8*L81)
L62D=-6*L62+E*C*(A7*L32+A6*L112+A8*L82)
L63D=-6*L63+E*C*(A7*L33+A6*L113+A8*L83)
L64D=-6*L64+E*C*(A7*L34+A6*L114+A8*L84)
L65D=-6*L65+E*C*(A7*L35+A6*L115+A8*L85)
L66D=-6*L66+E*C*(A7*L36+A6*L116+A8*L86)+...
       E*C*X11
L67D=-6*L67+E*C*(A7*L37+A6*L117+A8*L87)+...
```

```
      E*C*X3
L68D=-6*L68+E*C*(A7*L38+A6*L118+A8*L88)+...
      E*C*X8

Y6D=-6*Y6+E*C*(A71*Y3+A61*Y11+A81*Y8)
*.........................................................................*

X7D=-15*X7+KC*(15-5*F)*X5+F*KC*(.755-8*D)*X4+F*KC*D*17.6*PHC-...
      F*KC*D*17.6*X12-(D+E)*F*KC*C*(A7*X3+A6*X11+A8*X8)-...
      F*KC*(.755/CONV)*X2+KC*(6*F-15)*X6
L71D=-15*L71+KC*(15-5*F)*L51+F*KC*(.755-8*D)*L41-...
      F*KC*D*17.6*L121/CONV-(D+E)*F*KC*C*(A7*L31+A6*L111+A8*L81)-...
      F*KC*(.755/CONV)*L21+KC*(6*F-15)*L61
L72D=-15*L72+KC*(15-5*F)*L52+F*KC*(.755-8*D)*L42-...
      F*KC*D*17.6*L122/CONV-(D+E)*F*KC*C*(A7*L32+A6*L112+A8*L82)-...
      F*KC*(.755/CONV)*L22+KC*(6*F-15)*L62
L73D=-15*L73+KC*(15-5*F)*L53+F*KC*(.755-8*D)*L43-...
      F*KC*D*17.6*L123/CONV-(D+E)*F*KC*C*(A7*L33+A6*L113+A8*L83)-...
      F*KC*(.755/CONV)*L23+KC*(6*F-15)*L63
L74D=-15*L74+KC*(15-5*F)*L54+F*KC*(.755-8*D)*L44-...
      F*KC*D*17.6*L124/CONV-(D+E)*F*KC*C*(A7*L34+A6*L114+A8*L84)-...
      F*KC*(.755/CONV)*L24+KC*(6*F-15)*L64
L75D=-15*L75+KC*(15-5*F)*L55+F*KC*(.755-8*D)*L45-...
      F*KC*D*17.6*L125/CONV-(D+E)*F*KC*C*(A7*L35+A6*L115+A8*L85)-...
      F*KC*(.755/CONV)*L25+KC*(6*F-15)*L65
L76D=-15*L76+KC*(15-5*F)*L56+F*KC*(.755-8*D)*L46-...
      F*KC*D*17.6*L126/CONV-(D+E)*F*KC*C*(A7*L36+A6*L116+A8*L86)-...
      F*KC*(.755/CONV)*L26+KC*(6*F-15)*L66-...
      (D+E)*F*KC*C*X11
L77D=-15*L77+KC*(15-5*F)*L57+F*KC*(.755-8*D)*L47-...
      F*KC*D*17.6*L127/CONV-(D+E)*F*KC*C*(A7*L37+A6*L117+A8*L87)-...
      F*KC*(.755/CONV)*L27+KC*(6*F-15)*L67-...
      (D+E)*F*KC*C*X3
L78D=-15*L78+KC*(15-5*F)*L58+F*KC*(.755-8*D)*L48-...
      F*KC*D*17.6*L128/CONV-(D+E)*F*KC*C*(A7*L38+A6*L118+A8*L88)-...
      F*KC*(.755/CONV)*L28+KC*(6*F-15)*L68-...
      (D+E)*F*KC*C*X8

Y7D=-15*Y7+KC*(15-5*F)*Y5+F*KC*(.755-8*D)*Y4+F*KC*D*17.6*PHC-...
      F*KC*D*17.6*Y12-(D+E)*F*KC*C*(A71*Y3+A61*Y11+A81*Y8)-...
      F*KC*(.755/CONV)*Y2+KC*(6*F-15)*Y6
*.........................................................................*

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
*    YAW CHANNEL                                                      *
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

*.........................................................................*
```

```
X9D=-(1./T1)*X9+K1*A/T1*(A2*X3+A1*X11)
L91D=-(1./T1)*L91+K1*A/T1*(A2*L31+A1*L111)+...
       K1*A/T1*X11
L92D=-(1./T1)*L92+K1*A/T1*(A2*L32+A1*L112)+...
       K1*A/T1*X3
L93D=-(1./T1)*L93+K1*A/T1*(A2*L33+A1*L113)
L94D=-(1./T1)*L94+K1*A/T1*(A2*L34+A1*L114)
L95D=-(1./T1)*L95+K1*A/T1*(A2*L35+A1*L115)
L96D=-(1./T1)*L96+K1*A/T1*(A2*L36+A1*L116)
L97D=-(1./T1)*L97+K1*A/T1*(A2*L37+A1*L117)
L98D=-(1./T1)*L98+K1*A/T1*(A2*L38+A1*L118)

Y9D=-(1./T1)*Y9+K1*A/T1*(A21*Y3+A11*Y11)
*...............................................................*

X10D=K2/10*((K1*A/T1*A2+B*A4-H*ALPHAB*C*A7)*X3+...
     ((K1*A/T1)*A1+B*A5-H*ALPHAB*C*A6)*X11+(B*A3-H*ALPHAB*C*A8)*X8)+...
      K2/CONV*X1-K2*H*ALPHAB/CONV*X2+K2*(1.-1./(10*T1))*X9

L101D=K2/10*((K1*A/T1*A2+B*A4-H*ALPHAB*C*A7)*L31+...
 ((K1*A/T1)*A1+B*A5-H*ALPHAB*C*A6)*L111+(B*A3-H*ALPHAB*C*A8)*L81)+...
 K2/CONV*L11-K2*H*ALPHAB/CONV*L21+K2*(1.-1./(10*T1))*L91+...
 K2*K1*A/(10*T1)*X11
L102D=K2/10*((K1*A/T1*A2+B*A4-H*ALPHAB*C*A7)*L32+...
 ((K1*A/T1)*A1+B*A5-H*ALPHAB*C*A6)*L112+(B*A3-H*ALPHAB*C*A8)*L82)+...
 K2/CONV*L12-K2*H*ALPHAB/CONV*L22+K2*(1.-1./(10*T1))*L92+...
 K2*K1*A/(10*T1)*X3
L103D=K2/10*((K1*A/T1*A2+B*A4-H*ALPHAB*C*A7)*L33+...
 ((K1*A/T1)*A1+B*A5-H*ALPHAB*C*A6)*L113+(B*A3-H*ALPHAB*C*A8)*L83)+...
 K2/CONV*L13-K2*H*ALPHAB/CONV*L23+K2*(1.-1./(10*T1))*L93+...
 K2/10*B*X8
L104D=K2/10*((K1*A/T1*A2+B*A4-H*ALPHAB*C*A7)*L34+...
 ((K1*A/T1)*A1+B*A5-H*ALPHAB*C*A6)*L114+(B*A3-H*ALPHAB*C*A8)*L84)+...
 K2/CONV*L14-K2*H*ALPHAB/CONV*L24+K2*(1.-1./(10*T1))*L94+...
 K2/10*B*X3
L105D=K2/10*((K1*A/T1*A2+B*A4-H*ALPHAB*C*A7)*L35+...
 ((K1*A/T1)*A1+B*A5-H*ALPHAB*C*A6)*L115+(B*A3-H*ALPHAB*C*A8)*L85)+...
 K2/CONV*L15-K2*H*ALPHAB/CONV*L25+K2*(1.-1./(10*T1))*L95+...
 K2/10*B*X11

L106D=K2/10*((K1*A/T1*A2+B*A4-H*ALPHAB*C*A7)*L36+...
 ((K1*A/T1)*A1+B*A5-H*ALPHAB*C*A6)*L116+(B*A3-H*ALPHAB*C*A8)*L86)+...
 K2/CONV*L16-K2*H*ALPHAB/CONV*L26+K2*(1.-1./(10*T1))*L96-...
 K2/10*H*ALPHAB*C*X11
L107D=K2/10*((K1*A/T1*A2+B*A4-H*ALPHAB*C*A7)*L37+...
 ((K1*A/T1)*A1+B*A5-H*ALPHAB*C*A6)*L117+(B*A3-H*ALPHAB*C*A8)*L87)+...
 K2/CONV*L17-K2*H*ALPHAB/CONV*L27+K2*(1.-1./(10*T1))*L97-...
 K2/10*H*ALPHAB*C*X3
```

```
L108D=K2/10*((K1*A/T1*A2+B*A4-H*ALPHAB*C*A7)*L38+...
  ((K1*A/T1)*A1+B*A5-H*ALPHAB*C*A6)*L118+(B*A3-H*ALPHAB*C*A8)*L88)+...
  K2/CCNV*L18-K2*H*ALPHAB/CONV*L28+K2*(1.-1./(10*T1))*L98-...
  K2/10*H*ALPHAB*C*X8

Y10D=K2/10*((K1*A/T1*A21+B*A41-H*ALPHAB*C*A71)*Y3+...
((K1*A/T1)*A11+B*A51-H*ALPHAB*C*A61)*Y11+(B*A31-H*ALPHAB*C*A81)*Y8)+...
      K2/CONV*Y1-K2*H*ALPHAB/CONV*Y2+K2*(1.-1./(10*T1))*Y9

*.........................................................................*

X11D=-188.4*X11+188.4*CONV*X10
L111D=-188.4*L111+188.4*CONV*L101
L112D=-188.4*L112+188.4*CONV*L102
L113D=-188.4*L113+188.4*CONV*L103
L114D=-188.4*L114+188.4*CONV*L104
L115D=-188.4*L115+188.4*CONV*L105
L116D=-188.4*L116+188.4*CONV*L106
L117D=-188.4*L117+188.4*CONV*L107
L118D=-188.4*L118+188.4*CONV*L108

Y11D=-188.4*Y11+188.4*CONV*Y10
*.........................................................................*

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
* ACTUATOR EQUATION                                                   *
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

*.........................................................................*
X8D=-188.4*X8+188.4*CONV*X7
L81D=-188.4*L81+188.4*CONV*L71
L82D=-188.4*L82+188.4*CONV*L72
L83D=-188.4*L83+188.4*CONV*L73
L84D=-188.4*L84+188.4*CONV*L74
L85D=-188.4*L85+188.4*CONV*L75
L86D=-188.4*L86+188.4*CONV*L76
L87D=-188.4*L87+188.4*CONV*L77
L88D=-188.4*L88+188.4*CONV*L78

Y8D=-188.4*Y8+188.4*CONV*Y7
*
X1=INTGRL(0.0,X1D)
X2=INTGRL(0.0,X2D)
X3=INTGRL(0.0,X3D)
X4=INTGRL(0.0,X4D)
X5=INTGRL(0.0,X5D)
X6=INTGRL(0.0,X6D)
X7=INTGRL(0.0,X7D)
```

```
X8=INTGRL(0.0,X8D)
X9=INTGRL(0.0,X9D)
X10=INTGRL(0.0,X10D)
X11=INTGRL(0.0,X11D)
X12=INTGRL(0.0,X12D)

L11=INTGRL(0.0,L11D)
L21=INTGRL(0.0,L21D)
L31=INTGRL(0.0,L31D)
L41=INTGRL(0.0,L41D)
L51=INTGRL(0.0,L51D)
L61=INTGRL(0.0,L61D)
L71=INTGRL(0.0,L71D)
L81=INTGRL(0.0,L81D)
L91=INTGRL(0.0,L91D)
L101=INTGRL(0.0,L101D)
L111=INTGRL(0.0,L111D)
L121=INTGRL(0.0,L121D)

L12=INTGRL(0.0,L12D)
L22=INTGRL(0.0,L22D)
L32=INTGRL(0.0,L32D)
L42=INTGRL(0.0,L42D)
L52=INTGRL(0.0,L52D)
L62=INTGRL(0.0,L62D)
L72=INTGRL(0.0,L72D)
L82=INTGRL(0.0,L82D)
L92=INTGRL(0.0,L92D)
L102=INTGRL(0.0,L102D)
L112=INTGRL(0.0,L112D)
L122=INTGRL(0.0,L122D)

L13=INTGRL(0.0,L13D)
L23=INTGRL(0.0,L23D)
L33=INTGRL(0.0,L33D)
L43=INTGRL(0.0,L43D)
L53=INTGRL(0.0,L53D)
L63=INTGRL(0.0,L63D)
L73=INTGRL(0.0,L73D)
L83=INTGRL(0.0,L83D)
L93=INTGRL(0.0,L93D)
L103=INTGRL(0.0,L103D)
L113=INTGRL(0.0,L113D)
L123=INTGRL(0.0,L123D)

L14=INTGRL(0.0,L14D)
L24=INTGRL(0.0,L24D)
L34=INTGRL(0.0,L34D)
```

```
L44=INTGRL(0.0,L44D)
L54=INTGRL(0.0,L54D)
L64=INTGRL(0.0,L64D)
L74=INTGRL(0.0,L74D)
L84=INTGRL(0.0,L84D)
L94=INTGRL(0.0,L94D)
L104=INTGRL(0.0,L104D)
L114=INTGRL(0.0,L114D)
L124=INTGRL(0.0,L124D)

L15=INTGRL(0.0,L15D)
L25=INTGRL(0.0,L25D)
L35=INTGRL(0.0,L35D)
L45=INTGRL(0.0,L45D)
L55=INTGRL(0.0,L55D)
L65=INTGRL(0.0,L65D)
L75=INTGRL(0.0,L75D)
L85=INTGRL(0.0,L85D)
L95=INTGRL(0.0,L95D)
L105=INTGRL(0.0,L105D)
L115=INTGRL(0.0,L115D)
L125=INTGRL(0.0,L125D)

L16=INTGRL(0.0,L16D)
L26=INTGRL(0.0,L26D)
L36=INTGRL(0.0,L36D)
L46=INTGRL(0.0,L46D)
L56=INTGRL(0.0,L56D)
L66=INTGRL(0.0,L66D)
L76=INTGRL(0.0,L76D)
L86=INTGRL(0.0,L86D)
L96=INTGRL(0.0,L96D)
L106=INTGRL(0.0,L106D)
L116=INTGRL(0.0,L116D)
L126=INTGRL(0.0,L126D)

L17=INTGRL(0.0,L17D)
L27=INTGRL(0.0,L27D)
L37=INTGRL(0.0,L37D)
L47=INTGRL(0.0,L47D)
L57=INTGRL(0.0,L57D)
L67=INTGRL(0.0,L67D)
L77=INTGRL(0.0,L77D)
L87=INTGRL(0.0,L87D)
L97=INTGRL(0.0,L97D)
L107=INTGRL(0.0,L107D)
L117=INTGRL(0.0,L117D)
L127=INTGRL(0.0,L127D)
```

```
L18=INTGRL(0.0,L18D)
L28=INTGRL(0.0,L28D)
L38=INTGRL(0.0,L38D)
L48=INTGRL(0.0,L48D)
L58=INTGRL(0.0,L58D)
L68=INTGRL(0.0,L68D)
L78=INTGRL(0.0,L78D)
L88=INTGRL(0.0,L88D)
L98=INTGRL(0.0,L98D)
L108=INTGRL(0.0,L108D)
L118=INTGRL(0.0,L118D)
L128=INTGRL(0.0,L128D)

Y1=INTGRL(0.0,Y1D)
Y2=INTGRL(0.0,Y2D)
Y3=INTGRL(0.0,Y3D)
Y4=INTGRL(0.0,Y4D)
Y5=INTGRL(0.0,Y5D)
Y6=INTGRL(0.0,Y6D)
Y7=INTGRL(0.0,Y7D)
Y8=INTGRL(0.0,Y8D)
Y9=INTGRL(0.0,Y9D)
Y10=INTGRL(0.0,Y10D)
Y11=INTGRL(0.0,Y11D)
Y12=INTGRL(0.0,Y12D)

DX1=DEL1*L11+DEL2*L12+DEL3*L13+DEL4*L14+DEL5*L15+...
    DEL6*L16+DEL7*L17+DEL8*L18
DX2=DEL1*L21+DEL2*L22+DEL3*L23+DEL4*L24+DEL5*L25+...
    DEL6*L26+DEL7*L27+DEL8*L28
DX3=DEL1*L31+DEL2*L32+DEL3*L33+DEL4*L34+DEL5*L35+...
    DEL6*L36+DEL7*L37+DEL8*L38
DX4=DEL1*L41+DEL2*L42+DEL3*L43+DEL4*L44+DEL5*L45+...
    CEL6*L46+DEL7*L47+DEL8*L48
DX5=DEL1*L51+DEL2*L52+DEL3*L53+DEL4*L54+DEL5*L55+...
    DEL6*L56+DEL7*L57+DEL8*L58
DX6=DEL1*L61+DEL2*L62+DEL3*L63+DEL4*L64+DEL5*L65+...
    DEL6*L66+DEL7*L67+DEL8*L68
DX7=DEL1*L71+DEL2*L72+DEL3*L73+DEL4*L74+DEL5*L75+...
    DEL6*L76+DEL7*L77+DEL8*L78
DX8=DEL1*L81+DEL2*L82+DEL3*L83+DEL4*L84+DEL5*L85+...
    DEL6*L86+DEL7*L87+DEL8*L88
DX9=DEL1*L91+DEL2*L92+DEL3*L93+DEL4*L94+DEL5*L95+...
    DEL6*L96+DEL7*L97+DEL8*L98
DX10=DEL1*L101+DEL2*L102+DEL3*L103+DEL4*L104+DEL5*L105+...
    DEL6*L106+DEL7*L107+DEL8*L108
```

```
DX11=DEL1*L111+DEL2*L112+DEL3*L113+DEL4*L114+DEL5*L115+...
     DEL6*L116+DEL7*L117+DEL8*L118
DX12=DEL1*L121+DEL2*L122+DEL3*L123+DEL4*L124+DEL5*L125+...
     DEL6*L126+DEL7*L127+DEL8*L128

Y1S=X1+DX1
Y2S=X2+DX2
Y3S=X3+DX3
Y4S=X4+DX4
Y5S=X5+DX5
Y6S=X6+DX6
Y7S=X7+CX7
Y8S=)8+DX8
Y9S=X9+DX9
Y10S=X10+DX10
Y11S=X11+DX11
Y12S=X12+DX12

TIMER FINTIM=3.2,OUTDEL=.02
*PRINT  DEL1,DEL2,DEL3,DEL4,DEL5,DEL6,DEL7,DEL8
*PRINT  A1,A2,A3,A4,A5,A6,A7,A8
*PRINT  A11,A21,A31,A41,A51,A61,A71,A81
*PRINT  X1,X2,X3,X4,X5,X6,X7,X8,X9,X10,X11,X12
*PRINT  DX1,DX2,DX3,DX4,DX5,DX6,DX7,DX8,DX9,DX10,CX11,DX12
*PRINT  Y1,Y2,Y3,Y4,Y5,Y6,Y7,Y8,Y9,Y10,Y11,Y12
*PRINT  Y1S,Y2S,Y3S,Y4S,Y5S,Y6S,Y7S,Y8S,Y9S,Y10S,Y11S,Y12S
*OUTPUT TIME,Y1,Y1S
*LABEL
*PAGE XYPLOT
*OUTPUT TIME,Y2,Y2S
*LABEL
*PAGE XYPLOT
OUTPUT TIME,Y3(-0.60,0.60),Y3S(-0.60,0.60)
LABEL
PAGE XYPLOT
*OUTPUT TIME,Y4,Y4S
*LABEL
*PAGE XYPLOT
*OUTPUT TIME,Y5,Y5S
*LABEL
*PAGE XYPLOT
*OUTPUT TIME,Y6,Y6S
*LABEL
*PAGE XYPLOT
*OUTPUT TIME,Y7,Y7S
*LABEL
*PAGE XYPLOT
*OUTPUT TIME,Y8,Y8S
```

```
*LABEL
*PAGE XYPLOT
*OUTPUT TIME,Y9,Y9S
*LABEL
*PAGE XYPLOT
*OUTPUT TIME,Y10,Y10S
*LABEL
*PAGE XYPLOT
OUTPUT TIME,Y11(-2.5,1.5),Y11S(-2.5,1.5)
LABEL
PAGE XYPLOT
*OUTPUT TIME,Y12,Y12S
*LABEL
*PAGE XYPLOT
*OUTPUT TIME,X11,L115,L116,L117,L118
*LABEL
*PAGE XYPLOT
*OUTPUT TIME,X12,L121,L122,L123,L124
*LABEL
*PAGE XYPLOT
*OUTPUT TIME,X12,L125,L126,L127,L128
*LABEL
*PAGE XYPLOT
END
STOP
/*
```

APPENDIX F

AERODYNAMIC DERIVATIVES - FORTRAN PROGRAMS

```
$JOB
      REAL*8 X(8),Y(4),C(2,8,8),WK(80),F(8,4),DX(8,4),
     1 PDS1(8,4),PDS2(8,4),PDS3(8,4),PDS(6),R(8,4),DY(8,4),
     2 R1(1,8,4),R2(1,8,4),R3(1,8,4),X1(8),Y1(4),F1(8,4)
      REAL*8 XL1,XL2,XL3,XL4,YL1,YL2,YL3,YL4,YL5,XL,YL,C1
      INTEGER I,J,IER,IC,NX,NY,N,NX1,NY1,K,L
      IC=8
      NX=8
      NY=4
      WRITE(6,100)
      WRITE(6,200)
      DO 1 I=1,NX
      DO 2 J=1,NY
      X(1)=-4.
      X(2)=0.
      X(3)=4.
      X(4)=8.
      X(5)=12.
      X(6)=16.
      X(7)=20.
      X(8)=24.
      Y(1)=-10.
      Y(2)=-5.
      Y(3)=0.
      Y(4)=10.
      F(1,1)=.485
      F(2,1)=.55
      F(3,1)=.625
      F(4,1)=.71
      F(5,1)=.85
      F(6,1)=.95
      F(7,1)=1.
      F(8,1)=1.
      F(1,2)=.28
      F(2,2)=.275
      F(3,2)=.35
      F(4,2)=.43
      F(5,2)=.525
      F(6,2)=.575
      F(7,2)=.622
```

```
      F(8,2)=.587                                                       CMA00490
      F(1,3)=-.075                                                      CMA00500
      F(2,3)=0.0                                                        CMA00510
      F(3,3)=.075                                                       CMA00520
      F(4,3)=0.15                                                       CMA00530
      F(5,3)=0.2                                                        CMA00540
      F(6,3)=0.2                                                        CMA00550
      F(7,3)=0.245                                                      CMA00560
      F(8,3)=0.175                                                      CMA00570
      F(1,4)=-.63                                                       CMA00580
      F(2,4)=-.55                                                       CMA00590
      F(3,4)=-.48                                                       CMA00600
      F(4,4)=-.43                                                       CMA00610
      F(5,4)=-.39                                                       CMA00620
      F(6,4)=-.40                                                       CMA00630
      F(7,4)=-.50                                                       CMA00640
      F(8,4)=-.73                                                       CMA00650
      XL=X(I)                                                           CMA00660
      YL=Y(J)                                                           CMA00670
      CALL IBCCCU(F,X,NX,Y,NY,C,IC,WK,IER)                              CMA00680
      CALL DBCEVL(X,NX,Y,NY,C,IC,XL,YL,PDS,IER)                         CMA00690
C         PDS1(I,J)=PDS(1)                                              CMA00700
C         R1(1,I,J)=PDS1(I,J)                                           CMA00710
C         PDS2(I,J)=PDS(2)                                              CMA00720
C         R2(1,I,J)=PDS2(I,J)                                           CMA00730
C         PDS3(I,J)=PDS(3)                                              CMA00740
C         R3(1,I,J)=PDS3(I,J)                                           CMA00750
      WRITE(6,25)X(I),Y(J),PDS(1),PDS(2),PDS(3)                         CMA00760
    2 CONTINUE                                                          CMA00770
    1 CONTINUE                                                          CMA00780
C     DO 290 I=1,NX                                                     CMA00790
C         X1(I)=X(I)                                                    CMA00800
C     WRITE(6,25)X(I)                                                   CMA00810
C90       CONTINUE                                                      CMA00820
C         DO 295 J=1,NY                                                 CMA00830
C         Y1(J)=Y(J)                                                    CMA00840
C     WRITE(6,25)Y(J)                                                   CMA00850
C95       CONTINUE                                                      CMA00860
C     DO 333 I=1,NX                                                     CMA00870
C         DO 334 J=1,NY                                                 CMA00880
C         Y1(I)=Y(I)                                                    CMA00890
C         R(I,J)=R1(1,I,J)                                              CMA00900
C         DX(I,J)=R2(1,I,J)                                             CMA00910
C         DY(I,J)=R3(1,I,J)                                             CMA00920
C     WRITE(6,25)R(I,J),DX(I,J),DY(I,J)                                 CMA00930
C334  CONTINUE                                                          CMA00940
C333  CONTINUE                                                          CMA00950
      STOP                                                              CMA00960
```

```
25     FORMAT(5(F12.4))
100    FORMAT('1','3-D  PLOT AND DERIVATIVES'//)
200    FORMAT(15X,'      F           DF/DX          DF/DY   '//)
C300   FORMAT(15X,3(F12.5,3X)//)
C310   FORMAT(1X,F12.5//)
C320   FORMAT(1X,F12.5//)
C330   FORMAT(1X,F12.5//)
       END
C   IMSL ROUTINE NAME   - IBCCCU
C
C-----------------------------------------------------------------------
C
C   COMPUTER            - IBM/DOUBLE
C
C   LATEST REVISION     - JUNE 1, 1982
C
C   PURPOSE             - BICUBIC SPLINE TWO-DIMENSIONAL COEFFICIENT
C                           CALCULATOR
C
C   USAGE               - CALL IBCCCU (F,X,NX,Y,NY,C,IC,WK,IER)
C
C   ARGUMENTS    F      - NX BY NY MATRIX CONTAINING THE FUNCTION
C                           VALUES. (INPUT) F(I,J) IS THE FUNCTION VALUE
C                           AT THE POINT (X(I),Y(J)) FOR I=1,...,NX AND
C                           J=1,...,NY.
C                X      - VECTOR OF LENGTH NX. (INPUT) X MUST BE
C                           ORDERED SO THAT X(I) .LT. X(I+1) FOR
C                           I=1,...,NX-1.
C                NX     - NUMBER OF ELEMENTS IN X. (INPUT) NX MUST BE
C                           .GE. 4.
C                Y      - VECTOR OF LENGTH NY. (INPUT) Y MUST BE
C                           ORDERED SO THAT Y(J) .LT. Y(J+1) FOR
C                           J=1,...,NY-1.
C                NY     - NUMBER OF ELEMENTS IN Y. (INPUT) NY MUST BE
C                           .GE. 4.
C                         NOTE - THE COORDINATE PAIRS (X(I),Y(J)), FOR
C                           I=1,...,NX AND J=1,...,NY, GIVE THE POINTS
C                           WHERE THE FUNCTION VALUES F(I,J) ARE
C                           DEFINED.
C                C      - ARRAY OF SPLINE COEFFICIENTS. (OUTPUT)
C                           C IS OF DIMENSION 2 BY NX BY 2 BY NY.
C                           AT THE POINT (X(I),Y(J))
C                             C(1,I,1,J) = S
C                             C(2,I,1,J) = DS/DX
C                             C(1,I,2,J) = DS/DY
C                             C(2,I,2,J) = D(DS/DX)/DY
C                           WHERE S(X,Y) IS THE SPLINE APPROXIMATION.
C                           (NOTE - C IS TREATED INTERNALLY AS A
```

```
C                                  2 BY NX BY 2*NY ARRAY BECAUSE CERTAIN      CMA01450
C                                  ENVIRONMENTS DC NOT PERMIT QUADRUPLY-      CMA01460
C                                  DIMENSIONED ARRAYS.  IN THESE              CMA01470
C                                  ENVIRONMENTS THE CALLING PROGRAM MAY       CMA01480
C                                  DIMENSICN C IN THE SAME MANNER.)           CMA01490
C                   IC     - ROW DIMENSION OF MATRIX F AND SECOND             CMA01500
C                                DIMENSION OF ARRAY C EXACTLY AS              CMA01510
C                                SPECIFIED IN THE DIMENSION STATEMENT.        CMA01520
C                                (INPUT). IC MUST BE .GE. NX.                 CMA01530
C                   WK     - WORK VECTOR OF LENGTH                            CMA01540
C                                2*NX*NY+2*MAX(NX,NY)                         CMA01550
C                   IER    - ERROR PARAMETER. (OUTPUT)                        CMA01560
C                              TERMINAL ERROR                                 CMA01570
C                                IER = 129, IC IS LESS THAN NX                CMA01580
C                                IER = 130, NX IS LESS THAN 4                 CMA01590
C                                IER = 131, NY IS LESS THAN 4                 CMA01600
C                                IER = 132, X OR Y ARE NOT ORDERED SO THAT    CMA01610
C                                  X(I) .LT. X(I+1) ANC                       CMA01620
C                                  Y(I) .LT. Y(I+1)                           CMA01630
C                                                                             CMA01640
C   PRECISION/HARDWARE   - SINGLE AND DOUBLE/H32                              CMA01650
C                        - SINGLE/H36,H48,H60                                 CMA01660
C                                                                             CMA01670
C   REQD. IMSL ROUTINES  - IBCDCU,UERTST,UGETIC                               CMA01680
C                                                                             CMA01690
C   NOTATION             - INFORMATION CN SPECIAL NOTATION AND                CMA01700
C                            CONVENTIONS IS AVAILABLE IN THE MANUAL           CMA01710
C                            INTRODUCTION OR THROUGH IMSL ROUTINE UHELP       CMA01720
C                                                                             CMA01730
C   COPYRIGHT            - 1982 BY IMSL, INC. ALL RIGHTS RESERVED.            CMA01740
C                                                                             CMA01750
C   WARRANTY             - IMSL WARRANTS CNLY THAT IMSL TESTING HAS BEEN      CMA01760
C                            APPLIED TO THIS CODE. NO OTHER WARRANTY,         CMA01770
C                            EXPRESSED OR IMPLIED, IS APPLICABLE.             CMA01780
C                                                                             CMA01790
C-----------------------------------------------------------------------------CMA01800
C                                                                             CMA01810
      SUBROUTINE IBCCCU (F,X,NX,Y,NY,C,IC,WK,IER)                             CMA01820
C                                  SPECIFICATIONS FOR ARGUMENTS               CMA01830
      INTEGER            NX,NY,IC,IER                                         CMA01840
      DOUBLE PRECISION   F(IC,1),X(1),Y(1),C(2,IC,1),WK(1)                    CMA01850
C                                  SPECIFICATIONS FOR LOCAL VARIABLES         CMA01860
      INTEGER            IWK                                                  CMA01870
C                                  FIRST EXECUTABLE STATEMENT                 CMA01880
      IER = 129                                                               CMA01890
      IF (IC .LT. NX) GO TO 9000                                              CMA01900
      IER = 130                                                               CMA01910
      IF (NX .LT. 4) GO TO 9000                                               CMA01920
```

```
      IER = 131
      IF (NY .LT. 4) GO TO 9000
      IWK = 2*NY*NX
      CALL IBCDCU(X,F,NX,NY,WK(IWK+1),WK,IC,NY,IER)
      IF (IER .GT. 0) GO TO 9000
      CALL IBCDCU(Y,WK,NY,2*NX,WK(IWK+1),C,NY,2*IC,IER)
      IF (IER .EQ. 0) GO TO 9005
 9000 CONTINUE
      CALL UERTST(IER,6HIBCCCU)
 9005 RETURN
      END
C   IMSL ROUTINE NAME   - IBCDCU
C
C-----------------------------------------------------------------------
C
C   COMPUTER            - IBM/DOUBLE
C
C   LATEST REVISION     - JUNE 1, 1982
C
C   PURPOSE             - NUCLEUS CALLED ONLY BY IMSL SUBROUTINE IBCCCU
C
C   PRECISION/HARDWARE  - SINGLE AND DOUBLE/H32
C                       - SINGLE/H36,H48,H60
C
C   REQD. IMSL ROUTINES - NONE REQUIRED
C
C   NOTATION            - INFORMATION ON SPECIAL NOTATION AND
C                           CONVENTIONS IS AVAILABLE IN THE MANUAL
C                           INTRODUCTION OR THROUGH IMSL ROUTINE UHELP
C
C   COPYRIGHT           - 1982 BY IMSL, INC. ALL RIGHTS RESERVED.
C
C   WARRANTY            - IMSL WARRANTS ONLY THAT IMSL TESTING HAS BEEN
C                           APPLIED TO THIS CODE. NO OTHER WARRANTY,
C                           EXPRESSED OR IMPLIED, IS APPLICABLE.
C
C-----------------------------------------------------------------------
C
      SUBROUTINE IBCDCU (TAU,GTAU,N,M,W,VS,IC1,IC2,IER)
C                                  SPECIFICATIONS FOR ARGUMENTS
      INTEGER            N,M,IC1,IC2,IER
      DOUBLE PRECISION   TAU(N),GTAU(IC1,1),W(N,2),VS(IC2,2,1)
C                                  SPECIFICATIONS FOR LOCAL VARIABLES
      INTEGER            I,JJ,JM1,JP1,J,K,LIM,LL,LP1,NM1
      DOUBLE PRECISION   AA,BB,C1,C2,CC,CD,DTAU,G,H,RATIO,U,XILIM
C                                  FIRST EXECUTABLE STATEMENT
      LIM = N-3
      NM1 = N-1
```

```
      LP1 = LIM+1                                                       CMA02410
      IER = 1:2                                                         CMA02420
      W(2,1) = TAU(3)-TAU(1)                                            CMA02430
      IF (W(2,1).LE.0.0D0) RETURN                                       CMA02440
      DO 5 K=1,M                                                        CMA02450
         VS(K,1,1) = GTAU(1,K)                                          CMA02460
    5 CONTINUE                                                          CMA02470
      XILIM = TAU(1)                                                    CMA02480
      IF (LIM.LT.2) GO TO 20                                            CMA02490
      XILIM = TAU(N-2)                                                  CMA02500
      DO 15 I=2,LIM                                                     CMA02510
         J = I+1                                                        CMA02520
         W(J,1) = TAU(I+2)-TAU(J)                                       CMA02530
         IF (W(J,1).LE.0.0D0) RETURN                                    CMA02540
         DO 10 K=1,M                                                    CMA02550
   10    VS(K,1,I) = GTAU(J,K)                                          CMA02560
   15 CONTINUE                                                          CMA02570
   20 W(LP1,1) = TAU(N)-XILIM                                           CMA02580
      IF (W(LP1,1).LE.0.0D0) RETURN                                     CMA02590
      DO 25 K=1,M                                                       CMA02600
   25 VS(K,1,LP1) = GTAU(N,K)                                           CMA02610
      DO 35 I=2,LP1                                                     CMA02620
         DO 30 K=1,M                                                    CMA02630
   30    VS(K,2,I) = (VS(K,1,I)-VS(K,1,I-1))/W(I,1)                     CMA02640
   35 CONTINUE                                                          CMA02650
      DTAU = TAU(2)-TAU(1)                                              CMA02660
      RATIO = DTAU/W(2,1)                                               CMA02670
      W(1,2) = (RATIO-1.D0)**2                                          CMA02680
      W(1,1) = RATIO*(RATIO-1.D0)                                       CMA02690
      C1 = RATIO*(2.D0*RATIO-3.D0)                                      CMA02700
      DO 40 K=1,M                                                       CMA02710
   40 VS(K,2,1) = (GTAU(2,K)-GTAU(1,K))/DTAU+VS(K,2,2)*C1               CMA02720
      IF (LIM.LT.2) GO TO 55                                            CMA02730
      DO 50 I=2,LIM                                                     CMA02740
         J = I+1                                                        CMA02750
         JJ = I-1                                                       CMA02760
         G = -W(J,1)/W(JJ,2)                                            CMA02770
         C1 = 3.D0*W(I,1)                                               CMA02780
         C2 = 3.D0*W(J,1)                                               CMA02790
         DO 45 K=1,M                                                    CMA02800
   45    VS(K,2,I) = G*VS(K,2,JJ)+C1*VS(K,2,J)+C2*VS(K,2,I)             CMA02810
         W(I,2) = G*W(JJ,1)+2.D0*(W(I,1)+W(J,1))                        CMA02820
   50 CONTINUE                                                          CMA02830
   55 DTAU = TAU(N-1)-XILIM                                             CMA02840
      RATIO = DTAU/W(LP1,1)                                             CMA02850
      G = -(RATIO-1.D0)**2/W(LIM,2)                                     CMA02860
      W(LP1,2) = RATIO*(RATIO-1.D0)                                     CMA02870
      C1 = RATIO*(2.D0*RATIO-3.D0)                                      CMA02880
```

```
      DO 60 K=1,M                                                       CMA02890
   60 VS(K,2,LP1) = (GTAU(N-1,K)-VS(K,1,LIM))/DTAL+VS(K,2,LP1)*C1       CMA02900
      W(LP1,2) = G*W(LIM,1)+W(LP1,2)                                    CMA02910
      DO 65 K=1,M                                                       CMA02920
   65 VS(K,2,LP1) = (G*VS(K,2,LIM)+VS(K,2,LP1))/W(LP1,2)                CMA02930
      J = LIM                                                           CMA02940
   70 DO 75 K=1,M                                                       CMA02950
   75 VS(K,2,J) = (VS(K,2,J)-W(J,1)*VS(K,2,J+1))/W(J,2)                 CMA02960
      J = J-1                                                           CMA02970
      IF (J.GT.0) GO TO 70                                              CMA02980
      DO 95 K=1,M                                                       CMA02990
         DO 85 JJ=1,N                                                   CMA03000
            J = N+1-JJ                                                  CMA03010
            JM1 = J-1                                                   CMA03020
            IF (J.EQ.N) JM1 = J-2                                       CMA03030
            IF (J.EQ.1) JM1 = J                                         CMA03040
            DO 80 LL=1,2                                                CMA03050
               VS(K,LL,J) = VS(K,LL,JM1)                                CMA03060
   80       CONTINUE                                                    CMA03070
   85    CONTINUE                                                       CMA03080
         DO 90 J=2,NM1,LIM                                              CMA03090
            JM1 = J-1                                                   CMA03100
            JP1 = J+1                                                   CMA03110
            IF (JM1.EQ.2) JM1 = 1                                       CMA03120
            IF (JP1.EQ.NM1) JP1 = N                                     CMA03130
            H = TAU(JP1)-TAU(JM1)                                       CMA03140
            U = TAU(J)-TAU(JM1)                                         CMA03150
            AA = VS(K,1,JM1)                                            CMA03160
            BB = VS(K,2,JM1)                                            CMA03170
            CC = (3.D0*(VS(K,1,JP1)-VS(K,1,JM1))/H-(VS(K,2,JP1)+        CMA03180
     *      2.D0*VS(K,2,JM1)))/H                                        CMA03190
            DD = (2.D0*(VS(K,1,JM1)-VS(K,1,JP1))/H+(VS(K,2,JP1)+        CMA03200
     *      VS(K,2,JM1)))/H**2                                          CMA03210
            VS(K,1,J) = AA+U*(BB+U*(CC+DD*U))                           CMA03220
            VS(K,2,J) = BB+U*(2.D0*CC+3.D0*DD*U)                        CMA03230
   90    CONTINUE                                                       CMA03240
   95 CONTINUE                                                          CMA03250
      IER = 0                                                           CMA03260
      RETURN                                                            CMA03270
      END                                                               CMA03280
C   IMSL ROUTINE NAME   - UERTST                                        CMA03290
C                                                                       CMA03300
C-----------------------------------------------------------------------CMA03310
C                                                                       CMA03320
C   COMPUTER            - IBM/SINGLE                                    CMA03330
C                                                                       CMA03340
C   LATEST REVISION     - JUNE 1, 1982                                  CMA03350
C                                                                       CMA03360
```

```
C   PURPOSE             - PRINT A MESSAGE REFLECTING AN ERROR CONDITION CMA03370
C                                                                       CMA03380
C   USAGE               - CALL UERTST (IER,NAME)                        CMA03390
C                                                                       CMA03400
C   ARGUMENTS    IER    - ERROR PARAMETER. (INPUT)                      CMA03410
C                           IER = I+J WHERE                             CMA03420
C                             I = 128 IMPLIES TERMINAL ERROR MESSAGE,   CMA03430
C                             I =  64 IMPLIES WARNING WITH FIX MESSAGE, CMA03440
C                             I =  32 IMPLIES WARNING MESSAGE.          CMA03450
C                             J = ERROR CODE RELEVANT TO CALLING        CMA03460
C                                 ROUTINE.                              CMA03470
C                NAME   - A CHARACTER STRING OF LENGTH SIX PROVIDING    CMA03480
C                           THE NAME OF THE CALLING ROUTINE. (INPUT)    CMA03490
C                                                                       CMA03500
C   PRECISION/HARDWARE  - SINGLE/ALL                                    CMA03510
C                                                                       CMA03520
C   REQD. IMSL ROUTINES - UGETIO,USPKD                                  CMA03530
C                                                                       CMA03540
C   NOTATION            - INFORMATION ON SPECIAL NOTATION AND           CMA03550
C                           CONVENTIONS IS AVAILABLE IN THE MANUAL      CMA03560
C                           INTRODUCTION OR THROUGH IMSL ROUTINE UHELP  CMA03570
C                                                                       CMA03580
C   REMARKS      THE ERROR MESSAGE PRODUCED BY UERTST IS WRITTEN        CMA03590
C                TO THE STANDARD OUTPUT UNIT. THE OUTPUT UNIT           CMA03600
C                NUMBER CAN BE DETERMINED BY CALLING UGETIO AS          CMA03610
C                FOLLOWS..   CALL UGETIO(1,NIN,NOUT).                   CMA03620
C                THE OUTPUT UNIT NUMBER CAN BE CHANGED BY CALLING       CMA03630
C                UGETIO AS FOLLOWS..                                    CMA03640
C                                NIN = 0                                CMA03650
C                                NOUT = NEW OUTPUT UNIT NUMBER          CMA03660
C                                CALL UGETIO(3,NIN,NOUT)                CMA03670
C                SEE THE UGETIO DOCUMENT FOR MORE DETAILS.              CMA03680
C                                                                       CMA03690
C   COPYRIGHT           - 1982 BY IMSL, INC. ALL RIGHTS RESERVED.       CMA03700
C                                                                       CMA03710
C   WARRANTY            - IMSL WARRANTS ONLY THAT IMSL TESTING HAS BEEN CMA03720
C                           APPLIED TO THIS CODE. NO OTHER WARRANTY,    CMA03730
C                           EXPRESSED OR IMPLIED, IS APPLICABLE.        CMA03740
C                                                                       CMA03750
C-----------------------------------------------------------------------CMA03760
C                                                                       CMA03770
      SUBROUTINE UERTST (IER,NAME)                                      CMA03780
C                                  SPECIFICATIONS FOR ARGUMENTS         CMA03790
      INTEGER            IER                                            CMA03800
      INTEGER            NAME(1)                                        CMA03810
C                                  SPECIFICATIONS FOR LOCAL VARIABLES   CMA03820
      INTEGER            I,IEQ,IEQDF,IOUNIT,LEVEL,LEVOLD,NAMEQ(6),      CMA03830
     *                   NAMSET(6),NAMUPK(6),NIN,NMTB                   CMA03840
```

```
      DATA               NAMSET/1HU,1HE,1HR,1HS,1HE,1HT/
      DATA               NAMEQ/6*1H /
      DATA               LEVEL/4/,IEQDF/0/,IEQ/1H=/
C                                  UNPACK NAME INTO NAMUPK
C                                  FIRST EXECUTABLE STATEMENT
      CALL USPKD (NAME,6,NAMUPK,NMTB)
C                                  GET OUTPUT UNIT NUMBER
      CALL UGETIO(1,NIN,IOUNIT)
C                                  CHECK IER
      IF (IER.GT.999) GO TO 25
      IF (IER.LT.-32) GO TO 55
      IF (IER.LE.128) GO TO 5
      IF (LEVEL.LT.1) GO TO 30
C                                  PRINT TERMINAL MESSAGE
      IF (IEQDF.EQ.1) WRITE(IOUNIT,35) IER,NAMEQ,IEQ,NAMUPK
      IF (IEQDF.EQ.0) WRITE(IOUNIT,35) IER,NAMUPK
      GO TO 30
    5 IF (IER.LE.64) GO TO 10
      IF (LEVEL.LT.2) GO TO 30
C                                  PRINT WARNING WITH FIX MESSAGE
      IF (IEQDF.EQ.1) WRITE(IOUNIT,40) IER,NAMEQ,IEQ,NAMUPK
      IF (IEQDF.EQ.0) WRITE(IOUNIT,40) IER,NAMUPK
      GO TO 30
   10 IF (IER.LE.32) GO TO 15
C                                  PRINT WARNING MESSAGE
      IF (LEVEL.LT.3) GO TO 30
      IF (IEQDF.EQ.1) WRITE(IOUNIT,45) IER,NAMEQ,IEQ,NAMUPK
      IF (IEQDF.EQ.0) WRITE(IOUNIT,45) IER,NAMUPK
      GO TO 30
   15 CONTINUE
C                                  CHECK FOR UERSET CALL
      DO 20 I=1,6
         IF (NAMUPK(I).NE.NAMSET(I)) GO TO 25
   20 CONTINUE
      LEVOLD = LEVEL
      LEVEL = IER
      IER = LEVOLD
      IF (LEVEL.LT.0) LEVEL = 4
      IF (LEVEL.GT.4) LEVEL = 4
      GO TO 30
   25 CONTINUE
      IF (LEVEL.LT.4) GO TO 30
C                                  PRINT NON-DEFINED MESSAGE
      IF (IEQDF.EQ.1) WRITE(IOUNIT,50) IER,NAMEQ,IEQ,NAMUPK
      IF (IEQDF.EQ.0) WRITE(IOUNIT,50) IER,NAMUPK
   30 IEQDF = 0
      RETURN
   35 FORMAT(19H *** TERMINAL ERROR,10X,7H(IER = ,I3,
```

```
     1         20H) FROM IMSL ROUTINE ,6A1,A1,6A1)
  40 FORMAT(27H *** WARNING WITH FIX ERROR,2X,7H(IER = ,I3,
     1         20H) FROM IMSL ROUTINE ,6A1,A1,6A1)
  45 FORMAT(18H *** WARNING ERROR,11X,7H(IER = ,I3,
     1         20H) FROM IMSL ROUTINE ,6A1,A1,6A1)
  50 FORMAT(20H *** UNDEFINED ERROR,9X,7H(IER = ,I5,
     1         20H) FROM IMSL ROUTINE ,6A1,A1,6A1)
C
C                                  SAVE P FOR P = R CASE
C                                    P IS THE PAGE NAMUPK
C                                    R IS THE ROUTINE NAMUPK
  55 IEQDF = 1
     DO 60 I=1,6
  60 NAMEQ(I) = NAMUPK(I)
  65 RETURN
     END
C   IMSL ROUTINE NAME   - UGETIO
C
C-----------------------------------------------------------------------
C
C   COMPUTER            - IBM/SINGLE
C
C   LATEST REVISION     - JUNE 1, 1981
C
C   PURPOSE             - TO RETRIEVE CURRENT VALUES AND TO SET NEW
C                           VALUES FOR INPUT AND OUTPUT UNIT
C                           IDENTIFIERS.
C
C   USAGE               - CALL UGETIO(IOPT,NIN,NOUT)
C
C   ARGUMENTS    IOPT   - OPTION PARAMETER. (INPUT)
C                           IF IOPT=1, THE CURRENT INPUT AND OUTPUT
C                           UNIT IDENTIFIER VALUES ARE RETURNED IN NIN
C                           AND NOUT, RESPECTIVELY.
C                           IF IOPT=2, THE INTERNAL VALUE OF NIN IS
C                           RESET FOR SUBSEQUENT USE.
C                           IF IOPT=3, THE INTERNAL VALUE OF NOUT IS
C                           RESET FOR SUBSEQUENT USE.
C                NIN    - INPUT UNIT IDENTIFIER.
C                           OUTPUT IF IOPT=1, INPUT IF IOPT=2.
C                NOUT   - OUTPUT UNIT IDENTIFIER.
C                           OUTPUT IF IOPT=1, INPUT IF IOPT=3.
C
C   PRECISION/HARDWARE  - SINGLE/ALL
C
C   REQD. IMSL ROUTINES - NONE REQUIRED
C
C   NOTATION            - INFORMATION ON SPECIAL NOTATION AND
```

```
C                               CONVENTIONS IS AVAILABLE IN THE MANUAL
C                               INTRODUCTION OR THROUGH IMSL ROUTINE UHELP
C
C   REMARKS        EACH IMSL ROUTINE THAT PERFCRMS INPUT AND/OR OUTPUT
C                  CPERATIONS CALLS UGETIO TO CBTAIN THE CURRENT UNIT
C                  IDENTIFIER VALUES. IF UGETIC IS CALLED WITH IOPT=2 OR
C                  IOPT=3, NEW UNIT IDENTIFIER VALUES ARE ESTABLISHED.
C                  SUBSEQUENT INPUT/OUTPUT IS PERFORMEC ON THE NEW UNITS.
C
C   COPYRIGHT           - 1978 BY IMSL, INC. ALL RIGHTS RESERVED.
C
C   WARRANTY            - IMSL WARRANTS ONLY THAT IMSL TESTING HAS BEEN
C                           APPLIED TO THIS CODE. NO OTHER WARRANTY,
C                           EXPRESSED OR IMPLIED, IS APPLICABLE.
C
C-----------------------------------------------------------------------
C
      SUBROUTINE UGETIO(IOPT,NIN,NOUT)
C                                  SPECIFICATIONS FOR ARGUMENTS
      INTEGER            IOPT,NIN,NOUT
C                                  SPECIFICATIONS FOR LOCAL VARIABLES
      INTEGER            NIND,NOUTD
      DATA               NIND/5/,NOUTD/6/
C                                  FIRST EXECLTABLE STATEMENT
      IF (IOPT.EQ.3) GO TO 10
      IF (IOPT.EQ.2) GO TC 5
      IF (IOPT.NE.1) GO TO 9005
      NIN = NIND
      NOUT = NOUTD
      GO TO 9005
    5 NIND = NIN
      GO TO 9005
   10 NOUTD = NOUT
 9005 RETURN
      END
C   IMSL ROUTINE NAME   - DBCEVL
C
C-----------------------------------------------------------------------
C
C   COMPUTER            - IBM/DOUBLE
C
C   LATEST REVISICN     - JUNE 1, 1982
C
C   PURPOSE             - BICUBIC SPLINE MIXEC PARTIAL DERIVATIVE
C                           EVALUATOR
C
C   USAGE               - CALL DBCEVL (X,NX,Y,NY,C,IC,XL,YL,PDS,IER)
C
```

```
C   ARGUMENTS      X      - VECTOR OF LENGTH NX. (INPUT) X MUST BE        CMA05290
C                              ORDERED SO THAT X(I) .LT. X(I+1) FOR        CMA05300
C                              I=1,...,NX-1.                               CMA05310
C                  NX     - NUMBER OF ELEMENTS IN X. (INPUT) NX MUST BE   CMA05320
C                              .GE. 2.                                     CMA05330
C                  Y      - VECTOR OF LENGTH NY. (INPUT) Y MUST BE        CMA05340
C                              ORDERED SO THAT Y(J) .LT. Y(J+1) FOR        CMA05350
C                              J=1,...,NY-1.                               CMA05360
C                  NY     - NUMBER OF ELEMENTS IN Y. (INPUT) NY MUST BE   CMA05370
C                              .GE. 2.                                     CMA05380
C                           NOTE - THE COORDINATE PAIRS (X(I),Y(J)), FOR   CMA05390
C                              I=1,...,NX AND J=1,...,NY, GIVE THE POINTS  CMA05400
C                              WHERE THE FUNCTION VALUES ARE DEFINED.      CMA05410
C                  C      - ARRAY OF SPLINE COEFFICIENTS. (INPUT)         CMA05420
C                              C IS OF DIMENSION 2 BY NX BY 2 BY NY.       CMA05430
C                              THE SPLINE COEFFICIENTS CAN BE COMPUTED BY  CMA05440
C                              IMSL SUBROUTINE IBCCCU.                     CMA05450
C                              (NOTE - C IS TREATED INTERNALLY AS A        CMA05460
C                                2 BY NX BY 2*NY ARRAY BECAUSE CERTAIN     CMA05470
C                                ENVIRONMENTS DO NOT PERMIT QUADRUPLY-     CMA05480
C                                DIMENSIONED ARRAYS.  IN THESE             CMA05490
C                                ENVIRONMENTS THE CALLING PROGRAM MAY      CMA05500
C                                DIMENSION C IN THE SAME MANNER.)          CMA05510
C                  IC     - SECOND DIMENSION OF ARRAY C EXACTLY AS        CMA05520
C                              SPECIFIED IN THE DIMENSION STATEMENT        CMA05530
C                              (INPUT).  IC MUST BE .GE. NX.               CMA05540
C                  XL,YL  - (XL,YL) IS THE POINT AT WHICH THE MIXED       CMA05550
C                              PARTIAL DERIVATIVES OF THE SPLINE ARE TO BE CMA05560
C                              EVALUATED. (INPUT)                          CMA05570
C                  PDS    - VECTOR OF LENGTH 6 CONTAINING THE PARTIAL     CMA05580
C                              DERIVATIVES OF THE BICUBIC SPLINE, S(X,Y),  CMA05590
C                              EVALUATED AT X=XL AND Y=YL. (OUTPUT)        CMA05600
C                                PDS(1) = S(XL,YL)                         CMA05610
C                                PDS(2) = DS/DX                            CMA05620
C                                PDS(3) = DS/DY                            CMA05630
C                                PDS(4) = D(DS/DX)/DY                      CMA05640
C                                PDS(5) = D(DS/DX)/DX                      CMA05650
C                                PDS(6) = D(DS/DY)/DY.                     CMA05660
C                  IER    - ERROR PARAMETER. (OUTPUT)                     CMA05670
C                           WARNING ERROR                                  CMA05680
C                              IER = 33, XL IS LESS THAN X(1).             CMA05690
C                              IER = 34, YL IS LESS THAN Y(1).             CMA05700
C                              IER = 35, XL IS GREATER THAN X(NX).         CMA05710
C                              IER = 36, YL IS GREATER THAN Y(NY).         CMA05720
C                                                                          CMA05730
C   PRECISION/HARDWARE    - SINGLE AND DOUBLE/H32                         CMA05740
C                         - SINGLE/H36,H48,H60                            CMA05750
C                                                                          CMA05760
```

```
C   REQD. IMSL ROUTINES - UERTST,UGETIO                               CMA05770
C                                                                     CMA05780
C   NOTATION            - INFORMATION ON SPECIAL NOTATION AND         CMA05790
C                           CONVENTIONS IS AVAILABLE IN THE MANUAL    CMA05800
C                           INTRODUCTION OR THROUGH IMSL ROUTINE UHELP CMA05810
C                                                                     CMA05820
C   COPYRIGHT           - 1982 BY IMSL, INC. ALL RIGHTS RESERVED.     CMA05830
C                                                                     CMA05840
C   WARRANTY            - IMSL WARRANTS ONLY THAT IMSL TESTING HAS BEEN CMA05850
C                           APPLIED TO THIS CODE. NO OTHER WARRANTY,  CMA05860
C                           EXPRESSED OR IMPLIED, IS APPLICABLE.      CMA05870
C                                                                     CMA05880
C-----------------------------------------------------------------------CMA05890
C                                                                     CMA05900
      SUBROUTINE CBCEVL (X,NX,Y,NY,C,IC,XL,YL,PDS,IER)                CMA05910
C                                  SPECIFICATIONS FOR ARGUMENTS       CMA05920
      INTEGER            NX,NY,IC,IER                                 CMA05930
      DOUBLE PRECISION   X(1),Y(1),C(2,IC,1),XL,YL,PDS(6)             CMA05940
C                                  SPECIFICATIONS FOR LOCAL VARIABLES CMA05950
      INTEGER            I,J,K,KM1,KP1,KP2,LXPL,LX,LY,L,LXP1          CMA05960
      DOUBLE PRECISION   HX,HY,SUX(2),SUY(2),SU(2),SVX(2),SV(2),SXY(2), CMA05970
     *                   U,V,SPLNO,SPLN1,SPLN2,SO,SH,SPO,SPH,H,D      CMA05980
      SPLNO(SO,SH,SPO,SPH,H,D) = SO+D*(H*SPO+D*(3.DO*(SH-SO)-         CMA05990
     * (SPH+2.DO*SPO)*H+D*(2.DO*(SO-SH)+(SPH+SPO)*H)))                CMA06000
      SPLN1(SO,SH,SPO,SPH,H,D) = SPO+D*(6.DO*(SH-SO)/H-2.DO*          CMA06010
     * (SPH+2.DO*SPO)+3.DO*D*(2.DO*(SO-SH)/H+(SPH+SPO)))              CMA06020
      SPLN2(SO,SH,SPO,SPH,H,D) = 6.DO*(SH-SO)/H**2-2.DO*              CMA06030
     * (SPH+2.DO*SPO)/H+D*(2.DO*(SO-SH)/H**2+(SPH+SPO)/H)*6.DO        CMA06040
C                                  FIRST EXECUTABLE STATEMENT         CMA06050
      IER = 0                                                         CMA06060
      IF (XL.LT.X(1)) IER = 33                                        CMA06070
      DO 5 I=2,NX                                                     CMA06080
         LX = I-1                                                     CMA06090
         IF (XL.LE.X(I)) GO TO 10                                     CMA06100
    5 CONTINUE                                                        CMA06110
      IER = 35                                                        CMA06120
   10 IF (YL.LT.Y(1)) IER = 34                                        CMA06130
      DO 15 J=2,NY                                                    CMA06140
         LY = J-1                                                     CMA06150
         IF (YL.LE.Y(J)) GO TO 20                                     CMA06160
   15 CONTINUE                                                        CMA06170
      IER = 36                                                        CMA06180
   20 LXP1 = LX+1                                                     CMA06190
      HX = X(LXP1)-X(LX)                                              CMA06200
      HY = Y(LY+1)-Y(LY)                                              CMA06210
      U = (XL-X(LX))/HX                                               CMA06220
      V = (YL-Y(LY))/HY                                               CMA06230
      K = 2*LY                                                        CMA06240
```

```
      KP1 = K+1
      KP2 = K+2
      KM1 = K-1
      DO 25 L=1,2
         LXPL = LX-1+L
         I = 2*(LY-1+L)
         J = I-1
         SUX(L) = SPLN1(C(1,LX,J),C(1,LXF1,J),C(2,LX,J),
     *   C(2,LXP1,J),HX,U)
         SXY(L) = SPLN1(C(1,LX,I),C(1,LXF1,I),C(2,LX,I),
     *   C(2,LXP1,I),HX,U)
         SU(L) = SPLNO(C(1,LX,J),C(1,LXP1,J),C(2,LX,J),
     *   C(2,LXP1,J),HX,U)
         SUY(L) = SPLNO(C(1,LX,I),C(1,LXP1,I),C(2,LX,I),
     *   C(2,LXP1,I),HX,U)
         SV(L) = SPLNC(C(1,LXPL,KM1),C(1,LXPL,KP1),C(1,LXPL,K),
     *   C(1,LXFL,KP2),HY,V)
         SVX(L) = SPLNO(C(2,LXPL,KM1),C(2,LXPL,KP1),C(2,LXPL,K),
     *   C(2,LXFL,KP2),HY,V)
   25 CONTINUE
      PDS(1) = SPLNO(SV(1),SV(2),SVX(1),SVX(2),HX,U)
      PDS(2) = SPLN1(SV(1),SV(2),SVX(1),SVX(2),HX,U)
      PDS(3) = SPLN1(SU(1),SU(2),SUY(1),SUY(2),HY,V)
      PDS(4) = SPLN1(SUX(1),SUX(2),SXY(1),SXY(2),HY,V)
      PDS(5) = SPLN2(SV(1),SV(2),SVX(1),SVX(2),HX,U)
      PDS(6) = SPLN2(SU(1),SU(2),SUY(1),SUY(2),HY,V)
      IF (IER.GT.0) CALL UERTST(IER,6HDBCEVL)
      RETURN
      END
$ENTRY
```

```
$JOB
      REAL*8 X6(9),CNB(9),DCNB(9)
      INTEGER NDIM,I
      NDIM=9
      DO 1 I=1,NDIM
      X6(1)=-4.0
      X6(2)=0.0
      X6(3)=4.0
      X6(4)=6.2
      X6(5)=8.0
      X6(6)=10.0
      X6(7)=12.6
      X6(8)=16.0
      X6(9)=20.0
      CNB(1)=0.024
      CNB(2)=0.024
      CNB(3)=0.024
      CNB(4)=0.024
      CNB(5)=0.024
      CNB(6)=0.024
      CNB(7)=0.027
      CNB(8)=0.029
      CNB(9)=0.032
      CALL DDGT3(X6,CNB,DCNB,NDIM,IER)
1     CONTINUE
      DO 2 I=1,NDIM
      WRITE(8,25)X6(I),CNB(I),DCNB(I)
2     CONTINUE
      STOP
25    FORMAT(F12.4,3X,F12.4,3X,F12.4)
      END
C
C     ..................................................................
C
C        SUBROUTINE DDGT3
C
C        PURPOSE
C           TO COMPUTE A VECTOR OF DERIVATIVE VALUES GIVEN VECTORS OF
C           ARGUMENT VALUES AND CORRESPONDING FUNCTION VALUES.
C
C        USAGE
C           CALL DDGT3(X,Y,Z,NDIM,IER)
C
C        DESCRIPTION OF PARAMETERS
C           X      -  GIVEN VECTOR OF DOUBLE PRECISION ARGUMENT VALUES
C                     (DIMENSION NDIM)
C           Y      -  GIVEN VECTOR OF DOUBLE PRECISION FUNCTION VALUES
C                     CORRESPONDING TO X (DIMENSION NDIM)
```

```
C        Z      -   RESULTING VECTOR OF DOUBLE PRECISION DERIVATIVE
C                   VALUES (DIMENSION NDIM)
C        NDIM   -   DIMENSION OF VECTORS X,Y AND Z
C        IER    -   RESULTING ERROR PARAMETER
C                   IER  = -1  - NDIM IS LESS THAN 3
C                   IER  =  0  - NO ERROR
C                   IER POSITIVE  - X(IER) = X(IER-1) OR X(IER) =
C                                   X(IER-2)
C
C     REMARKS
C        (1)   IF IER = -1,2,3, THEN THERE IS NO COMPUTATION.
C        (2)   IF IER =  4,...,N, THEN THE DERIVATIVE VALUES Z(1)
C              ,..., Z(IER-1) HAVE BEEN COMPUTED.
C        (3)   Z CAN HAVE THE SAME STORAGE ALLOCATION AS X OR Y.  IF
C              X OR Y IS DISTINCT FROM Z, THEN IT IS NOT DESTROYED.
C
C     SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C        NONE
C
C     METHOD
C        EXCEPT AT THE ENDPOINTS X(1) AND X(NDIM), Z(I) IS THE
C        DERIVATIVE AT X(I) OF THE LAGRANGIAN INTERPOLATION
C        POLYNOMIAL OF DEGREE 2 RELEVANT TO THE 3 SUCCESSIVE POINTS
C        (X(I+K),Y(I+K)) K = -1,0,1. (SEE HILDEBRAND, F.B.,
C        INTRODUCTION TO NUMERICAL ANALYSIS, MC GRAW-HILL, NEW YORK/
C        TORONTO/LONDON, 1956, PP. 64-68.)
C
C     ..................................................................
C
      SUBROUTINE DDGT3(X,Y,Z,NDIM,IER)
C
C
      DIMENSION X(1),Y(1),Z(1)
      DOUBLE PRECISION X,Y,Z,DY1,DY2,DY3,A,B
C
C        TEST OF DIMENSION AND ERROR EXIT IN CASE NDIM IS LESS THAN 3
      IER=-1
      IF(NDIM-3)8,1,1
C
C        PREPARE DIFFERENTIATION LOOP
    1 A=X(1)
      B=Y(1)
      I=2
      DY2=X(2)-A
      IF(DY2)2,9,2
    2 DY2=(Y(2)-B)/DY2
C
C        START DIFFERENTIATION LOOP
```

```
      DO 6 I=3,NDIM                                                     CNB00970
      A=X(I)-A                                                          CNB00980
      IF(A)3,9,3                                                        CNB00990
    3 A=(Y(I)-B)/A                                                      CNB01000
      B=X(I)-X(I-1)                                                     CNB01010
      IF(B)4,9,4                                                        CNB01020
    4 DY1=DY2                                                           CNB01030
      DY2=(Y(I)-Y(I-1))/B                                               CNB01040
      DY3=A                                                             CNB01050
      A=X(I-1)                                                          CNB01060
      B=Y(I-1)                                                          CNB01070
      IF(I-3)5,5,6                                                      CNB01080
    5 Z(1)=DY1+DY3-DY2                                                  CNB01090
    6 Z(I-1)=DY1+DY2-DY3                                                CNB01100
C          END OF DIFFERENTIATION LOOP                                  CNB01110
C                                                                       CNB01120
C          NORMAL EXIT                                                  CNB01130
      IER=0                                                             CNB01140
      I=NDIM                                                            CNB01150
    7 Z(I)=DY2+DY3-DY1                                                  CNB01160
    8 RETURN                                                            CNB01170
C                                                                       CNB01180
C          ERROR EXIT IN CASE OF IDENTICAL ARGUMENTS                    CNB01190
    9 IER=I                                                             CNB01200
      I=I-1                                                             CNB01210
      IF(I-2)8,8,7                                                      CNB01220
      END                                                               CNB01230
$ENTRY                                                                  CNB01240
```

APPENDIX G

NONLINEAR AUTOPILOT _ CSMP PROGRAM

```
//ZURRA JOB (0314,1797),'THESIS',CLASS=J
// EXEC CSMPXV
//X.COMPRINT DD DUMMY
//X.SYSPRINT DD DUMMY
//X.FLOTPARM DD *
 &PLOT SCALE=.5 &END
//X.SYSIN DD *
*/*MAIN ORG=NPGVM1.1797P
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
*   NAME : TIAGO DA SILVA RIBEIRO                        WT 21      *
*                                                                   *
*   PARAMETER SENSITIVITY ANALYSIS OF A BANK TO TURN MISSILE        *
*                                                                   *
*   TITLE: NONLINEAR AUTOPILOT                                      *
*          ELLIPTICAL CASE                                          *
*                                                                   *
*                                                                   *
* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
*
*
PARAM IXX=110.,IYY=790.,IZZ=853.,W=2475.,...
      S=3.1416,D=2.,QB=1650.,KB=.48,K=4.17

PARAM CONV=57.3,C2=3.1416,C3=188.4,C4=150.,C5=6.,C6=.530544,C7=.48,...
         C8=.38375,C9=3.07,C10=5.,C11=.05033,C12=.755,C13=8.,...
         C14=17.6,C15=6.,C16=.078,C17=15.,C18=4.,C19=.913
PARAM T1=.25,K1=.839,K2=6.08,KYP=1.

*INCON ICX1=-1.,ICX2=-.0105,ICX3=.01148
*INCON ICX4=.658,ICX6=2.41,ICX7=.0636

C0=QB*S*D/IXX
C1=QB*S*D/IYY
C01=CB*S*D/IZZ
C02=QB*S/W


*
* FUNCTION GENERATION ....... ( 2-D )..................................
```

```
*
FUNCTION CLB=(-4.,.011),(0.0,0.0),(4.0,-.011),(5.2,-.0135),...
         (8.0,-.022),(10.0,-.027),(12.6,-.031),(16.0,-.031),(20.0,-.041)

FUNCTION CNB=(-4.,.024),(0.C,0.024),(4.0,.024),(5.2,.024),...
         (8.0,.024),(10.0,.024),(12.6,.027),(16.0,.029),(20.0,.032)

FUNCTION CYB=(-4.,-.049),(0.0,-.044),(4.0,-.048),(5.2,-.049),...
         (8.0,-.052),(10.0,-.055),(12.6,-.058),(16.0,-.061),(20.0,-.065)

FUNCTION CLDY=(-4.,.0025),(0.0,0.0),(4.0,-.002),(5.2,-.004),...
         (8.0,-.0075),(10.0,-.01),(12.6,-.014),(16.0,-.019),(20.0,-.024)

FUNCTION CNDY=(-4.,-.04),(0.0,-.04),(4.0,-.04),(5.2,-.04),...
         (8.0,-.04),(10.0,-.04),(12.6,-.04),(16.0,-.042),(20.0,-.0453)

FUNCTION CYDY=(-4.,.015),(0.0,0.015),(4.0,.015),(5.2,.015),...
         (8.0,.015),(10.0,.015),(12.6,.015),(16.0,.0173),(20.0,.0195)

FUNCTION CLDR=(-4.,.0232),(0.0,0.0232),(4.0,.0232),(5.2,.0232),...
         (8.0,.0232),(10.0,.0232),(12.6,.025),(16.0,.027),(20.0,.0298)

FUNCTION CNDR=(-4.,-.0045),(0.0,0.0),(4.0,.0045),(5.2,.0055),...
         (8.0,.011),(10.0,.0145),(12.6,.0195),(16.0,.025),(20.0,.032)

AN3=AFGEN(CLDR,Y6)
AN4=AFGEN(CNDY,Y6)
AN5=AFGEN(CNDR,Y6)
AN6=AFGEN(CYDY,Y6)
AN7=AFGEN(CLDY,Y6)
AN8=AFGEN(CLB,Y6)
AN9=AFGEN(CNB,Y6)
ANA=AFGEN(CYB,Y6)

A3=AFGEN(CLDR,X6)
A4=AFGEN(CNDY,X6)
A5=AFGEN(CNDR,X6)
A6=AFGEN(CYDY,X6)
A7=AFGEN(CLDY,X6)
A8=AFGEN(CLB,X6)
A9=AFGEN(CNB,X6)
AA=AFGEN(CYB,X6)

*
* GENERATION OF 2-D DERIVATIVES
*
FUNCTION DCLB=(-4.,-.0027),(.0,-.0027),(4.,-.0022),(5.2,-.0024),...
         (8.,-.0027),(10.,-.0021),(12.6,-.0015),(16.,-.0014),(20.,-.0011)
```

```
FUNCTION DCNB=(-4.,.0000),(.0,.0000),(4.,.0030),(5.2,.0000),...
        (8.,.0000),(10.,.0005),(12.6,.0009),(16.,.0007),(20.,.0008)
FUNCTION DCYB=(-4.,.0024),(.0,.0001),(4.,-.0009),(5.2,-.0009),...
        (8.,-.0013),(10.,-.0013),(12.6,-.0010),(16.,-.0009),(20.,-.0011)
FUNCTION DCLDY=(-4.,-.0007),(.0,-.0006),(4.,-.0014),(5.2,-.0015),...
        (8.,-.0013),(10.,-.0014),(12.6,-.0015),(16.,-.0014),(20.,-.0011)
FUNCTION DCNDY=(-4.,.0000),(.0,.0000),(4.,.0000),(5.2,.0000),...
        (8.,.0000),(10.,.0000),(12.6,-.0003),(16.,-.0007),(20.,-.0010)
FUNCTION DCYDY=(-4.,.0000),(.0,.0000),(4.,.0300),(5.2,.0000),...
        (8.,.0000),(10.,.0000),(12.6,.0003),(16.,.0006),(20.,.0006)
FUNCTION DCLDR=(-4.,.0000),(.0,.0000),(4.,.0000),(5.2,.0000),...
        (8.,.0000),(10.,.0003),(12.6,.0006),(16.,.0006),(20.,.0008)
FUNCTION DCNDR=(-4.,.0011),(.0,.0011),(4.,.0009),(5.2,.0012),...
        (8.,.0018),(10.,.0018),(12.6,.0018),(16.,.0017),(20.,.0018)
*
*
DAN3=AFGEN(DCLDR,Y6)
DAN4=AFGEN(DCNDY,Y6)
DAN5=AFGEN(DCNDR,Y6)
DAN6=AFGEN(DCYDY,Y6)
DAN7=AFGEN(DCLDY,Y6)
DAN8=AFGEN(DCLB,Y6)
DAN9=AFGEN(DCNB,Y6)
CANA=AFGEN(DCYB,Y6)


DA3=AFGEN(DCLDR,X6)
DA4=AFGEN(DCNDY,X6)
DA5=AFGEN(DCNDR,X6)
DA6=AFGEN(DCYDY,X6)
DA7=AFGEN(DCLDY,X6)
DA8=AFGEN(DCLB,X6)
DA9=AFGEN(DCNB,X6)
DAA=AFGEN(DCYB,X6)
*
* FUNCTION GENERATION........( 3-D )................................

FUNCTION CM,-10.0=(-4.0,0.485),(0.0,.55),(4.0,.625),(8.0,.71),...
        (12.0,.85),(16.0,.95),(20.0,1.0),(24.0,1.0)
FUNCTION CM,0.0=(-4.0,-.075),(0.0,0.0),(4.0,.075),(8.0,.15),...
        (12.0,.2),(16.0,.2),(20.0,.245),(24.0,.175)
FUNCTION CM,10.=(-4.0,-.63),(0.0,-.55),(4.0,-.48),(8.0,-.43),...
        (12.0,-.39),(16.0,-.40),(20.0,-.50),(24.0,-.73)

A1=TWOVAR(CM,X6,X4)
AN1=TWOVAR(CM,Y6,Y4)

FUNCTION CN,-10.0=(-4.0,-1.05),(0.0,-.25),(4.0,.6),(8.0,1.45),...
```

```
                (12.0,2.38),(16.0,3.3),(20.0,4.3),(24.0,5.4)
FUNCTION CN,0.0=(-4.0,-.8),(0.0,-.08),(4.0,.8),(8.0,1.7),...
                (12.0,2.6),(16.0,3.5),(20.0,4.55),(24.0,5.7)
FUNCTION CN,10.=(-4.0,-.6),(0.0,.2),(4.0,1.),(8.0,1.9),...
                (12.0,2.8),(16.0,3.8),(20.0,4.9),(24.0,6.)

A2=TWOVAR(CN,X6,X4)
AN2=TWOVAR(CN,Y6,Y4)

* GENERATION OF 3-D DERIVATIVES

FUNCTION CM6,-10.0=(-4.0,.0130),(0.0,.0185),(4.0,.0180),(8.0,.0294),...
                (12.0,.0331),(16.0,.0182),(20.0,.0065),(24.0,-.0068)
FUNCTION CM6,-5.0=(-4.0,-.0200),(0.0,.0131),(4.0,.0200),(8.0,.0232),...
                (12.0,.0184),(16.0,.0120),(20.0,.0065),(24.0,-.0290)
FUNCTION CM6,0.0=(-4.0,.0190),(0.0,.0186),(4.0,.0190),(8.0,.0178),...
                (12.0,.0034),(16.0,.0059),(20.0,.0067),(24.0,-.0516)
FUNCTION CM6,10.0=(-4.0,.0197),(0.0,.0195),(4.0,.0147),(8.0,.0118),...
                (12.0,.0057),(16.0,-.0122),(20.0,-.0395),(24.0,-.0772)
*
DAN16=TWOVAR(CM6,Y6,Y4)
DA16=TWOVAR(CM6,X6,X4)

FUNCTION CM4,-10.0=(-4.0,-.0159),(0.0,-.055),(4.0,-.0551),...
           (8.0,-.0563),(12.0,-.064),(16.0,-.0725),(20.0,-.0756),(24.0,-0841)
FUNCTION CM4,-5.0=(-4.0,-.061),(0.0,-.055),(4.0,-.055),(8.0,-.0558),...
                (12.0,-.0655),(16.0,-.0763),(20.0,-.0755),(24.0,-.0818)
FUNCTION CM4,0.0=(-4.0,-.0759),(0.0,-.055),(4.0,-.0551),(8.0,-.0563),...
                (12.0,-.0640),(16.0,-.0725),(20.0,-.0752),(24.0,-.0837)
FUNCTION CM4,10.0=(-4.0,-.0149),(.0,-.055),(4.0,-.0561),(8.0,-.0603),...
                (12.0,-.0520),(16.C,-.0425),(20.0,-.0736),(24.0,-.1001)

DAN14=TWOVAR(CM4,Y6,Y4)
DA14=TWOVAR(CM4,X6,X4)

FUNCTION CN6,-10.0=(-4.0,.1842),(0.0,.211),(4.0,.2092),(8.0,.2272),...
                (12.0,.2321),(16.0,.2319),(20.0,.2653),(24.0,.2819)
FUNCTION CN6,0.0=(-4.0,.1453),(0.0,.2074),(4.0,.2253),(8.0,.2265),...
                (12.0,.2188),(16.0,.2482),(20.0,.2884),(24.0,.2482)
FUNCTION CN6,5.0=(-4.0,.2496),(0.0,.1827),(4.0,.2196),(8.0,.2288),...
                (12.0,.2153),(16.0,.2599),(20.0,.2825),(24.0,.2599)
FUNCTION CN6,10.0=(-4.0,.2141),(0.0,.1925),(4.0,.2141),(8.0,.2256),...
                (12.0,.2335),(16.0,.2654),(20.0,.2798),(24.0,.2654)

DAN26=TWOVAR(CN6,Y6,Y4)
DA26=TWOVAR(CN6,X6,X4)
```

```
FUNCTION CN4,-10.0=(-4.0,.0308),(0.0,-.0332),(4.0,.02),(8.0,.0042),...
         (12.0,.02),(16.0,.0083),(20.0,.03),(24.0,.03)
FUNCTION CN4,0.0=(-4.0,.0208),(0.0,.0448),(4.0,.0200),(8.0,.0342),...
         (12.0,.0200),(16.0,.0283),(20.0,.0300),(24.0,.0300)
FUNCTION CN4,5.0=(-4.0,.0196),(0.0,.0336),(4.0,.0200),(8.0,.0342),...
         (12.0,.0200),(16.0,.0308),(20.0,.0300),(24.0,.0300)
FUNCTION CN4,10.0=(-4.0,.0208),(0.0,-.0112),(4.0,.02),(8.0,-.0058),...
         (12.0,.0200),(16.0,.0283),(20.0,.0300),(24.0,.0300)

DAN24=TWOVAR(CN4,Y6,Y4)
DA24=TWCVAR(CN4,X6,X4)

DEL1=.1*A1
DEL2=.1*A2
DEL3=.1*A3

AN11=1.1*AN1
AN21=1.1*AN2
AN31=1.1*AN3

* ACTUAL EQUATIONS
NZC=-2.0*STEP(2.0)+2.0*STEP(5.0)
NC=NZC-C19*COS(Y7)

*.................... PITCH EQUATIONS ..................................

* CONTROL LAW

Y1D=-C4*Y1-C02*C4*AN21

Y2D=-C5*Y2-C6*NZC+C6*C19*COS(Y7)+C7*Y1

Y3D=(C5*C8-C9)*Y2+C6*C8*NZC-C6*C8*C19*COS(Y7)-C7*C8*Y1+...
     (C8/CONV**2)*Y17*Y18+C1*C8*AN11+(C9/CONV)*Y5

Y4D=-C3*Y4+C3*CONV*Y3

* AERODYNAMIC

NZB=-C02*AN21

Y5D=Y17*Y18/CONV+C1*CONV*AN11

Y6D=Y5-KB*C02*AN21-Y16*Y18/CONV

Y7D=Y5*COS(Y19)/CCNV-Y17*SIN(Y19)/CONV

*..................ROLL-YAW EQUATIONS..................................
```

```
* CCNTROL LAW

NOSORT
IF(TIME.LE.5.0) GO TO 10
PHC=C2*STEP(5.0)
   IF (Y6.GT.1.0) GO TO 20
      YY6=1.0
      GO TO 30
20 CONTINUE
      YY6=Y6
30 CONTINUE

Y8D=-C10*Y8+(C12-C13*C11)*Y10+C11*C14*PHC-C11*C14*Y19-...
     C11*C0*(AN8*Y16+AN7*Y15+AN31*Y13)-C12/CONV*Y18

Y9D=-Y9/T1+K1/T1*CO2*(ANA*Y16+AN6*Y15)

Y10D=-C13*Y10+C14*PHC-C14*Y19

Y12D=-C17*Y12+K*(C17-C10/C18)*Y8+K/C18*(C12-C11*C13)*Y10+...
      K*C11*C14/C18*PHC-K*C11*C14/C18*Y19-K/C18*C0*AN8*...
      (C11+C16)*Y16-K/C18*C0*AN7*(C11+C16)*Y15-...
      K/C18*C0*AN31*(C11+C16)*Y13-K*C12/(C18*CONV)*Y18+...
      K*(C15/C18-C17)*Y11

Y13D=-C3*Y13+C3*CONV*Y12

Y14D=K2/10*(-Y9/T1+K1/T1*CO2*(ANA*Y16+AN6*Y15)-Y15*Y18/CONV**2+...
    CO1*(AN9*Y16+AN4*Y15+AN5*Y13)-KYP/CONV**2*(Y5-KB*CO2*AN21-...
    Y16*Y18/CONV)*Y18-KYP/CONV*YY6*C0*(AN8*Y16+AN7*Y15+AN31*Y13))...
    +K2/CONV*Y17+K2*Y9-K2*KYP/CONV**2*YY6*Y18

Y15D=-C3*Y15+C3*CONV*Y14

* AERODYNAMIC

NYB=CO2*(ANA*Y16+AN6*Y15)
Y16D=KB*CO2*(ANA*Y16+AN6*Y15)+Y6*Y18/CONV-Y17

Y17D=-Y5*Y18/CONV+CO1*CONV*(AN9*Y16+AN4*Y15+AN5*Y13)

Y18D=C0*CONV*(AN8*Y16+AN7*Y15+AN31*Y13)

Y19D=Y18/CONV

Y1=INTGRL(-1.0,Y1D)
Y2=INTGRL(-0.0105,Y2D)
```

```
Y3=INTGRL(0.01148,Y3D)
Y4=INTGRL(0.658,Y4D)
Y5=INTGRL(0.0,Y5D)
Y6=INTGRL(2.41,Y6D)
Y7=INTGRL(0.0636,Y7D)
Y8=INTGRL(0.0,Y8D)
Y9=INTGRL(0.0,Y9D)
Y10=INTGRL(0.0,Y10D)
Y11=INTGRL(0.0,Y11D)
Y12=INTGRL(0.0,Y12D)
Y13=INTGRL(0.0,Y13D)
Y14=INTGRL(0.0,Y14D)
Y15=INTGRL(0.0,Y15D)
Y16=INTGRL(0.0,Y16D)
Y17=INTGRL(0.0,Y17D)
Y18=INTGRL(0.0,Y18D)
Y19=INTGRL(0.0,Y19D)

10 CONTINUE

* X.......X1
* Y.......X2
* DPC.....X3
* DP......X4
* Q.......X5
* A.......X6
* THETA...X7
* Y1......X8
* Y2......X9
* X1......X10
* X2......X11
* DRC.....X12
* DR......X13
* DYC.....X14
* DY......X15
* B.......X16
* R.......X17
* P.......X18
* PHI.....X19

* NOMINAL AND SENSITIVITY EQUATIONS

*NZC=-2.0*STEP(2.0)+2.0*STEP(5.0)

*.................. PITCH EQUATIONS ................................

* CONTROL LAW
```

```
X1D=-C4*X1-C02*C4*A2
L11D=-C4*L11-C02*C4*(DA26*L61+DA24*L41)
L12D=-C4*L12-C02*C4*(DA26*L62+DA24*L42+1.)
L13D=-C4*L13-C02*C4*(DA26*L63+DA24*L43)

X2D=-C5*X2-C6*NZC+C6*C19*CCS(X7)+C7*X1
L21D=-C5*L21-C6*C19*SIN(X7)*L71+C7*L11
L22D=-C5*L22-C6*C19*SIN(X7)*L72+C7*L12
L23D=-C5*L23-C6*C19*SIN(X7)*L73+C7*L13

X3D=(C5*C8-C9)*X2+C6*C8*NZC-C6*C8*C19*COS(X7)-C7*C8*X1+...
     (C8/CONV**2)*X17*X18+C1*C8*A1+(C9/CONV)*X5
L31D=(C5*C8-C9)*L21+C6*C8*C19*SIN(X7)*L71-C7*C8*L11+C8*(X18*L171+...
     X17*L181)/(CCNV**2)+C1*C8*(DA16*L61+DA14*L41+1.)+C9*L51/CONV
L32D=(C5*C8-C9)*L22+C6*C8*C19*SIN(X7)*L72-C7*C8*L12+C8*(X18*L172+...
     X17*L182)/(CCNV**2)+C1*C8*(DA16*L62+DA14*L42)+C9*L52/CONV
L33D=(C5*C8-C9)*L23+C6*C8*C19*SIN(X7)*L73-C7*C8*L13+C8*(X18*L173+...
     X17*L183)/(CONV**2)+C1*C8*(DA16*L63+DA14*L43)+C9*L53/CONV

X4D=-C3*X4+C3*CONV*X3
L41D=-C3*L41+C3*CCNV*L31
L42D=-C3*L42+C3*CCNV*L32
L43D=-C3*L43+C3*CONV*L33

* AERODYNAMIC

NZB=-C02*A2

X5D=X17*X18/CONV+C1*CONV*A1
L51D=(X17*L181+X18*L171)/CCNV+C1*CONV*(DA16*L61+DA14*L41+1.)
L52D=(X17*L182+X18*L172)/CONV+C1*CONV*(DA16*L62+DA14*L42)
L53D=(X17*L183+X18*L173)/CONV+C1*CONV*(DA16*L63+DA14*L43)

X6D=X5-KB*C02*A2-X16*X18/CONV
L61D=L51-KB*C02*(DA26*L61+DA24*L41)-(X16*L181+X18*L161)/CONV
L62D=L52-KB*C02*(DA26*L62+DA24*L42+1.)-(X16*L182+X18*L162)/CONV
L63D=L53-KB*C02*(DA26*L63+DA24*L43)-(X16*L183+X18*L163)/CONV

X7D=X5*COS(X19)/CCNV-X17*SIN(X19)/CONV
L71D=(COS(X19)*L51-X5*SIN(X19)*L191)/CONV-(SIN(X19)*L171+...
     X17*COS(X19)*L171)/CONV
L72D=(COS(X19)*L52-X5*SIN(X19)*L192)/CONV-(SIN(X19)*L172+...
     X17*COS(X19)*L172)/CONV
L73D=(COS(X19)*L53-X5*SIN(X19)*L193)/CONV-(SIN(X19)*L173+...
     X17*COS(X19)*L173)/CONV

*..................ROLL-YAW EQUATIONS...............................
```

```
* CCNTROL LAW

NOSORT
IF(TIME.LE.5.0) GO TO 1
PHC1=C2*STEP(5.0)
   IF (X6.GT.1.0) GO TO 2
        XX6=1.0
        LL61=0.
        LL62=0.
        LL63=0.
        GO TO 3
 2 CONTINUE
        XX6=X6
        LL61=L61
        LL62=L62
        LL63=L63
 3 CONTINUE

X8D=-C10*X8+(C12-C13*C11)*X10+C11*C14*PHC1-C11*C14*X19-...
     C11*C0*(A8*X16+A7*X15+A3*X13)-C12/CONV*X18
L81D=-C10*L81+(C12-C11*C13)*L101-C11*C14*L191-C11*C0*(A8*L161+...
     A7*L151+A3*L131+L61*(X16*DA8+X15*DA7+X13*DA3))-...
     C12*L181/CONV
L82D=-C10*L82+(C12-C11*C13)*L102-C11*C14*L192-C11*C0*(A8*L162+...
     A7*L152+A3*L132+L62*(X16*DA8+X15*DA7+X13*DA3))-...
     C12*L182/CONV
L83D=-C10*L83+(C12-C11*C13)*L103-C11*C14*L193-C11*C0*(A8*L163+...
     A7*L153+A3*L133+L63*(X16*DA8+X15*DA7+X13*DA3)+X13)-...
     C12*L183/CONV

X9D=-X9/T1+K1/T1*C02*(AA*X16+A6*X15)
L91D=-L91/T1+K1/T1*C02*(AA*L161+A6*L151+L61*(X16*DAA+...
     X15*DA6))
L92D=-L92/T1+K1/T1*C02*(AA*L162+A6*L152+L62*(X16*DAA+...
     X15*DA6))
L93D=-L93/T1+K1/T1*C02*(AA*L163+A6*L153+L63*(X16*DAA+...
     X15*DA6))

X10D=-C13*X10+C14*PHC1-C14*X19
L101C=-C13*L101-C14*L191
L102D=-C13*L102-C14*L192
L103D=-C13*L103-C14*L193

X11D=-C15*X11+C16*C0*(A8*X16+A7*X15+A3*X13)
L111D=-C15*L111+C16*C0*(A8*L161+A7*L151+A3*L131+...
     L61*(X16*DA8+X15*DA7+X13*DA3))
L112D=-C15*L112+C16*C0*(A8*L162+A7*L152+A3*L132+...
     L62*(X16*DA8+X15*DA7+X13*DA3))
```

```
L113D=-C15*L113+C16*C0*(A8*L163+A7*L153+A3*L133+...
      L63*(X16*DA8+X15*DA7+X13*DA3)+X13)

X12D=-C17*X12+K*(C17-C10/C18)*X8+K/C18*(C12-C11*C13)*X10+...
       K*C11*C14/C18*PHC1-K*C11*C14/C18*X19-K/C18*C0*A8*...
       (C11+C16)*X16-K/C18*C0*A7*(C11+C16)*X15-...
       K/C18*C0*A3*(C11+C16)*X13-K*C12/(C18*CONV)*X18+...
       K*(C15/C18-C17)*X11
L121D=-C17*L121+K*(C17-C10/C18)*L81+K/C18*(C12-C11*C13)*L101-...
       K*C11*C14/C18*L191-K/C18*C0*(C11+C16)*(A8*L161+A7*L151+...
       A3*L131+L61*(X16*DA8+X15*DA7+X13*DA3))-...
       K*C12/(C18*CONV)*L181+K*(C15/C18-C17)*L111
L122D=-C17*L122+K*(C17-C10/C18)*L82+K/C18*(C12-C11*C13)*L102-...
       K*C11*C14/C18*L192-K/C18*C0*(C11+C16)*(A8*L162+A7*L152+...
       A3*L132+L62*(X16*DA8+X15*DA7+X13*DA3))-...
       K*C12/(C18*CONV)*L182+K*(C15/C18-C17)*L112
L123D=-C17*L123+K*(C17-C10/C18)*L83+K/C18*(C12-C11*C13)*L103-...
       K*C11*C14/C18*L193-K/C18*C0*(C11+C16)*(A8*L163+A7*L153+...
       A3*L133+L63*(X16*DA8+X15*DA7+X13*DA3)+X13)-...
       K*C12/(C18*CONV)*L183+K*(C15/C18-C17)*L113

X13D=-C3*X13+C3*CCNV*X12
L131D=-C3*L131+C3*CONV*L121
L132D=-C3*L132+C3*CONV*L122
L133D=-C3*L133+C3*CONV*L123

X14D=K2/10*(-X9/T1+K1/T1*C02*(AA*X16+A6*X15)-X15*X18/CONV**2+...
     C01*(A9*X16+A4*X15+A5*X13)-KYP/CONV**2*(X5-KB*C02*A2-...
     X16*X18/CONV)*X18-KYP/CONV*XX6*C0*(A8*X16+A7*X15+A3*X13))...
     +K2/CONV*X17+K2*X9-K2*KYP/CJNV**2*XX6*X18
L141D=K2/10*(-L91/T1+K1/T1*C02*(AA*L161+A6*L151+L61*(X16*DAA+X15*...
     DA6))-(X15*L181+X18*L151)/(CONV**2)+C01*(A9*L161+A4*L151+A5*L131+...
     L61*(X16*DA9+X15*DA4+X13*DA5))-KYP/(CONV**2)*(X5*L181+X18*L51-KB*...
     C02*(A2*L181+X18*(DA26*L61+DA24*L41)-(X18**2*L161+2*X18*X16*L181)...
     /CONV))-KYP/CONV*C0*(XX6*(A8*L161+A7*L151+A3*L131+L61*(X16*DA8+...
     X15*DA7+X13*DA3))+A8*(XX6*L161+X16*LL61)+A7*(XX6*L151+X15*LL61)+...
     A3*(XX6*L131+X13*LL61)+X16*(XX6*DA8*L61+A8*LL61)+X15*(XX6*DA7*L61...
     +A7*LL61)+X13*(XX6*DA3*L61+A3*LL61)))+K2/CONV*L171+K2*L91-...
     K2*KYP/(CONV**2)*(XX6*L181+X18*LL61)
L142D=K2/10*(-L92/T1+K1/T1*C02*(AA*L162+A6*L152+L62*(X16*DAA+X15*...
    DA6))-(X15*L182+X18*L152)/(CONV**2)+C01*(A9*L162+A4*L152+A5*L132+...
    L62*(X16*DA9+X15*DA4+X13*DA5))-KYP/(CONV**2)*(X5*L182+X18*L52-KB*...
    C02*(A2*L182+X18*(DA26*L62+DA24*L42)-(X18**2*L162+2*X18*X16*L182)...
    /CONV)+1.)-KYP/CONV*C0*(XX6*(A8*L162+A7*L152+A3*L132+L62*(X16*DA8+...
    X15*DA7+X13*DA3))+A8*(XX6*L162+X16*LL62)+A7*(XX6*L152+X15*LL62)+...
    A3*(XX6*L132+X13*LL62)+X16*(XX6*DA8*L62+A8*LL62)+X15*(XX6*DA7*L62...
    +A7*LL62)+X13*(XX6*DA3*L62+A3*LL62)))+K2/CONV*L172+K2*L92-...
    K2*KYP/(CONV**2)*(XX6*L182+X18*LL62)
```

```
L143D=K2/10*(-L93/T1+K1/T1*C02*(AA*L163+A6*L153+L63*(X16*DAA+X15*...
 DA6))-(X15*L183+X18*L153)/(CONV**2)+C01*(A9*L163+A4*L153+A5*L133+...
 L63*(X16*DA9+X15*DA4+X13*DA5))-KYP/(CONV**2)*(X5*L183+X18*L53-KB*...
 C02*(A2*L183+X18*(DA26*L63+DA24*L43)-(X18**2*L163+2*X18*X16*L183)...
 /CONV))-KYP/CONV*C0*(XX6*(A8*L163+A7*L153+A3*L133+L63*(X16*DA8+...
 X15*DA7+X13*DA3)+X13)+A8*(XX6*L163+X16*LL63)+A7*(XX6*L153+X15*LL63)+...
 A3*(XX6*L133+X13*LL63)+X16*(XX6*DA8*L63+A8*LL63)+X15*(XX6*DA7*L63...
 +A7*LL63)+X13*(XX6*DA3*L63+A3*LL63)))+K2/CONV*L173+K2*L93-...
 K2*KYP/(CONV**2)*(XX6*L183+X18*LL63)

X15D=-C3*X15+C3*CONV*X14
L151D=-C3*L151+C3*CONV*L141
L152D=-C3*L152+C3*CONV*L142
L153D=-C3*L153+C3*CONV*L143

* AERODYNAMIC

NYB=C02*(AA*X16+A6*X15)
X16D=KB*C02*(AA*X16+A6*X15)+X6*X18/CONV-X17
L161D=KB*C02*(AA*L161+A6*L151+L61*(X16*DAA+X15*DA6))+(X6*...
      L181+X18*L61)/CONV-L171
L162D=KB*C02*(AA*L162+A6*L152+L62*(X16*DAA+X15*DA6))+(X6*...
      L182+X18*L62)/CONV-L172
L163D=KB*C02*(AA*L163+A6*L153+L63*(X16*DAA+X15*DA6))+(X6*...
      L183+X18*L63)/CONV-L173

X17D=-X5*X18/CONV+C01*CONV*(A9*X16+A4*X15+A5*X13)

L171D=-(X5*L181+X18*L51)/CONV+C01*CONV*(A9*L161+A4*L151+A5*...
      L131+L61*(X16*DA9+X15*DA4+X13*DA5))
L172D=-(X5*L182+X18*L52)/CONV+C01*CONV*(A9*L162+A4*L152+A5*...
      L132+L62*(X16*DA9+X15*DA4+X13*DA5))
L173D=-(X5*L183+X18*L53)/CONV+C01*CONV*(A9*L163+A4*L153+A5*...
      L133+L63*(X16*DA9+X15*DA4+X13*DA5))
X18D=C0*CONV*(A8*X16+A7*X15+A3*X13)
L181D=C0*CONV*(A8*L161+A7*L151+A3*L131+L61*(X16*DA8+...
      X15*DA7+X13*DA3))
L182D=C0*CONV*(A8*L162+A7*L152+A3*L132+L62*(X16*DA8+...
      X15*DA7+X13*DA3))
L183D=C0*CONV*(A8*L163+A7*L153+A3*L133+L63*(X16*DA8+...
      X15*DA7+X13*DA3)+X13)

X19D=X18/CONV
L191D=L181/CONV
L192D=L182/CONV
L193D=L183/CONV

X8=INTGRL(0.0,X8D)
```

```
X9=INTGRL(0.0,X9D)
X10=INTGRL(0.0,X10D)
X11=INTGRL(0.0,X11D)
X12=INTGRL(0.0,X12D)
X13=INTGRL(0.0,X13D)
X14=INTGRL(0.0,X14D)
X15=INTGRL(0.0,X15D)
X16=INTGRL(0.0,X16D)
X17=INTGRL(0.0,X17D)
X18=INTGRL(0.0,X18D)
X19=INTGRL(0.0,X19D)

L81=INTGRL(0.0,L81D)
L91=INTGRL(0.0,L91D)
L101=INTGRL(0.0,L101D)
L111=INTGRL(0.0,L111D)
L121=INTGRL(0.0,L121D)
L131=INTGRL(0.0,L131D)
L141=INTGRL(0.0,L141D)
L151=INTGRL(0.0,L151D)
L161=INTGRL(0.0,L161D)
L171=INTGRL(0.0,L171D)
L181=INTGRL(0.0,L181D)
L191=INTGRL(0.0,L191D)

L82=INTGRL(0.0,L82D)
L92=INTGRL(0.0,L92D)
L102=INTGRL(0.0,L102D)
L112=INTGRL(0.0,L112D)
L122=INTGRL(0.0,L122D)
L132=INTGRL(0.0,L132D)
L142=INTGRL(0.0,L142D)
L152=INTGRL(0.0,L152D)
L162=INTGRL(0.0,L162D)
L172=INTGRL(0.0,L172D)
L182=INTGRL(0.0,L182D)
L192=INTGRL(0.0,L192D)

L83=INTGRL(0.0,L83D)
L93=INTGRL(0.0,L93D)
L103=INTGRL(0.0,L103D)
L113=INTGRL(0.0,L113D)
L123=INTGRL(0.0,L123D)
L133=INTGRL(0.0,L133D)
L143=INTGRL(0.0,L143D)
L153=INTGRL(0.0,L153D)
L163=INTGRL(0.0,L163D)
L173=INTGRL(0.0,L173D)
```

```
L183=INTGRL(0.0,L183D)
L193=INTGRL(0.0,L193D)

DX8=DEL1*L81+DEL2*L82+DEL3*L83
DX9=DEL1*L91+DEL2*L92+DEL3*L93
DX10=DEL1*L101+DEL2*L102+DEL3*L103
DX11=DEL1*L111+DEL2*L112+DEL3*L113
DX12=DEL1*L121+DEL2*L122+DEL3*L123
DX13=DEL1*L131+DEL2*L132+DEL3*L133
DX14=DEL1*L141+DEL2*L142+DEL3*L143
DX15=DEL1*L151+DEL2*L152+DEL3*L153
DX16=DEL1*L161+DEL2*L162+DEL3*L163
DX17=DEL1*L171+DEL2*L172+DEL3*L173
DX18=DEL1*L181+DEL2*L182+DEL3*L183
DX19=DEL1*L191+DEL2*L192+DEL3*L193

Y8S=X8+DX8
Y9S=X9+DX9
Y10S=X10+DX10
Y11S=X11+DX11
Y12S=X12+DX12
Y13S=X13+DX13
Y14S=X14+DX14
Y15S=X15+DX15
Y16S=X16+DX16
Y17S=X17+DX17
Y18S=X18+DX18
Y19S=X19+DX19

1      CONTINUE
SORT
X1=INTGRL(-1.0,X1D)
X2=INTGRL(-0.0105,X2D)
X3=INTGRL(0.01148,X3D)
X4=INTGRL(0.658,X4D)
X5=INTGRL(0.0,X5D)
X6=INTGRL(2.41,X6D)
X7=INTGRL(0.0636,X7D)

L11=INTGRL(0.0,L11D)
L21=INTGRL(0.0,L21D)
L31=INTGRL(0.0,L31D)
L41=INTGRL(0.0,L41D)
L51=INTGRL(0.0,L51D)
L61=INTGRL(0.0,L61D)
L71=INTGRL(0.0,L71D)
```

```
L12=INTGRL(0.0,L12D)
L22=INTGRL(0.0,L22D)
L32=INTGRL(0.0,L32D)
L42=INTGRL(0.0,L42D)
L52=INTGRL(0.0,L52D)
L62=INTGRL(0.0,L62D)
L72=INTGRL(0.0,L72D)

L13=INTGRL(0.0,L13D)
L23=INTGRL(0.0,L23D)
L33=INTGRL(0.0,L33D)
L43=INTGRL(0.0,L43D)
L53=INTGRL(0.0,L53D)
L63=INTGRL(0.0,L63D)
L73=INTGRL(0.0,L73D)

DX1=DEL1*L11+DEL2*L12+DEL3*L13
DX2=DEL1*L21+DEL2*L22+DEL3*L23
DX3=DEL1*L31+DEL2*L32+DEL3*L33
DX4=DEL1*L41+DEL2*L42+DEL3*L43
DX5=DEL1*L51+DEL2*L52+DEL3*L53
DX6=DEL1*L61+DEL2*L62+DEL3*L63
DX7=DEL1*L71+DEL2*L72+DEL3*L73

Y1S=X1+DX1
Y2S=X2+DX2
Y3S=X3+DX3
Y4S=X4+DX4
Y5S=X5+DX5
Y6S=X6+DX6
Y7S=X7+DX7

NZ=NZB*COS(X19)+NYB*SIN(X19)
TIMER FINTIM=10.0,OUTDEL=.05,DELMIN=.5E-8
METHOD RKSFX
*OUTPUT TIME,Y3(-0.04,0.20),Y3S(-0.04,0.20)
*LABEL
*PAGE XYPLOT
*OUTPUT TIME,Y4(-1.5,10.5),Y4S(-1.5,10.5)
*LABEL
*PAGE XYPLOT
*OUTPUT TIME,Y5(-16.,16.),Y5S(-16.,16.)
*LABEL
*PAGE XYPLOT
*OUTPUT TIME,Y6(0.0,8.0),Y6S(0.0,8.0)
*LABEL
*PAGE XYPLOT
*OUTPUT TIME,Y12(-.06,.3),Y12S(-.06,.3)
```

```
*LABEL
*PAGE XYPLOT
OUTPUT TIME,Y15(-7.5,6.C),Y15S(-7.5,6.0)
LABEL
PAGE XYPLOT
OUTPUT TIME,Y16(-1.5,0.9),Y16S(-1.5,0.9)
LABEL
PAGE XYPLOT
END
STOP
ENDJOB
/*
```

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